Annals of the Academy of Romanian Scientists
Series on Physics and Chemistry
Online ISSN **2559 – 1061** Volume **11**, Number 1/**2026**

LOW-LYING STATES OF ^{67}Ge AND ^{67}As NUCLEI IN PARTICLE – QUADRUPOLE - PHONON MODEL (PTQM) (REVIEW)

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This scientific contribution, with some additions, is a review of our article published in Romanian Reports in Physics, Volume 55, Issue 4, P. 452-459, 2003.

Abstract. *The energy levels and electromagnetic properties in ^{67}Ge and ^{67}As nuclei on the basis of the SU(6) particle – quadrupole – phonon model (PTQM) were described. A rather good description of the low-lying negative and positive parity states was obtained using the parametrization in vibrational SU(5) limit of the ^{66}Ge core.*

Keywords: energy levels, electromagnetic properties, negative and positive parity states.

DOI : 10.56082/annalsarsciphyschem.2026.1.55

1. Introduction

A few years ago, the region of nuclei with N~Z and A~70 has been extensive theoretical and experimental studied. Since these nuclei are in the interesting transitional region, these nuclei exhibit a range of features, including oblate and prolate deformations as well as rapid variations in shape as a function of both spin and particle number. In the last time we have studied the duble-odd ^{68}As nucleus [1] via the ^{54}Fe(^{16}O,pn)^{68}As reaction. Nuclear excited states have been assigned from investigations of ɤ-rays using in-beam-spectroscopic techniques. For a theoretical interpretation of this nucleus it is essential to know the experimental and theoretical nuclear structure of the neighboring odd ^{67}Ge and ^{67}As nuclei. The even-odd ^{67}Ge and odd-even ^{67}As were investigated in-beam ɤ -ray spectroscopy [2, 3]. In the present work we tried to describe the energy levels and electromagnetic properties in ^{67}Ge and ^{67}As on the basis of the SU(6) particle quadrupole- phonon model (PTQM) [4] by coupling of the odd neutron and odd proton respectively in the $2p_{1/2}$, $2p_{3/2}$, $1f_{5/2}$ and $1g_{9/2}$ orbitals with the anharmonic quadrupole vibrational core, $^{66}Ge_{34}$. The calculations were performed with the computer code PTQM [5].

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2. Levels scheme of ^{66}GE in the TQM

The experimental levels scheme of ^{66}Ge nucleus is well known. Fig. 1. [6] Firstly we calculated the excited states for the even-even core ^{66}Ge nucleus, in the truncated quadrupole phonon model (TQM) [4], equivalent at the phenomenological level, with the interacting boson model (IBM) [7] which describes the collective states in terms of $J=0^+$ (s) and 2^+ (d) bosons. The anharmonicity parameters of the core, (h_i) have been fitted to the experimental energy spectrum in the rotational SU(3), vibrational SU(5) and O(6) limits, respectively. (Table 1).

