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PARTITION FUNCTIONS THAT REPEL PERFECT-POWERS*

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Abstract

A conjecture by Sun states that the partition function $p(n)$, for $n > 1$, is never a perfect power. Recent work by Merca et al. proposes generalizations of perfect-power repulsion for $p(n)$. In this note, we prove these generalizations for the functions $p_B(n)$, which count the number of partitions of n with the largest part $\leq B$. If $B \geq 4$ and $k \geq 3$, with $k \nmid (B-1)$, then we prove that there are only finitely many pairs (n, m) for which $|p_B(n) - m^k| \leq d$. These results support Sun and Merca et al.'s conjectures, as $p_B(n) \to p(n)$ when $B \to +\infty$. To prove this, we reduce the problem to Siegel's Theorem, which guarantees the finiteness of integral points on curves with genus ≥ 1.

Keywords: partition function, perfect powers.
MSC: 11P82, 05A17, 05A20.

1 Introduction and statement of results

The distribution of perfect powers is a classic topic in number theory. A milestone is Catalan's conjecture (1844), proven by Mihăilescu in 2002 [5], which asserts that the Diophantine equation

$$x^a - y^b = 1 \qquad (x, y, a, b > 1)$$

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only has the single solution $(x, y, a, b) = (3, 2, 2, 3)$, and so the only consecutive perfect powers are 8 and 9. Building on this classical narrative, attention has shifted from perfect powers to their relationship with number theoretic functions. Sun [6, 7] proposed that partition numbers $p(n)$ where $n > 1$, are never perfect-powers. This has led to further developments by Merca et al. [4].

We recall these conjectures. Throughout, $p(n)$ denotes the number of (unrestricted) integer partitions of n, where $p(0) = 1$. The generating function for this function is given by the Euler product

$$P(q) = \sum_{n=0}^{\infty} p(n)q^n = \prod_{n=1}^{\infty} \frac{1}{1-q^n}. \tag{1}$$

An integer is a *perfect power* if it has the form m^k with integers $m \geq 2$ and $k \geq 2$. For a fixed exponent $k \geq 2$, we define

$$\Delta_k(n) \ := \ \min_{m \geq 0} |\, p(n) - m^k \,| \tag{2}$$

for the distance from $p(n)$ to the nearest kth power. The works of Sun [6, 7] and Merca et al. [4] concern the following conjecture.

Conjecture (Sun, Merca et al.)**.** *The partition function $p(n)$ repels perfect-powers.*

1. *(No perfect-powers) For every $n \geq 2$ and $k \geq 2$, $p(n) \neq m^k$ for all integers $m \geq 2$.*
2. *(Perfect-power Repulsion) For $k \geq 2$ and $d \geq 0$, at most finitely many n satisfy $\Delta_k(n) \leq d$.*

Remark 1. *We note that these questions are similar in flavor to the work of Tengely and Ulas [8] on equal values of special partition functions that have Diophantine interpretations.*

We prove exact analogs of the conjecture for the sequence of partition functions $p_B(n)$ that converges to $p(n)$. Specifically, for $B \geq 2$, we consider the truncated Euler products

$$P_B(q) = \sum_{n \geq 0} p_B(n)\, q^n \ = \ \prod_{m=1}^{B} \frac{1}{1-q^m}. \tag{3}$$

One sees that $p_B(n)$ is the number of partitions of n whose largest part is at most B. Moreover, we also have that $p_B(n)$ is the number of partitions of n

with at most B parts by considering the usual Ferrers board conjugation. Of course, we have $\lim_{B\to+\infty} p_B(n) = p(n)$. In analogy with $\Delta_k(n)$, we define

$$\Delta_k^{(B)}(n) \;:=\; \min_{m\geq 0}\big|\, p_B(n) - m^k \,\big|, \tag{4}$$

which is the distance between $p_B(n)$ and the nearest kth power. We prove the following theorem that confirms perfect-power repulsion for these functions.

