Collective features of dipolar response in nuclear systems within schematic and transport models

Virgil Baran[1,2], Cosmin Cipu[1], and Darius-Matei Banu[1]

Abstract

The collective nature and the vibrational structure of the dipolar response in nuclear systems is investigated within a self-consistent mean-field model based on Vlasov equation. We study the role of the variation with density of the symmetry energy, in parametrizations based on Skyrme interactions, on the emergence oflow-lying, Pygmy Dipole Response (PDR) in neutron rich systems. The predictions of the transport model are then compared with the results obtained within a RPA aproaches based on extended separable interactions, able to encode the dependence of nuclear symmetry energy on density. Our results provide insights upon the transition from skin oscillation to a highly bulk collective dynamics.

keywords: Pygmy dipole resonance, Vlasov equation, Separable residual interactions

DOI: 10.56082/annalsarsciphyschem.2026.1.29

1 Introduction

In the presence of particle interaction and of specific quantum correlations, many-body systems manifest a complex dynamics, which includes single-particle excitations but also collective states, sometimes non-perturbative, when a coherent participation of many constituents manifests [1, 2].

The plasmons, the collective states in metallic clusters, and the Giant Resonances in atomic nuclei [3], which are exhausting most of the Energy Weighted Sum Rule (EWSR) associated with the specific excitation operator, are notable examples. An

[1] Faculty of Physics, University of Bucharest, Atomistilor 405, Magurele, Romania;
[2] Academy of Romanian Scientists, Strada Ilfov 3, Bucharest, Romania;
E-mail: `virgil.baran@unibuc.ro`.

understanding based on superposition of particle-hole excitations coupled by the residual interaction [4] allows for the identification of some collective degrees of freedom, whose macroscopic dynamics provides a relatively simple representation of these modes [1, 5].

The nature of the low-lying dipolar response in neutron-rich nuclei, below the Giant Dipole Resonance (GDR), observed as a resonance-like structure, exhausting a few percentages of the Energy Weighted Sum Rule (EWSR) [8, 9, 10] and associated to PDR states, presents a great interest theoretically as well as from experimental point of view. The experimental results indicate puzzling properties demanding more data which will allow to establish correlations between the features of PDR, such as energy position, EWSR, and contribution to polarizabilities, and the properties of nuclei, such as mass, isospin, and size of the neutron skin, or the parameters of the symmetry energy of the equation of state [6, 7]. The experimental facilities dedicated to nuclear photonics, either in operation or currently under commissioning, such as HIγS and ELI, can uncover the detailed structure of the PDR using next-generation gamma-ray sources.

In this work we explore the connection between the symmetry energy and the the response following an $E1$ perturbation in the PDR region. We place special emphasis on the role played by the density dependence of the symmetry energy on the development of the low-lying dipolar collective response, which show up an isoscalar character. The PDR and GDR will be discussed within a self-consistent mean-field approach based on Vlasov equation with Skyrme interactions manifesting specific density dependence of symmetry energy. A comparison with the predictions of schematic model which is a generalization of the Brown-Bolsterli approach [11, 12] will be discussed. Our quantum many-body model with separable interactions [17], is a generalization of the Brown-Bolsterli approach [11, 12], by introducing a density-dependent particle-hole residual interaction which encodes in a simplified manner the density dependence of symmetry energy below saturation.

2 Dipolar response within Vlasov transport model

The study the dipolar collective dynamics of neutron rich nuclei is based on a model considering two coupled Landau-Vlasov kinetic equations for neutrons and protons [13]:

$$\frac{\partial f_q}{\partial t} + \frac{\mathbf{p}}{m}\frac{\partial f_q}{\partial \mathbf{r}} - \frac{\partial U_q}{\partial \mathbf{r}}\frac{\partial f_q}{\partial \mathbf{p}} = I_{coll}[f_n, f_p], \tag{2.1}$$

