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DEFORMED NUCLEAR COLLECTIVE STATES DESCRIBED IN LABORATORY SYSTEM

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Abstract

Microscopic description of nuclear collective excitations is reviewed in terms of the deformed Quasiparticle Random Phase Approximation (dQRPA). To this purpose a deformed single particle basis with good angular momentum is used. The dQRPA equations for collectice states are formally similar to those given by the spherical formalism. It is shown that electromagnetic BE2-values in even-even deformed nuclei are satisfactorily described for both spherical and deformed nuclei. The generalization to proton-neutron excitations decribing beta decays in deformed nuclei is given in terms of pn-dQRPA. The analysis of the effective axial-vector strength g_A evidences shell effects.

keywords: Deformed Quasiparticle Random Phase Approximation, deformed nuclei, electromagnetic transitions, beta decays

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1 Introduction

The many-body description of deformed nuclei is usually performed by using single particle states (sp) in the intrinsic system of coordinates. One of the most popular approaches is provided the Nilsson model [1], describing sp dynamics in an intrinsic system of cylindrical symmetry. This basis is used to describe collective excitations within the Quasiparticle Random Phase Approximation (QRPA) [2]. Thus, the physical states with good angular momentum are given by the projection procedure

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[3]. This projection after variation method, was extensively used to describe a large variety of collective excitations, like the Gamow-Teller 1^+ resonance in the context of double-beta-decay processes [4, 5, 6, 7], low-lying states in neutron-rich nuclei [8, 9, 10], the linear response function [11], giant resonances [12, 13, 14], the Pygmy resonance [15] and magnetic properties of nuclei [16].

A more exact approach is the use of projected wave functions before the variational procedure, like in the VAMPIR method [17], able to describe complex like the shape coexistence effects [18]. On the other hand, the projected Generator Coordinate Method gives a precise analysis of first excited 2^+ state in even-even deformed nuclei [19, 20].

In Ref. [21] A.A. Raduta proposed a relative simple approach in terms of the product between one spherical sp orbital and core wave function described by a deformed coherent state, projected to a given angular momentum, describing a basis state for the many-body problem. In this way, many-particle-many-hole configurations are described in an effective way due to the fact that the sp states are "dressed" by core wave functions. The equations of motion have a similar form as for a spherical system. The method was used to describe various collective excitations in deformed nuclei, like the two-neutrino double beta decay [22, 23, 24], magnetic collective states [25] and collective excitation in atomic clusters [26].

Later on, in Ref. [27], we generalized this approach by considering all allowed spherical sp states in order to build a sp state "dressed by deformation". A particular case is the adiabatic limit, which is nothing else than the usual Nilsson wave function expressed in the laboratory frame. In the first part of this review we show that dQRPA satisfactorily describes BE2 values in deformed nuclei [27], while in the second part we use the pn-dQRPA to generalize our earlier results [28] on the problem of the effective axial-vector strength by using the available data on GT β^- and β^+/EC decay rates between 0^+ states in even-even and 1^+ states in neighboring odd-odd nuclei [29]. It turns out that an additional analysis of $2\nu\beta\beta$ half-lives is essential in determining the effective value of $g_{\rm A}$.

2 Theoretical background

In order to describe collective excitations we proceed with the steps described in Refs. [27, 29, 30, 31].

2.1 Single particle basis

1) We first build a deformed sp basis with good angular momentum starting from the standard Nilsson sp representation in the intrinsic frame, which is then transformed

in the laboratory system of coordinates [27]

$$\begin{aligned} |\tau j m) &= a^{\dagger}_{\tau j m}(\Omega)|0) \\ &= \sum_{J=even} \sum_{j_s \geq j} \mathcal{X}^{J k_s}_{\tau j} \left[Y_J(\Omega) \otimes |\tau k_s\rangle \right]_{jm}, \\ |\tau k_s \nu\rangle &= c^{\dagger}_{\tau k_s \nu}|0\rangle, \quad \tau = p, n, \end{aligned} \tag{2.1}$$

where Ω denotes the Euler angles of the intrinsic symmetry axis with respect to the laboratory system and $j \equiv (\epsilon, j^{\pi})$ (deformed eigenvalue, total spin$^{\rm parity}$). The creation operators $c^{\dagger}_{\tau k_s \nu}$ describe the eigenstates of a spherical nuclear plus proton Coulomb mean field having the quantum numbers $k_s \equiv (e, l, j_s)$, (spherical eigenvalue, orbital angular momentum, total spherical spin), with ν being the z projection of j_s. The expansion coefficients are proportional to standard Nilsson amplitudes with $j = K$, where K is the spin projection on the intrinsic symmetry axis

$$\mathcal{X}^{J k_s}_{\tau j} = \sqrt{2} \langle j j; j_s - j | J 0 \rangle x^{k_s}_{\tau j}, \tag{2.2}$$

and by the bra-ket product we denoted the Clebsch-Gordan coefficient. The amplitudes $x^{k_s}_{\tau j}$ in Eq. (2.2) are found by diagonalizing the quadrupole operator in the spherical Woods-Saxon basis. Let us mention that both the $\mathcal{X}$ and x amplitudes satisfy orthonormality relations and the creation sp operators (2.1) satisfy the following anticommutationb rule

$$\int d\Omega \left\{ a_{\tau j m}, a^{\dagger}_{\tau j' m'} \right\} = \delta_{jj'} \delta_{mm'} \ . \tag{2.3}$$