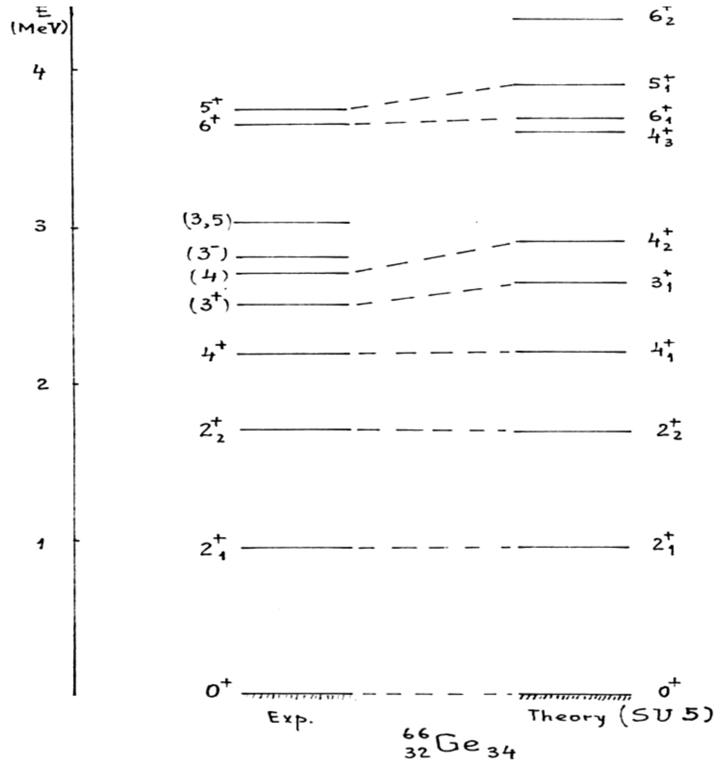

Fig. 1. Levels scheme of ^{66}Ge nucleus

In the interpretation of ^{66}Ge the total boson number N=5 has been included. This is equal to half the number of valance-shell proton particles and neutron particles in the 28 proton and neutron shell.

The TQM theoretical level spectrum is shown in fig.1 in the comparation with experimental results [6]. We remark that the level scheme of the even-even ^{66}Ge

(Z=32, N=34) nucleus is to be best explained in the vibrational SU(5) limit, with the fitted anharmonicity parameters to the experimental spectrum. (Table 1).

Table 1 - TQM parameters for the 66Ge core (Z=32, N=34) nucleus.

	The calculated parameters			The fitted parameters to the experimental spectrum		
	SU(3)	O(6)	SU(5)	SU(3)	O(6)	SU(5)
h_1	0.957	-0.181	0.957	0.957	-0.181	0.957
h_2	-0.141	-0.224	0	-0.141	-0.224	0
h_3	-0.379	0	0	-0.379	0	0
h_{40}	0.410	-0.020	-0.134	-0.010	-0.015	-0.098
h_{42}	-0.142	0.221	-0.273	-0.142	0.120	-0.240
h_{44}	1.167	1.123	0.344	0.167	1.020	0.334

3. PTQM calculations

Calculations for the levels scheme of the even-odd ^{67}Ge (Z=32, N=35) and odd-even ^{67}As (Z=33, N=34) nuclei, were performed in the SU(6) particle-quadrupole-phonon model (PTQM) [4] which is equivalent with the interaction boson-fermion model (IBFM) [7]. The IBFM introduces an odd particle in addition to boson. The Hamiltonian of the PTQM employed here is in the form: $H_{PTQM} = H_P + H_{TQM} + H_{PVI}$, where H_P is the spherical single-particle (or quasi-particle) hamiltonian; H_{TQM} denotes the TQM hamiltonian for the quadrupole vibrational even-even core nucleus [7] and H_{PVI} denotes the interaction between the odd quasiparticle and the quadrupole vibrations [7]. This interaction consists of three terms:
$H_{PVI} = H_{MON} + H_{DYN} + H_{EXC}$, where H_{MON} is the hamiltonian of the monopole interaction with the Γ_0 strength and H_{EXC} denotes the hamiltonian of the exchange interaction with the Λ_0 strength. Another parameter of the PTQM is χ = -1.323 [7].
The PTQM hamiltonian was diagonalized in the particle-phonon basis lj, n``I, J> where the quasiparticle l j> and the n-phonon state l n``I> are coupled to the total angular momentum J. Here is an additional quantum number to distinguish, if needed, between different states of the same n and I.
In the second step we calculated the negative and positive parity states of the even- odd ^{67}Ge nucleus was performed in the PTQM representation of IBFM by coupling of the odd neutron in the $p_{1/2}$, $p_{3/2}$, $f_{5/2}$ and $g_{9/2}$ orbital with the vibrational even-even core ^{66}Ge. For the calculation of negative-parity states following parameterization was used:

- the core ^{66}Ge excitations have been approximated with anharmonic vibration in the SU(5) limit (Table 1); The energy of the first 21^{+} states in ^{66}Ge was used as an effective phonon energy (hw2);

- the quasineutron energies (E, in MeV) and the occupation probabilities (ν^2) were taken from BCS calculations: E(ν p3/2)–E(ν p1/2)= 0.11; E(ν f5/2)– E(ν p 1/2)=0.20; v^2 (ν p 1/2) = 0.12; v^2 (ν p 3/2) = 0.82; v^2 (ν f5/2) = 0.71.