which determine the evolution of the one-body distribution functions $f_q(\vec{r},\vec{p},t)$, with $q = n,p$ [14]. Since we shall focus mainly on the dynamics of various collective modes and not on their collisional damping in the following we do not consider the collision integral. From the one-body distribution functions one obtains the local densities $\rho_q(\vec{r},t) = \int \frac{2d^3\mathbf{p}}{(2\pi\hbar)^3} f_q(\vec{r},\vec{p},t)$ and define the isoscalar density $\rho_{isoscalar} = \rho = \rho_n + \rho_p$

and the isospin or isovector density $\rho_{isovector} = \rho_i = \rho_n - \rho_p$. The nuclear mean-fields U_q appearing in Eq. (2.1) are derived within an Energy Density Functional (EDF) approach where the total energy of the system $E = \int d^3r \, (\mathcal{E}_{kin}(\mathbf{r}) + \mathcal{E}_{pot}(\mathbf{r}))$ is obtained starting from an effective interaction of Skyrme-like (SKM^*) type [15]. The interaction contribution to the energy density is:

$$\mathcal{E}_{pot}(\rho, \rho_i) = \frac{A}{2}\frac{\rho^2}{\rho_0} + \frac{B}{\sigma+1}\frac{\rho^{\sigma+1}}{\rho_0^{\sigma}} + \frac{C(\rho)}{2}\frac{\rho_i^2}{\rho_0} \quad , \tag{2.2}$$

The functional derivative of $\mathcal{E}_{pot}$ with respect to the proton (neutron) density $U_q(\rho) = \delta\mathcal{E}_{pot}(\mathbf{r})/\delta\rho_q(\mathbf{r})$ leads to:

$$U_p = A\frac{\rho}{\rho_0} + B\Big(\frac{\rho}{\rho_0}\Big)^{\sigma} - C(\rho)\frac{\rho_i}{\rho_0} + \frac{1}{2}\frac{dC(\rho)}{d\rho}\frac{\rho_i^2}{\rho_0} \quad , \tag{2.3}$$

$$U_n = A\frac{\rho}{\rho_0} + B\Big(\frac{\rho}{\rho_0}\Big)^{\sigma} + C(\rho)\frac{\rho_i}{\rho_0} + \frac{1}{2}\frac{dC(\rho)}{d\rho}\frac{\rho_i^2}{\rho_0} \quad . \tag{2.4}$$

In the spirit of EDF formalism the coefficients A, B and σ are chosen in order to reproduce the saturation properties of symmetric nuclear matter with $\rho_0 = 0.16 fm^{-3}$, $E/A = -16MeV$ and a compressibility modulus $K = 200MeV$. The resulting values are $A = -356MeV$, $B = 303MeV$, $\sigma = 7/6$. Even if the energy functional is obtained from an effective interaction and a many-body variational method, there are several subtle differences between HF and EDF approach since the coefficients factorizing terms which are functions of the local density (and possibly more general of the density gradients) are adjusted by requiring that a set of experimental properties such as the saturation density, the binding energy at saturation, the compressibility modulus, the symmetry energy at saturation and possibly other features are reproduced. Therefore, these coefficients encode more physics than those provided within HF theory. In the isovector sector, based on the philosophy of EDF, while keeping the value of symmetry energy at saturation almost the same, we shall allow for three different dependences with density away from equilibrium. Concerning the density dependence of the symmetry energy, we consider, in the mean-field structure, different parametrizations of $C(\rho)$. While keeping the value of symmetry energy at saturation almost the same, we shall allow for three different dependences with density away from equilibrium. For the asystiff EOS $C(\rho)$ is constant, $C(\rho) = 32MeV$. Then the symmetry energy $E_{sym}/A = \dfrac{\epsilon_F}{3} + \dfrac{C(\rho)}{2}\dfrac{\rho}{\rho_0}$ at saturation takes the value $E_{sym}/A = 28.3MeV$ while the slope parameter $L = 3\rho_0\dfrac{dE_{sym}/A}{d\rho}|_{\rho=\rho_0}$ is $L = 72MeV$.