Notice that in the spherical limit, where one has $x^{k_s}_{\tau j} = \delta_{j_s j}$, the operator (2.1) is proportional to the usual spherical sp creation operator with a "statistical" coefficient

$$\mathcal{X}^{J k_s}_{\tau j} = \sqrt{\frac{2}{2j+1}} \delta_{j_s j}, \tag{2.4}$$

expressing the fact that two particles with intrinsic projections $K = \pm j$ are distributed over $2j+1$ projections in the laboratory system of coordinates.

The particular case of laboratory sp amplitudes given by Eq. (2.2) is what we call "adiabatic approach", because we used the standard intrinsic Nilsson wave function with components x, having a given intrinsic angular momentum projection, but related to the laboratory frame according to the relation (2.1). Thus, the Coriolis mixing of angular momentum projections is neglected. In the general case, the amplitudes $\mathcal{X}$ entering Eq. (2.1) can be obtained by diagonalizing the QQ interaction in the laboratory frame. The particle occupancy of the obtained sp levels implies a more general rule than two particles/level and a quasiparticle is built in terms of all sp components replacing Eq. (2.13). Our analysis concerning B(E2)-values in well deformed nuclei [27] showed that the so-called adiabatic approach gives very good results. For this reason we also used this simplification in the analysis of beta decays.

2) Then we expressed the one-body particle-hole (ph) operators of the multipolarity λ as follows

$$Q_{\lambda\mu}(\tau) = \sum_{j_1 \leq j_2} \frac{(\tau j_1 || Q_\lambda || \tau j_2)}{\sqrt{(2\lambda+1)(1+\delta_{j_1 j_2})}} \left[a^\dagger_{\tau j_1} \otimes \tilde{a}_{\tau j_2} \right]_{\lambda\mu}, \tag{2.5}$$

for the same kind of particles $\tau = p, n$ and

$$Q_{\lambda\mu}(\tau_1\tau_2) = \sum_{j_1 j_2} \frac{(\tau_1 j_1 || Q_\lambda || \tau_2 j_2)}{\sqrt{(2\lambda+1)}} \left[a^\dagger_{\tau_1 j_1} \otimes \tilde{a}_{\tau_2 j_2} \right]_{\lambda\mu}, \tag{2.6}$$

for proton neutron excitations describing beta-decay processes. Here we dropped the Euler angles Ω for simplicity. The reduced matrix element in the deformed basis (2.1) is given by integration over Euler angles

$$\begin{aligned}
(\tau_1 j_1 || Q_\lambda || \tau_2 j_2) &= \hat{j}_1 \hat{j}_2 \sum_{J k_{s1} k_{s2}} \mathcal{X}^{J k_{s1}}_{\tau_1 j_1} \mathcal{X}^{J k_{s2}}_{\tau_2 j_2} \\
&\times (-)^{j_{s1}+j_2+\lambda-J} W(j_1 j_{s1} j_2 j_{s2}; J\lambda) \langle \tau_1 k_{s1} || Q_\lambda || \tau_2 k_{s2} \rangle,
\end{aligned} \tag{2.7}$$

in terms of the spherical reduced matrix elements. Here $\hat{j} = \sqrt{2j+1}$ and W is the Racah coefficient. One has a similar result for a particle-particle (pp) operator. For the monopole particle-number and pairing operators in the laboratory system we will consider the leading $J = 0$ component

$$\begin{aligned}
N_{\tau j} &\approx \left(x^j_{\tau j} \right)^2 \frac{2}{2j+1} \sum_m a^\dagger_{\tau j m} a_{\tau j m}, \\
P^\dagger_{\tau j} &\approx \left(x^j_{\tau j} \right)^2 \frac{2}{2j+1} \sum_m a^\dagger_{\tau j m} a^\dagger_{\tau j -m} (-)^{j-m}.
\end{aligned} \tag{2.8}$$

3) We use as Hamiltonian describing collective excitations monopole pairing plus a residual interaction. The pairing interaction is given by

$$\begin{aligned}
H_P &= \sum_p (\epsilon_p - \lambda_{prot}) N_p - \frac{G_{prot}}{4} \sum_{pp'} P^\dagger_p P_{p'} \\
&+ \sum_n (\epsilon_n - \lambda_{neut}) N_n - \frac{G_{neut}}{4} \sum_{nn'} P^\dagger_n P_{n'} .
\end{aligned} \tag{2.9}$$

where the meaning of the short-hand notation is $\tau \equiv (\tau, \epsilon j^\pi)$. Here, the chemical potential for protons (neutrons) is denoted by λ_{prot} (λ_{neut}).