- the boson-neutron interaction parameters A^{ν}_0 = 0.1, $\Gamma_0{}^{\nu}$= 0.4, $\Lambda^{\nu}{}_0$ = 5.7 (all in MeV and N=5) have been adjusted to the experimental negative – parity states of ^{67}Ge.

For the positive-parity states in ^{67}Ge the occupation probability ν (νg $_{9/2}$) = 0.10 is obtained from BCS model, the other parameters are the same as in the calculation of negative parity states.

In the third step we have undertaken the IBFM/PTQM investigations for odd-even ^{67}As, by coupling of the odd proton in the p 1/2, p 3/2, f 5/2 and g 9/2 orbital with vibrational even-even ^{66}Ge nucleus in the SU(5) limit. (Table 1). For the interpretation of negative-parity states in ^{67}As the following parameterization is used:

- the same boson core, ^{66}Ge, as in the calculation for ^{67}Ge;
- the BCS quasiproton energies (E, in MeV) and the occupation probabilities, (v^2)::
E(πp $_{3/2}$) – E(πp $_{1/2}$) = 0.037; E(πf $_{5/2}$) – E(πp $_{1/2}$) = 0.081; v2 (πp $_{1/2}$) = 0.100; v^2 (πp $_{3/2}$) = 0.690; v^2 (πf $_{5/2}$) = 0.250.
- the bosons-proton interactions parameters are the same as in the calculations for ^{69}As [8] with a slight modification in order to obtain better agreement with experimental data:
$A_0{}^{\pi}$ = 0.07, $\Gamma_0{}^{\pi}$= 0.4, $\Lambda_0{}^{\pi}$ =6.9 and χ= -1.323.
The positive-parity state in ^{67}As was fitted to the experimental levels using the occupation probability v^2 (g $_{9/2}$) = 0.09 as obtained from BCS model and the other parameters are the same as in the calculations of the negative-parity states.

4. Results and discussion

The calculated negative and positive parity spectra and the experimental energy spectrum of ^{67}Ge and ^{67}As are shown in Figs. 2 and 3. We can see that it is a relatively good agreement between the theoretical level schemes of ^{67}Ge and ^{67}As nuclei with the available experimental data. In Table 2 the IBFM/PTQM the wave functions amplitudes (ξ, nνI) of the lowest-lying calculated states are presented for both ^{67}Ge, ^{67}As nuclei, where the wave functions are expressed in the boson fermion coupled basis $|J\rangle = \xi_{jn\nu I}$ j, nI; J> (only squares amplitudes higher than 2% are listed).