The asysoft case corresponds to a Skyrme-like, SKM*, parametrisation with $\dfrac{C(\rho)}{\rho_0} = (482 - 1638\rho)MeV fm^3$ which leads to a small value of the slope parameter $L = 14.4MeV$. Lastly, for the asysuperstiff EOS, $\dfrac{C(\rho)}{\rho_0} = \dfrac{32}{\rho_0}\dfrac{2\rho}{(\rho+\rho_0)}$, the

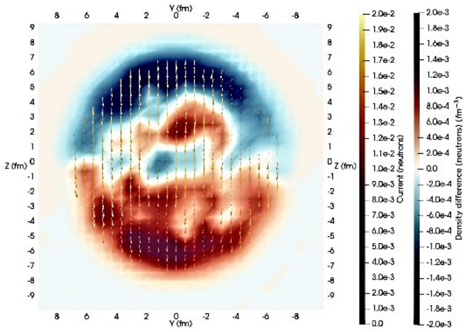

Figure 1: Neutron density distribution (in terms of deviations from equilibrium values) and neutron current density distribution soon after a dipolar perturbation along z-axis.

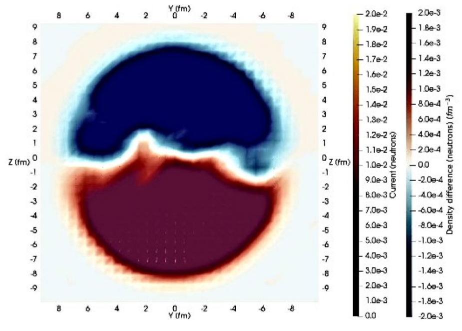

Figure 2: Neutron density distribution (in terms of deviations from equilibrium values) and neutron current density distribution after a quarter of period of oscillation.

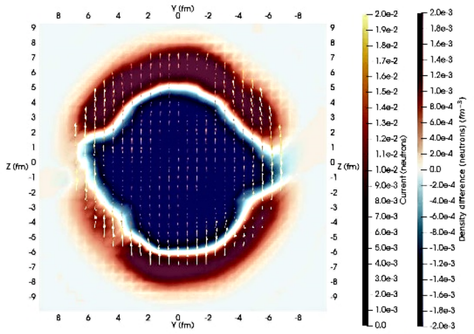

Figure 3: Neutron density distribution (in terms of deviations from equilibrium values) and neutron current density distribution after around half of period of oscillation.

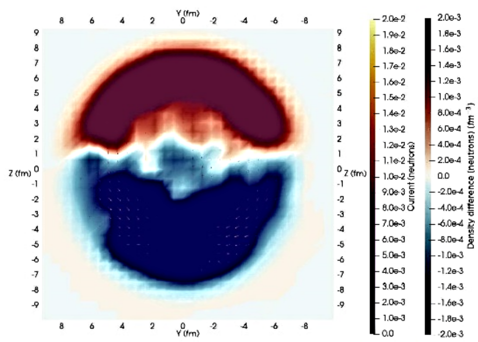

Figure 4: Neutron density distribution (in terms of deviations from equilibrium values) and neutron current density distribution after around three quarters of period of oscillation.

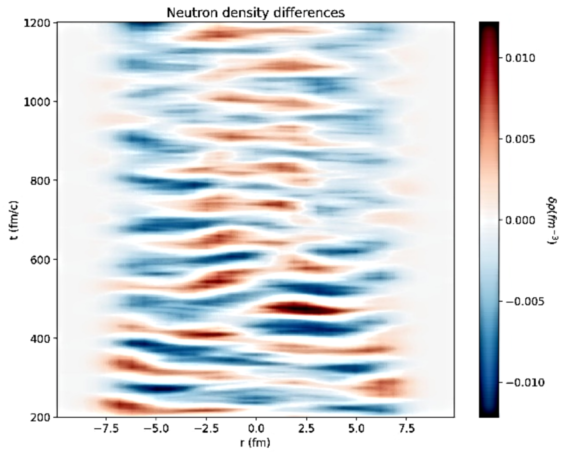

Figure 5: The time evolution of neutron density (in terms of deviations from equilibrium values).