For particle-hole (ph) excitations describing surface oscillations and rotations we use the following separable residual interaction

$$\begin{aligned} H &= H_P - H_Q \qquad (2.10) \\ &= H_P - \frac{1}{2}\sum_{\lambda}\sum_{\tau\tau'} F^{\lambda}_{\tau\tau'} \sum_{\mu} Q_{\lambda\mu}(\tau) Q_{\lambda-\mu}(\tau')(-)^{\mu} . \end{aligned}$$

In order to describe beta-decay GT decay processes use separable interaction with constant strengths in both the ph and pp channels

$$\begin{aligned} H &= H_P + H_{\beta} \qquad (2.11) \\ &= H_P + g_{\rm ph} \sum_{\mu} D^{-}_{1\mu} \left(D^{-}_{1\mu}\right)^{\dagger} - g_{\rm pp} \sum_{\mu} P^{-}_{1\mu} \left(P^{-}_{1\mu}\right)^{\dagger} . \end{aligned}$$

The strength parameters $g_{\rm ph}$ (particle-hole) and $g_{\rm pp}$ (particle-particle) are the ones of the corresponding spherical limit in Refs. [32, 33], and are given in units of MeV.

The GT operators are given by

$$\begin{aligned} D^{-}_{1\mu} &= \frac{1}{\sqrt{3}} \sum_{pn} (p||\sigma||n) \left[a^{\dagger}_{p} \otimes \tilde{a}_{n}\right]_{1\mu} , \qquad (2.12) \\ P^{-}_{1\mu} &= \frac{1}{\sqrt{3}} \sum_{pn} (p||\sigma||n) \left[a^{\dagger}_{p} \otimes a^{\dagger}_{n}\right]_{1\mu} , \end{aligned}$$

in terms of the Pauli operator σ_{μ}. The reduced matrix element in the deformed basis (2.1) is given in terms of the standard spherical matrix element by Eq. (2.7) with $\lambda = 1$.

2.2 Quasiparticle representation

4) The next step involves introducing a quasiparticle representation for protons and neutrons:

$$a^{\dagger}_{\tau m} = u_{\tau} \alpha^{\dagger}_{\tau m} + v_{\tau} \alpha_{\tau -m} (-)^{j_{\tau}-m}, \quad \tau = p, n, \qquad (2.13)$$

where u and v are the BCS vacancy and occupation amplitudes respectively. The BCS equations have formally the structure of the deformed case due to the "statistical" factors entering the particle-number and pairing operators (2.8). By using the quasiparticle representation (2.13) one obtains

$$Q_{\lambda\mu}(\tau) = \sum_{j_1 \le j_2} \xi^{\lambda}_{\tau_{12}} \left[\overline{A}^{\dagger}_{\lambda\mu}(\tau 12) + (-)^{\lambda-\mu} \overline{A}_{\lambda-\mu}(\tau 12)\right] , \qquad (2.14)$$

in terms of the normalized pair quasiparticle operator

$$\overline{A}^{\dagger}_{jm}(\tau_{12}) = \frac{1}{\sqrt{1+\delta_{j_1j_2}}}\left[\alpha^{\dagger}_{\tau j_1}\otimes\alpha^{\dagger}_{\tau j_2}\right]_{jm} , \tag{2.15}$$

where we used the short-hand notation $\tau_{12}=(\tau j_1 j_2)$ and

$$\xi^{\lambda}_{\tau_{12}} = \frac{\langle\tau j_1||Q_\lambda||\tau j_2\rangle}{\sqrt{(2\lambda+1)(1+\delta_{j_1j_2})}}(u_{\tau j_1}v_{\tau j_2}+v_{\tau j_1}u_{\tau j_2}) . \tag{2.16}$$