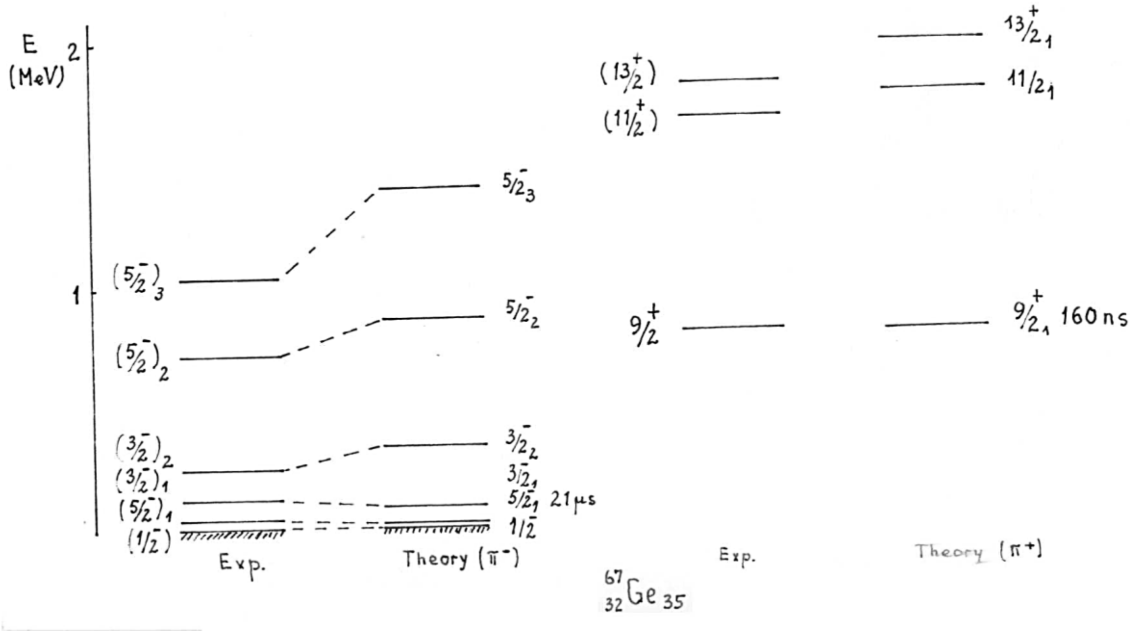

Fig. 2. Levels scheme of ^{67}Ge nucleus

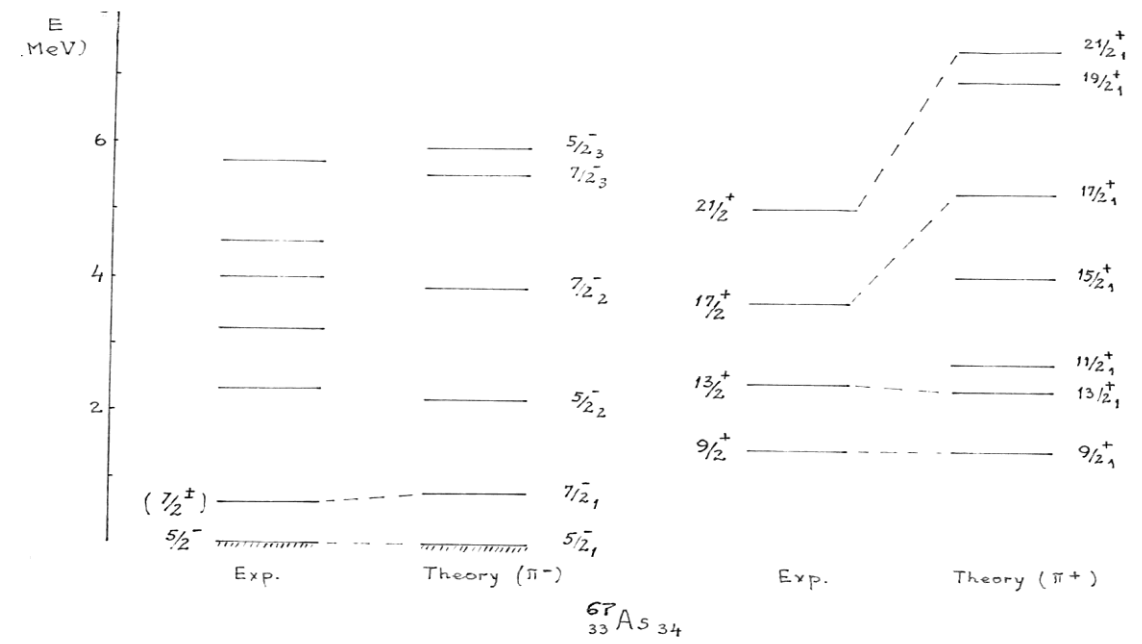

Fig. 3. Levels scheme of ^{67}As nucleus

Table 2. PTQM wave functions of the states in ^{67}Ge in the quadrupole-phonon model.