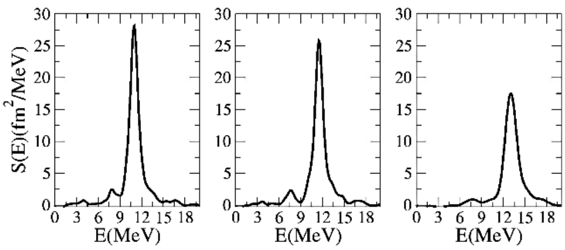

Figure 6: The strength function obtained from Vlasov transport model for ^{132}Sn by considering three parameterizations od asy-EOS.

symmetry term increases rapidly around saturation density being characterized by a value of the slope parameter $L = 96.6MeV$. The integration of the transport equations is based on a new numerical approach originating from the test-particle (or pseudo-particle) method, which will be presented elswere. The method is able to reproduce accurately the equation of state of nuclear matter and provide reliable results regarding the properties of nuclear surface [16].

The dynamics based on the Vlasov transport model represents the semi-classical limit of Time-Dependent Hartree-Fock (TDHF) and, for small-oscillations, of the RPA equations. While the model is unable to account for effects associated with the shell structure is suitable to describe robust quantum modes, of zero-sound type in nuclear systems. In Figures 1-4 we represent the neutron density distribution (in terms of deviation from equilibrium values) at four instants following a dipolar

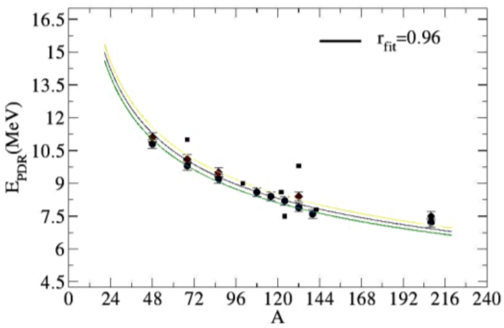

Figure 7: Mass number dependence of PDR centroid predicted by Vlasov transport model. The maroon squares are experimental data.

excitation along z-axis.The time evolution of the same quantity along a radial distribution is represented in Figure 5. This results demonstrate the vibrational character of the dipole response which is associated to GDR. However the calculated strength function unveil the existence of an additional mode with an energy centroid below the peak of GDR, see Figure 6 for the ^{132}Sn case, corresponding to the three asy-EOS introduce previously, which we associate with PDR. A sistematic study show that the position of PDR centroid follows a parameterization $E_{PDR} = 40A^{-1/3}$, see Figure 7.

3 A schematic model with generalized separable residual interaction

For the $E1$ excitations, the separable ansatz for the residual interaction $A_{ij} = \bar{V}_{ph',hp'} = V_{ph',hp'} - V_{ph',p'h}$, by neglecting the exchange term, is $A_{ij} = \lambda_1 D_{ph} D^*_{p'h'}$, where $D_{ph} = \langle ph|\hat{D}|0\rangle = \langle p|\hat{d}|h\rangle \equiv Q_i$ are the particle-hole matrix elements of single-particle terms associated to dipole operator $\hat{\vec{D}} = \frac{ZN}{A}(\frac{1}{Z}\sum_{i=1}^{Z}\hat{\vec{r}}_i - \frac{1}{N}\sum_{i=1}^{N}\hat{\vec{r}}_i)$ and i refers to a $p-h$ pair. For excitations along z-axis $\hat{D}$ is the one-body dipole operator $\hat{D} = \sum_{\mu,\nu} D_{\mu\nu} a^+_\mu a_\nu = \frac{ZN}{A}(\frac{1}{Z}\sum_{p,p'} < p|z|p' > a^+_p a_{p'} - \frac{1}{N}\sum_{n,n'} < n|z|n' > a^+_n a_{n'})$. The self-consistency requirement between the vibrating potential and the induced density variations for the dipolar field, fixes the coupling constant λ_1 to $\lambda_1(\rho_0) = \frac{A^2}{NZ}\frac{10 b^{(pot)}_{sym}(\rho_0)}{AR^2}$, with $R = 1.2A^{1/3}$, proportional to the potential symmetry energy $b^{(pot)}_{sym}(\rho_0)$ at saturation density, $\rho_0 = 0.16 fm^{-3}$ [3, 17].