For GT operators in the quasiparticle representation one obtains

$$\begin{aligned} D^{-}_{1\mu} &= \sum_{pn}\left[\xi_{pn}A^{\dagger}_{1\mu}(pn)+\overline{\xi}_{pn}A_{1-\mu}(pn)(-)^{1-\mu}\right], \\ P^{-}_{1\mu} &= \sum_{pn}\left[\eta_{pn}A^{\dagger}_{1\mu}(pn)-\zeta_{pn}A_{1-\mu}(pn)(-)^{1-\mu}\right], \end{aligned} \tag{2.17}$$

where

$$\begin{aligned} A^{\dagger}_{1\mu}(pn) &= \left[\alpha^{\dagger}_{p}\otimes\alpha^{\dagger}_{n}\right]_{1\mu} = (-)^{j_p+j_n}A^{\dagger}_{1\mu}(np), \\ A_{1\mu}(pn) &= \left(A^{\dagger}_{1\mu}(pn)\right)^{\dagger} = (-)^{1-\mu}\left[\tilde{\alpha}_n\otimes\tilde{\alpha}_p\right]_{1-\mu} \end{aligned} \tag{2.18}$$

depend also on the Euler angles Ω, and we have defined

$$\begin{aligned} \xi_{pn} &= \frac{(p||\sigma||n)}{\sqrt{3}}u_p v_n = (-)^{j_p-j_n}\overline{\xi}_{np}, \\ \overline{\xi}_{pn} &= \frac{(p||\sigma||n)}{\sqrt{3}}v_p u_n = (-)^{j_p-j_n}\xi_{np}, \\ \eta_{pn} &= \frac{(p||\sigma||n)}{\sqrt{3}}u_p u_n = (-)^{j_p-j_n}\eta_{np}, \\ \zeta_{pn} &= \frac{(p||\sigma||n)}{\sqrt{3}}v_p v_n = (-)^{j_p-j_n}\zeta_{np}. \end{aligned} \tag{2.19}$$

2.3 Collective excitations

5) Finally, we diagonalize the residual interaction in terms of collective dQRPA phonons of multipole λ

$$\Gamma^{\dagger}_{\lambda\mu}(\omega) = \sum_{\tau}\sum_{j_1\leq j_2}[\mathcal{X}^{\omega}_{\lambda}(\tau_{12})\overline{A}^{\dagger}_{\lambda\mu}(\tau_{12})-\mathcal{Y}^{\omega}_{\lambda}(\tau_{12})\overline{A}_{\lambda-\mu}(\tau_{12})(-)^{\lambda-\mu}] \tag{2.20}$$

or proton-neutron interaction within the pn-dQRPA framework

$$\Gamma^{\dagger}_{1\mu}(\omega) = \sum_{pn} \left[X^{\omega}_{pn} A^{\dagger}_{1\mu}(pn) - Y^{\omega}_{pn} A_{1-\mu}(pn)(-)^{1-\mu} \right] , \tag{2.21}$$

given in terms of the creation pair operator (2.18) with ω the eigenvalue index. Using the boson commutation rule for instance in the case of ph phonons

$$\int d\Omega \left[\Gamma_{\lambda\mu}(\omega), \Gamma^{\dagger}_{\lambda\mu}(\omega') \right] = \delta_{\omega,\omega'} , \tag{2.22}$$

one obtains the standard orthonormality condition for amplitudes

$$\sum_{\tau_{12}} \left(\mathcal{X}^{\omega}_{\lambda}(\tau_{12}) \mathcal{X}^{\omega'}_{\lambda}(\tau_{12}) - \mathcal{Y}^{\omega}_{\lambda}(\tau_{12}) \mathcal{Y}^{\omega'}_{\lambda}(\tau_{12}) \right) = \delta_{\omega,\omega'} . \tag{2.23}$$

Similar relations can be obtained for pn phonons. The equations of motion are derived from a projection procedure over Euler angles, i.e

$$\langle 0| \int d\Omega \left[A_{\lambda\mu}, \left[H, \Gamma^{\dagger}_{\lambda\mu}(\omega) \right] \right] |0\rangle = \omega \langle 0| \int d\Omega \left[A_{\lambda\mu}, \Gamma^{\dagger}_{\lambda\mu}(\omega) \right] |0\rangle , \tag{2.24}$$

where $|0\rangle$ is the vacuum state. One uses a similar commutatot with $A^{\dagger}_{\lambda-\mu}$.

The dQRPA amplitudes are given by equations of motion which are formally similar to those of the spherical version

$$\begin{pmatrix} \mathcal{A}_{\lambda}(\tau_{12}, \tau'_{12}) & \mathcal{B}_{\lambda}(\tau_{12}, \tau'_{12}) \\ -\mathcal{B}_{\lambda}(\tau_{12}, \tau'_{12}) & -\mathcal{A}_{\lambda}(\tau_{12}, \tau'_{12}) \end{pmatrix} \begin{pmatrix} \mathcal{X}^{\omega}_{\lambda}(\tau'_{12}) \\ \mathcal{Y}^{\omega}_{\lambda}(\tau'_{12}) \end{pmatrix} = \omega \begin{pmatrix} \mathcal{X}^{\omega}_{\lambda}(\tau_{12}) \\ \mathcal{Y}^{\omega}_{\lambda}(\tau_{12}) \end{pmatrix} . \tag{2.25}$$