J	j	n	I	$(\xi_{nv})^2$	J	j	n	I	$(\xi_{nv})^2$
$1/2^-$	1/2	0	0	0.360	$5/2^-$	3/2	4	2	0.048
	1/2	2	0	0.340		3/2	4	4	0.321
	1/2	4	0	0.090		5/2	4	0	0.088
	3/2	2	4	0.030		5/2	4	2	0.018
	5/2	2	0	0.020		5/2	4	4	0.143
$3/2^-_1$	3/2	2	0	0.063	$9/2^+$	9/2	0	0	0.498
	3/2	2	2	0.210		9/2	1	2	0.109
	5/2	2	2	0.079		9/2	2	2	0.240
	5/2	2	4	0.640					
$5/2^-_1$	3/2	2	2	0.058	$11/2^+$	9/2	1	2	0.322
	3/2	2	4	0.382		9/2	2	2	0.303
	5/2	2	0	0.061		9/2	3	2	0.118
	5/2	2	2	0.232		9/2	4	2	0.094
	1/2	2	2	0.029					
$5/2^-$	3/2	4	2	0.048	$13/2^+$	9/2	1	2	0.041
	3/2	4	4	0.321		9/2	2	2	0.803
	5/2	4	0	0.088		9/2	2	4	0.087
	5/2	4	2	0.018		9/2	3	2	0.058
$5/2^-$	½	2	2	0.03	$5/2^-$	3/2	4	2	0.051
	3/2	2	4	0.591		3/2	4	4	0.294
	5/2	2	0	0.089		5/2	2	2	0.440
	5/2	2	2	0.347		5/2	4	0	0.069
	5/2	2	4	0.029		5/2	4	2	0.022
						5/2	4	4	0.161
$7/2^-$	3/2	2	4	0.048	$11/2^+$	9/2	2	2	0.843
	3/2	3	4	0.492		9/2	2	4	0.122
	5/2	2	4	0.018		9/2	3	3	0.029
	5/2	3	3	0.057					
	5/2	3	4	0.181					
	1/2	3	3	0.029					
$5/2^-$	3/2	4	2	0.051	$13/2^+$	9/2	2	2	0.881
	3/2	4	4	0.294		9/2	2	4	0.087
	5/2	2	2	0.440		9/2	3	2	0.022
	5/2	4	0	0.069					

Employing these wave functions we have calculated the electromagnetic properties of the low-lying states. For the ^{67}Ge nucleus the following effective charges and gyromagnetic ratios were used: $e^{v}= 0.5$; $e^{vib}= 0.9$; $g_R =0.477$;$g_e=g_e^{free}=0$; g_s

$=0.7g_s^{free}= -2.68$ and for ^{67}As nucleus: $e^{\pi} =1$; $e^{vib} = 1$;ge=1g_R =Z/2 A=0.240; $g_s =0.5g_s^{free} = 2.78$.

The PTQM electromagnetic properties, B(E2) and B(M1) value, and branching ratios for the transitions between the low lying negative parity states and positive parity states were calculated and compared with the experimental data from [1] (Tables 3 a,b)
From the data given in [1] results that the experimental electromagnetic intensities for the low-lying states are statistic relatively well reproduced by the calculated values.
Concluding, the PTQM calculations give a rather good description of the low lying negative and positive parity states of ^{67}Ge and ^{67}As for parameterization in vibrational SU(5) limit of the ^{66}Ge core.

Acknowledgments

1.Thanks to Prof. Emilian Drăgulescu, from the National Institute of Physics and Nuclear Engineering (IFIN-HH) Măgurele, Romania, who allowed the use of the PTQM calculation program.

2. In memory of our collegue, Apolodor Răduță, from Academy of Romanian Science, Bucharest.

3. In memory of my colleague Teodor Bădică with whom I collaborated for about 40 years at IFIN-HH Măgurele.

R E F E R E N C E S

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