The reason for an extended separable interaction is related to the fact that the potential symmetry energy decreases with density, and therefore a smaller value for the coupling constant of the particle-hole interaction is expected at the lower densities of the nucleus. Because a fraction of the nucleons is localised in a less dense surface

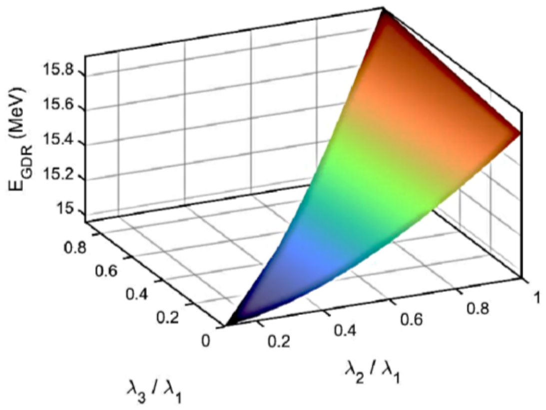

Figure 8: GDR energies as a function of parameters of the residual interaction.

region, we relax the condition of a unique coupling constant for all particle-hole interactions. We assume that only for a subsystem of particle-hole pairs, *i.e.*, $i, j \leq i_c$, at the saturation density, the interaction is given by $A_{ij} = \lambda_1(\rho_0) Q_i Q_j^*$, while for the subsystem at lower density ρ_e, *i.e.*, $i, j > i_c$, the interaction is characterised by a weaker strength $A_{ij} = \lambda_3(\rho_e) Q_i Q_j^*$. If $i \leq i_c$ and $j > i_c$ or $i > i_c$ and $j \leq i_c$, *i.e.*, for the coupling between the two subsystems, within a region of intermediate density, we consider $A_{ij} = \lambda_2(\rho_i) Q_i Q_j^*$. The values of λ_2 and λ_3 will reflect the behaviour of symmetry energy below normal density and therefore $\lambda_1(\rho_0) > \lambda_2(\rho_i) > \lambda_3(\rho_e)$.

In the Random Phase Approximation (RPA) the amplitudes which appear in the excitation operator:

$$\Omega_{RPA}^{+(n)} = \sum_{p,h} X_{ph}^{(n)} a_p^+ a_h + Y_{ph}^{(n)} a_h^+ a_p, \tag{3.1}$$

and which obey the normalization conditions $\sum_j |X_j^{(n)}|^2 - |Y_j^{(n)}|^2 = 1$ are obtained from the RPA equations

$$\epsilon_i X_i^{(n)} + \sum_j (A_{ij} X_j^{(n)} + B_{ij} Y_j^{(n)}) = E_n X_i^{(n)}, \tag{3.2}$$

$$\epsilon_i Y_i^{(n)} + \sum_j (B_{ij}^* X_j^{(n)} + A_{ij}^* Y_j^{(n)}) = -E_n Y_i^{(n)}, \tag{3.3}$$

with $B_{ij} = \bar{V}_{pp'hh'}$. The amplitudes Y_j are a measure of ground state correlations and by setting all $Y_j = 0$ we recover the TDA equations. For separable particle-hole

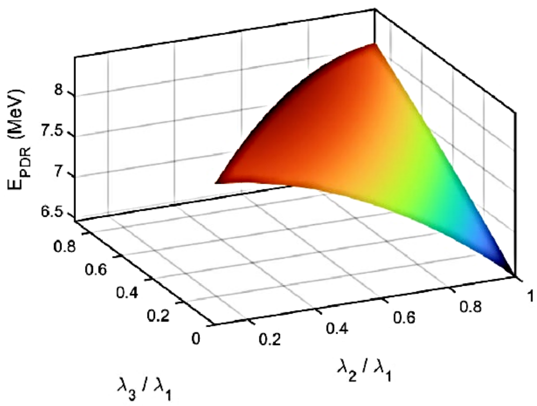

Figure 9: PDR energies as a function of parameters of the residual interaction.