The dQRPA matrix elements are given by the following symmetrized double commutators

$$\begin{aligned} \mathcal{A}_{\lambda}(\tau_{12}\tau'_{12}) &= \langle 0| \int d\Omega \left[\overline{A}_{\lambda\mu}(\tau_{12}), H, \overline{A}^{\dagger}_{\lambda\mu}(\tau'_{12}) \right] |0\rangle \\ &= \delta_{\tau_{12}\tau'_{12}} (E_{\tau j_1} + E_{\tau j_2}) - F^{\lambda}_{\tau\tau'} \xi^{\lambda}_{\tau_{12}} \xi^{\lambda}_{\tau'_{12}} \\ \mathcal{B}_{\lambda}(\tau_{12}\tau'_{12}) &= -\langle 0| \int d\Omega \left[\overline{A}_{\lambda\mu}(\tau_{12}), H, \overline{A}_{\lambda-\mu}(\tau'_{12})(-)^{\lambda-\mu} \right] |0\rangle \\ &= -F^{\lambda}_{\tau\tau'} \xi^{\lambda}_{\tau_{12}} \xi^{\lambda}_{\tau'_{12}} . \end{aligned} \tag{2.26}$$

Here we used an approximation analogous to the Hartree-Fock factorized ansatz of two-body density into a product between one-body densities. They contain quasiparticle energies $E_{\tau j} = \sqrt{(e_{\tau j} - \lambda_{\tau})^2 + \Delta^2_{\tau}}$, and the factors $\xi^{\lambda}_{\tau'_{12}}$ (2.16) featuring the deformed matrix elements. As we will show latter, in spite of the similarity to the spherical QRPA, this formalism is more general. It is able to describe in an unified way low-lying excitations not only in spherical but also in deformed systems.

The pn-dQRPA equations of motion

$$\begin{pmatrix} \mathcal{A}_{pn,p'n'} & \mathcal{B}_{pn,p'n'} \\ -\mathcal{B}_{pn,p'n'} & -\mathcal{A}_{pn,p'n'} \end{pmatrix} \begin{pmatrix} X^{\omega}_{p'n'} \\ Y^{\omega}_{p'n'} \end{pmatrix} = \omega \begin{pmatrix} X^{\omega}_{pn} \\ Y^{\omega}_{pn} \end{pmatrix} \tag{2.27}$$

are also formally given by the usual spherical relations

$$\begin{aligned} \mathcal{A}_{pn,p'n'} &= \delta_{pp'}\delta_{nn'}\left(E_p + E_n\right) \\ &+ 2g_{ph}(\xi_{pn}\xi_{p'n'} + \overline{\xi}_{pn}\overline{\xi}_{p'n'}) \\ &- 2g_{pp}(\eta_{pn}\eta_{p'n'} + \zeta_{pn}\zeta_{p'n'}), \\ \mathcal{B}_{pn,p'n'} &= 2g_{ph}(\xi_{pn}\overline{\xi}_{p'n'} + \overline{\xi}_{pn}\xi_{p'n'}) \\ &+ 2g_{pp}(\eta_{pn}\zeta_{p'n'} + \zeta_{pn}\eta_{p'n'}), \end{aligned} \tag{2.28}$$

but in terms of deformed quasiparticle spectra E_p, E_n and reduced matrix elements multiplied by BCS amplitudes (2.19). Thus, in the present approach the dQRPA vacuum is spherical, in contrast to the approximations adopted earlier where the spherical symmetry of the phonon was restored after deriving the equations of motion, still leaving the vacuum itself deformed.

2.4 Transitions

The strength of an electric transition, connecting the dQRPA eigenstate ω to the ground state, is characterized by the $B(\mathrm{E}\lambda : \omega)$ value defined by

$$B(\mathrm{E}\lambda : \omega \to 0) = \frac{1}{2\lambda+1}\left|\left(0||\hat{T}_\lambda||\nu\right)\right|^2 , \tag{2.29}$$

with the reduced matrix element

$$\left(0||\hat{T}_\lambda||\omega\right) = \sqrt{2\lambda+1}\sum_{\tau} e_\tau \sum_{j_1\le j_2} \xi^{\lambda}_{\tau_{12}}[\mathcal{X}^{\omega}_{\lambda}(\tau_{12}) + \mathcal{Y}^{\omega}_{\lambda}(\tau_{12})] , \tag{2.30}$$

where e_τ are the effective charges

$$e_\pi = e(1+\chi) , \quad e_\nu = \chi , \tag{2.31}$$

expressed in terms of the polarization parameter χ.