Theorem 1. *If $B \geq 4$ and $k \geq 3$, with $k \nmid (B-1)$, then the following are true.*

1. *There are at most finitely many n for which $p_B(n) = m^k$.*
2. *If $d \geq 0$, then we have*

$$\#\{\, n \geq 1 : \; \Delta_k^{(B)}(n) \leq d \,\} < +\infty.$$

Example 1. *The partition function $p_2(n)$ does not repel perfect-powers. To see this, we note that*

$$\sum_{n\geq 0} p_2(n)\, q^n \;=\; \frac{1}{(1-q)(1-q^2)} \;=\; \Big(\sum_{a\geq 0} q^a\Big)\Big(\sum_{b\geq 0} q^{2b}\Big).$$

Taking the Cauchy product gives

$$\frac{1}{(1-q)(1-q^2)} = \sum_{n\geq 0} \Big(\#\{(a,b) \in \mathbb{Z}_{\geq 0}^2 : \; a + 2b = n\}\Big)\, q^n.$$

For a fixed n, writing $n = a + 2b$ with $a, b \geq 0$ is the same as choosing $b = 0, 1, \ldots, \lfloor n/2 \rfloor$, and then setting $a = n - 2b$. Therefore, we have that

$$p_2(n) = \lfloor n/2 \rfloor + 1 \qquad (n \geq 0).$$

In particular, for any integer $m \geq 1$ and any $k \geq 2$, if we take $n := 2(m^k - 1)$, then

$$p_2(n) = \Big\lfloor \frac{2(m^k-1)}{2} \Big\rfloor + 1 = (m^k - 1) + 1 = m^k.$$

Thus, $p_2(n)$ takes every perfect kth power.

Example 2. *If $B = 3$, then we have*

$$P_3(q) = \frac{1}{(1-q)(1-q^2)(1-q^3)} = \sum_{n \geq 0} p_3(n) q^n,$$

Along each residue class $n \equiv r \pmod 6$, we have

$$p_3(6t + r) = Q_r(t) \quad (t \in \mathbb{Z}_{\geq 0}),$$

where the six quadratic polynomials $Q_r \in \mathbb{Z}[t]$ are

$$\begin{aligned} &Q_0(t) = 3t^2 + 3t + 1, \quad &&Q_1(t) = 3t^2 + 4t + 1, \quad &&Q_2(t) = 3t^2 + 5t + 2, \\ &Q_3(t) = 3t^2 + 6t + 3, \quad &&Q_4(t) = 3t^2 + 7t + 4, \quad &&Q_5(t) = 3t^2 + 8t + 5. \end{aligned}$$

For $r \in \{0, 1, 4, 5\}$, there are infinitely many $t \geq 0$ such that $Q_r(t)$ is a perfect square. Consequently, $p_3(n)$ is a square for infinitely many n in each of the residue classes $r \in \{0, 1, 4, 5\} \pmod 6$. Each diophantine equation $Q_r(t) = m^2$ reduces to a Pell-type equation in (x, m) with discriminant 12 after completing the square in t.

(Case $r = 0$) *From $3t^2 + 3t + 1 = m^2$,*

$$(6t + 3)^2 - 12m^2 = -3.$$

This is the negative Pell equation $x^2 - 12m^2 = -3$ (with $x = 6t + 3$), which has infinitely many integer solutions. For instance, starting with $(x, m) = (3, 1)$ and multiplying by the fundamental unit $7 + 2\sqrt{12}$ yields an infinite family. The first few t are $0, 7, 104, 1455, \ldots$, giving

$$p_3(6t) \in \{1, 13^2, 181^2, 2521^2, \ldots\}.$$

(Case $r = 1$) *From $3t^2 + 4t + 1 = m^2$,*

$$(6t + 4)^2 - 12m^2 = 4.$$

The Pell-type equation $x^2 - 12m^2 = 4$ has infinitely many solutions; e.g. $t = 0, 8, 120, 1680, \ldots$ produce $p_3(6t + 1) = 1, 15^2, 209^2, 2911^2, \ldots$.

(Case $r = 4$) *From $3t^2 + 7t + 4 = m^2$,*

$$(6t + 7)^2 - 12m^2 = 1,$$

the (positive) Pell equation with fundamental solution $(x, m) = (7, 2)$, hence infinitely many solutions. We obtain $t = 0, 15, 224, \ldots$ and $p_3(6t + 4) = 2^2, 28^2, 390^2, \ldots$.

(Case $r = 5$) *From* $3t^2 + 8t + 5 = m^2$,

$$(6t + 8)^2 - 12m^2 = 4,$$

again yielding infinitely many solutions; e.g. $t = 1, 31, 449, \ldots$ *and* $p_3(6t + 5) = 4^2, 56^2, 780^2, \ldots$.