interactions $A_{ij} = \lambda Q_i Q_j^*$ and $B_{ij} = \lambda Q_i Q_j$ we get the dispersion relation:

$$\sum_i \frac{2\epsilon_i |Q_i|^2}{E_n^2 - \epsilon_i^2} = \frac{1}{\lambda}, \tag{3.4}$$

By considering the extended separable interactions described above, in the degenerate case, $\epsilon_i = \epsilon$, the nontrivial solutions for amplitudes are obtained if:

$$(E_n^2 - \epsilon^2)^2 - 2\epsilon(\lambda_1\alpha + \lambda_3\beta)(E_n^2 - \epsilon^2) + 4\epsilon^2(\lambda_1\lambda_3 - \lambda_2^2)\alpha\beta = 0. \tag{3.5}$$

We employ the EWSR associated with the isovector dipolar field corresponding to the unperturbed case: $m_1 = \hbar\omega_0(\alpha + \beta) = \frac{\hbar^2}{2m}\frac{NZ}{A}$. The values for α and β are related to the number of protons (Z_c) and neutrons (N_c) which belong to core, ($A_c = N_c + Z_c$) and the number of neutrons considered in excess, *i.e.* nucleons at much lower density (N_e), respectively. The approach that includes extended separable interactions predicts the appearance of two dipolar non-trivial states, which we associated with PDR and GDR, that share all $E1$ EWSR, thereby manifesting specific collective features both in a TDA and RPA treatments [17]. We show in Figures 8 and 9 the predictions of the model for the energies of the two modes as a function of the values of the parameters of separable interaction and notice a nice agreeemt with experimental data and the predictions based on Vlasov equation. As expected, in the limiting case when all coupling constants return to the value associated to saturation density, as in the original Brown-Bolsterli model. In this situation EWSR exhausted by the PDR state goes to zero, suggesting that the density dependence of symmetry energy is influencing the appearence of a collective response below GDR.

4 Conclusions

In this work, by considering two different many-body treatments to dipolar dynamics in nuclear systems we evidenced the existence of a mode below GDR response manifesting collective fetures and which can be associated to PDR observed experimentally in the same energy region.

References

[1] P. Ring, P. Schuck, *The Nuclear Many-Body Problem* (Springer, New-York, 1980).

[2] U. Fano, Rev. Mod. Phys. **64**, 313 (1992).

[3] A. Bohr, B. R. Mottelson, *Nuclear Structure* vol II, p. 481 (World Scientific, Singapore, 1998).

[4] G.E. Brown, *Unified Theory of Nuclear Models* (North-Holland Publishing Company, Amsterdam, 1964).

[5] D.M. Brink, Nucl. Phys. A **4**, 215 (1957).

[6] T. Aumann and T. Nakamura, Phys. Scr. **T 152**, 014012 (2013).

[7] D. Savran, T. Aumann and A. Zilges, Prog. Part. Nucl. Phys. **70**, 210 (2013).

[8] P. Adrich et al., Phys. Rev. Lett. **95**, 132501 (2005).

[9] O. Wieland et al., Phys. Rev. Lett. **102**, 092502 (2009); O. Wieland, A. Bracco, Prog. Part. Nucl. Phys. **66**, 374 (2011).

[10] D.M. Rossi et al., Phys. Rev. Lett. **111**, 242503 (2013).

[11] G.E. Brown, M. Bolsterli, Phys. Rev. Lett. **3**, 472 (1959).

[12] G.E. Brown, J.A. Evans, D.J. Thouless, Nucl. Phys. A **24**, 1 (1961).

[13] V. Baran, B. Frecus, M. Colonna, M Di Toro, Phys. Rev.**C 85**, 051601 (2012).

[14] V.Baran, M. Colonna, M. Di Toro, V. Greco, Phys. Rep. **410**, 335 (2005).

[15] M. Colonna, M. Di Toro, A. B. Larionov, Phys. Lett. B **428** 1 (1998).

[16] V. Baran, M. Colonna, M Di Toro, A. Croitoru, D. Dumitru, Phys. Rev. C **88**, 044610 (2013).

[17] V. Baran, DI Palade, M. Colonna, M Di Toro, A. Croitoru, AI Nicolin, Phys. Rev. C **91**, 054303 (2015).