We define the GT β-decay transition matrix elements as follows [34]:

$$\begin{aligned} \beta^-_{\omega 0} &= \beta^+_{0\omega} \equiv (\omega||\beta^-||0) = \sqrt{3}\sum_{pn}\left(\xi_{pn}X^{\omega}_{pn} + \overline{\xi}_{pn}Y^{\omega}_{pn}\right) , \\ \beta^+_{\omega 0} &= \beta^-_{0\omega} \equiv (\omega||\beta^+||0) = \sqrt{3}\sum_{pn}\left(\overline{\xi}_{pn}X^{\omega}_{pn} + \xi_{pn}Y^{\omega}_{pn}\right) . \end{aligned} \tag{2.32}$$

These transitions are described within the pn-dQRPA formalism and they are schematically shown in Fig. 1. We find the eigenvalues ω and amplitudes X, Y by using a standard diagonalisation procedure.

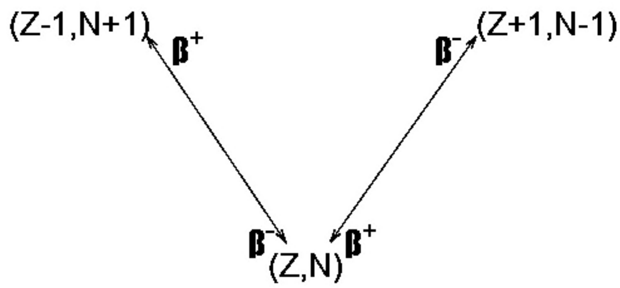

Figure 1: Weak processes described by the pn-dQRPA [29].

3 Numerical applications

3.1 Particle-hole collective excitations

In Ref. [27] we have applied the dQRPA approach to describe the B(E2) values of decays from the 2_1^+ collective state to the ground state in superfluid vibrational, transitional and rotational nuclei. Nilsson states in the intrinsic system of coordinates were provided by diagonalizing the axially symmetric quadrupole interaction $\beta m_N\omega^2 r^2 Y_{20}(\theta')$. As a basis we used the eigenstates of the spherical Woods-Saxon potential with a universal set of parameters [35] plus a Coulomb interaction for protons. The deformation parameters were taken from Ref. [36] and the experimental pairing gaps were estimated by using binding-energy data [36] of neighboring nuclei. The quadrupole-quadrupole coupling strengths $F_{\tau\tau'}$, defined by Eq. (2.10), were considered equal, as in the classical Ref. [37]. The common value was determined from the experimental 2_1^+ energy [38] for each nucleus by using the standard dispersion-relation technique for separable interactions. For the polarization parameter in Eq. (2.31) we adopted the (universal) value $\chi = 0.2$.

In order to analyze the differences between the spherical and deformed approaches, we have investigated the E2 decay properties of the 2_1^+ states in superfluid even-even nuclei for the region $50 < Z \leq 100$. We have divided the data into two sets separated by the magic number $Z = 82$. In Fig. 2 (a) we give by the open symbols the experimental B(E2) values [38] in the region $50 \leq Z \leq 82$ (111 values), while by the filled symbols we indicate the B(E2) values for $82 < Z \leq 100$ (27 values). The correspondence between the charge number of the isotopic chains we analyzed, the associate neutron numbers and the interval of the index values n is given in Table I of Ref. [27]. In Fig. 2 (b) are plotted the results given by the

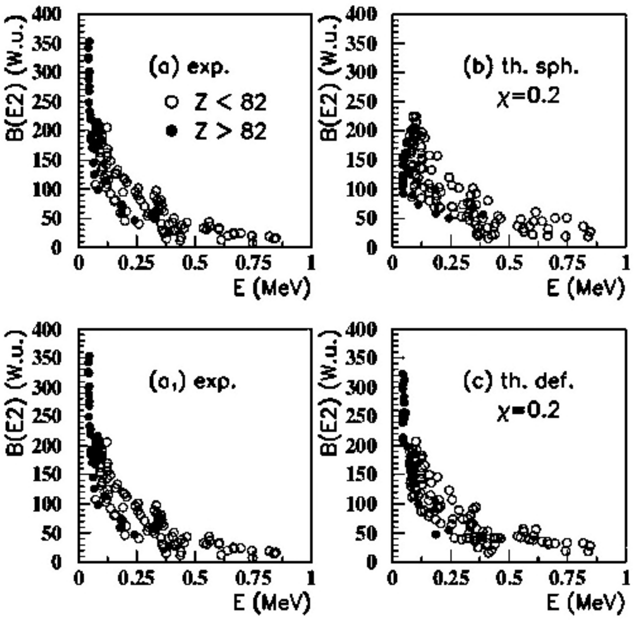

Figure 2: Theoretical B(E2) values given by the standard spherical QRPA (b) and the dQRPA (c) versus excitation energy [27]. For a better comparison in (a) and (a_1) are given the same experimental B(E2) values [38].

spherical QRPA code. They reproduce the order of magnitude of the B(E2)s in the region $50 \leq Z \leq 82$ (open symbols), containing mostly vibrational and transitional nuclei. The relative mean square error in this region is about 30%. In the region $82 < Z \leq 100$ (filled symbols), where most nuclei have small 2_1^+ energies and rotational spectra, the computed B(E2) values are by a factor $2-3$ lower, the mean square error being about 100%. So, as expected, the spherical QRPA fails in reproducing the B(E2) values for rotational nuclei. On the contrary, in Fig. 2 (c) this region is much better reproduced by the dQRPA (filled symbols). Indeed, here the mean square error decreases down to 25%. In order to enable a clear comparison, in Fig. 2 (a_1) we show again the experimental values. The better agreement of the dQRPA results with the data can be explained by the summation over core states J in the reduced matrix element given by Eq. (2.7). Indeed, by considering only the lowest yrast core state the B(E2) results become lower, comparable to the spherical method. Thus, the core states play an important role in describing well deformed nuclei.

3.2 Proton-neutron collective excitations

A major issue is the unknown effective value of the weak axial-vector coupling strength $g_{\rm A}$ entering the $0\nu\beta\beta$-decay amplitude as $g_{\rm A}^2$ and the decay half-life as $g_{\rm A}^4$. This problem is similar to the use of an effective charge to describe electromagnetic excitations in nuclei. A common conclusion of several studies evidenced s relatively large scattering of $g_{\rm A}$ values, in the range $g_{\rm A} \approx 0.25 - 0.82$. The effective value of $g_{\rm A}$ depends on the used nuclear many-body framework, like the IBA-2, IBFFM-2, pnQRPA, SM etc. [39]. There are also several other sources that affect the value of $g_{\rm A}$ [40, 41].

In order to investigate the effective value of g_A we analyzed in Ref. [29] experimental data concerning β^- and β^+/EC transitions from/to 0^+ ground states of even nuclei to/from 1^+ ground states of neighboring odd-odd nuclei. The experimental data are taken from [42]. We used as spherical sp states $c^\dagger_{\tau k_s \nu}$ the eigenstates of the spherical Woods-Saxon plus proton Coulomb mean field with the universal parameterization of Ref. [35]. The deformed eigenstates $a^\dagger_{\tau jm}$, given by Eq. (2.1), are obtained by diagonalizing the quadrupole-quadrupole interaction in the adiabatic limit, thus neglecting the Coriolis forces. The quadrupole deformation parameters were taken from Ref. [36]. The u_τ and v_τ amplitudes were determined by solving the BCS equations for protons and neutrons with monopole interaction reproducing the experimental pairing gaps.

It is known that the so-called Ikeda sum rule

$$\mathrm{ISR} = \sum_\omega \left[\left(\beta^-_{\omega 0}\right)^2 - \left(\beta^+_{\omega 0}\right)^2 \right] = 3(N-Z), \tag{3.1}$$

is automatically fulfilled within the spherical pnQRPA [34]. In order to check the accuracy of the rule in our case, we use in this relation the transition operators given by the deformed approach (2.32). In Ref. [29] we analyzed the ratio ISR/[3($N-Z$)] against the mass number A. One sees that the rule is reasonably fulfilled with a mean value of 1.02 and an overall deviation of less than 5%. Although we do not have an analytical form of ISR within pn-dQRPA, it is quite remarkable that by increasing the pair basis to 200 states one obtains a small increase of the above mentioned mean ratio ISR/[3(N-Z)], up to 1.06. Thus, the saturation property is fulfilled within our approach.

In Ref. [29] we analyzed transition strengths as functions of neutron number by using the extracted $\log ft$ values defined by [34]

$$g_{\rm A}\beta^{\pm}_{\rm exp}=\sqrt{\frac{6147\ (2J_i+1)}{10^{\log ft}}}, \tag{3.2}$$

where J_i is the spin of the initial nucleus and $g_{\rm A}$ denotes the effective axial-vector coupling strength. We evidenced larger values of transitions strength that above $N=50$ and $N=82$ magic numbers. Similar shell effects were found for the α-decay reduced width (proportional to the spectroscopic factor) in regions close to these magic numbers [43]. The α-decay reduced width used here is given by the leading monopole component defined as

$$\Gamma=2P_0\gamma_0^2, \tag{3.3}$$

in terms of the total α-decay width Γ and monopole penetrability through the Coulomb barrier P_0.

The similar behaviour of the experimental α-decay reduced width and EC transition probabilities where we notice the obvious linear correlations between the two mentioned quantities for transitions from even-even emitters. Large α-decay reduced widths above magic nuclei, with $100\gamma_0^2\geq 1$, are explained by an enhanced α clustering in this region. For EC processes, an explanation of large peaks above magic number with $\left(\beta^{+}_{\rm exp}\right)^2\geq 0.5$ are connected to the fact that the few protons and neutrons above closed shells have similar properties to free nucleons.

Our schematic approach depends on two Hamiltonian dipole parameters, namely $g_{\rm ph}$ and $g_{\rm pp}$. It is well known that the energy of the giant Gamow-Teller resonance is very sensitive to the particle-hole strength $g_{\rm ph}$. Therefore, we adjusted this parameter in order to fulfill the semi-empirical rule describing the centroid of the Gamow-Teller resonance as a function of the charge and neutron numbers [34]. The fitting line describing this law is given by the following relation

$$g_{\rm ph}=0.00072A+0.09504,\quad \sigma=0.047. \tag{3.4}$$

Furthermore, the position of the lowest pn excitation described within the pn-dQRPA is sensitive to the particle-particle strength $g_{\rm pp}$. The critical values of this strength can be fitted by

$$g_{\rm pp}({\rm crit})=3.3A^{-0.7},\quad \sigma=0.080. \tag{3.5}$$

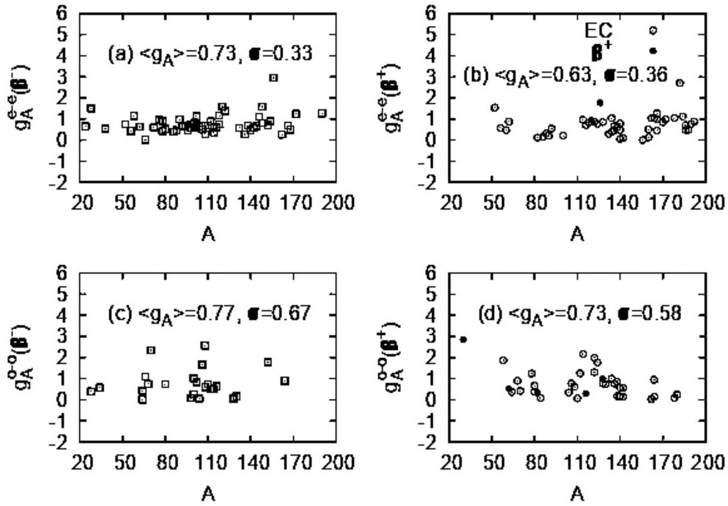

Figure 3: Effective axial-vector strength versus the mass number A for even-even [panels (a) and (b)] and odd-odd emitters [panels (c) and (d)] for $g_{\rm pp} = 0.3 g_{\rm pp}(\rm crit)$ [29]. The averaged values with their rms deviations are given in each panel.

We performed in Ref. [29] a systematic analysis of the effective value of $g_{\rm A}$ versus the mass number for β^- and β^+/EC transitions from/to 0^+ ground states of even-even nuclei to/from 1^+ ground states of the corresponding neighboring odd-odd nuclei. In Fig. 3 we give a typical example for transitions from even-even nuclei [panels (a) and (b)] and odd-odd nuclei [panels (c) and (d)]. One notices that the transitions from even-even nuclei provide less scattered values of $g_{\rm A}$ ($\sigma \sim 0.3$) than from odd-odd emitters ($\sigma \sim 0.6$). This approach then provides a mean value of the effective axial-vector coupling $\langle g_{\rm A} \rangle = 0.68$ with larger values for emitters above magic nuclei.

4 Conclusions

We reviewed the description of collective excitations in deformed nuclei in the laboratory frame of coordinates. Thus, the physical collective states are directly determined, without any additional projection procedure. First we described the quadrupole collective excitations in even-even nuclei within a common formalism called deformed QRPA (dQRPA), by using a sp basis with good angular momentum provided by the Nilsson states rotated into the laboratory frame of coordinates. We analyzed E2 transitions from the collective 2_1^+ state to the ground state for superfluid nuclei in the charge-number range $50 < Z \leq 100$ by using a single value χ=0.2 for the polarization charge. It turns out that the dQRPA improves dramatically the description of the E2 transitions for deformed nuclei.

Then we analyzed the experimental Gamow-Teller β^- and β^+/EC decay rates between 0^+ ground states of even-even nuclei and 1^+ ground states in neighboring odd-odd nuclei in order to derive an average effective value of the weak axial-vector strength g_A. The pn-QRPA equations of motion were derived in terms of a single-particle basis with projected angular momentum provided by the diagonalization of a deformed mean field plus a schematic dipole residual interaction. The parameters involved are mass dependent particle-hole (ph) and particle-particle (pp) strengths. It turned out that the collectivity of transition amplitudes increases linearly with the increasing mass number. This behaviour has an important consequence, namely that the structure of ground states in light odd-odd nuclei is mainly given by the pn pair occupying single-particle states closest to the Fermi level.

We also evidenced an important experimental fact: the available EC transition matrix elements are proportional to the α-decay reduced widths from even-even emitters. Our analysis led to a quenched average effective value $\langle g_A \rangle \approx 0.7$, with a root mean square deviation of $\sigma \sim 0.3$ for transitions from even-even emitters and $\sigma \sim 0.6$ for transitions from odd-odd emitters.

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