In each case the corresponding Pell or Pell-type equation has infinitely many solutions because the unit group of $\mathbb{Z}[\sqrt{12}]$ *is infinite; iterating by the fundamental unit* $7 + 2\sqrt{12}$ *produces an infinite family. Therefore* $p_3(n)$ *assumes square values infinitely often in the residue classes* $n \equiv 0, 1, 4, 5 \pmod 6$.

To prove Theorem 1, we use the fact that the partition functions $p_B(n)$ are quasipolynomial for large n, meaning each behaves like fixed polynomials along specific arithmetic progressions. This regularity allows us to reduce the conjectures to curves with genus ≥ 1, which then allows us to appeal to Siegel's Theorem. Summing over these progressions implies Theorem 1.

2 Nuts and bolts and the proof of Theorem 1

This section gathers the necessary structural inputs for our partition function perfect-power repulsion theorem and then demonstrates Theorem 1. Subsection 2.1 records the quasipolynomial description of $p_B(n)$ on residue classes, its growth, discrete spacing, and includes two Diophantine lemmas that manage the proximity to perfect kth powers. Subsection 2.2 combines these tools to prove Theorem 1.

2.1 Nuts and bolts

We start with the quasipolynomial (QP) structure for the counting function $p_B(n)$ and the resulting growth and spacing on arithmetic progressions. Throughout, we assume that $B \geq 4$ and $k \geq 3$ are fixed integers, where $k \nmid (B - 1)$.

Lemma 1 (QP structure, degree, and spacing)**.** *If we let* $L := \mathrm{lcm}(1, 2, \ldots, B)$, *then there are polynomials* $Q_0, \ldots, Q_{L-1} \in \mathbb{Q}[x]$ *of the same degree* $B - 1$ *with positive leading coefficients such that for all* n *we have*

$$p_B(Ln + r) = Q_r(n) \qquad (0 \leq r < L).$$

Moreover, for each r, one has

$$\begin{aligned} Q_r(n) &= \alpha_r\, n^{B-1} + O(n^{B-2}) \qquad (\alpha_r > 0),\\ Q_r(n+1) - Q_r(n) &= (B-1)\alpha_r\, n^{B-2} + O(n^{B-3}). \end{aligned}$$

Proof. Consider the generating function (3)

$$P_B(q) = \sum_{n\ge 0} p_B(n) q^n = \prod_{m=1}^{B} \frac{1}{1-q^m},$$

and let $L := \operatorname{lcm}(1,2,\dots,B)$. All of the poles of $P_B(q)$ lie at the Lth roots of unity ζ. The pole at $\zeta = 1$ has order B, and every other pole has order at most $B-1$. Therefore, we can write the (finite) partial fraction decomposition

$$P_B(q) = \sum_{\zeta^L=1} \sum_{j=1}^{e(\zeta)} \frac{R_{\zeta,j}(q)}{(1-\zeta q)^j},$$

where $R_{\zeta,j}(q)$ are polynomials and $e(\zeta) \le B$, with $e(1) = B$. Extracting coefficients via

$$[q^N](1-\zeta q)^{-j} = \zeta^N \binom{N+j-1}{j-1},$$

we obtain

$$p_B(N) = \sum_{\zeta^L=1} \sum_{j=1}^{e(\zeta)} \zeta^N P_{\zeta,j}(N),$$

where each $P_{\zeta,j} \in \mathbb{Q}[N]$ has degree at most $j-1$. Fixing a residue class $N \equiv r \pmod{L}$, then we have that each $\zeta^N = \zeta^r$ is constant, and we conclude that $p_B(Ln+r)$ agrees (for all $n \ge 0$) with a polynomial $Q_r(n) \in \mathbb{Q}[n]$ of degree at most $B-1$.

The degree is exactly $B-1$ with positive leading coefficient, because the only term contributing in top degree is the pole at $\zeta = 1$ and $j = B$. Near $q = 1$, we have

$$\prod_{m=1}^{B} \frac{1}{1-q^m} \sim \frac{1}{\prod_{m=1}^{B} m} \cdot (1-q)^{-B} \qquad (q \to 1),$$

so $[q^N](1-q)^{-B} = \binom{N+B-1}{B-1} \sim \dfrac{N^{B-1}}{(B-1)!}$. Therefore, the leading coefficient equals

$$\alpha_r = \frac{L^{B-1}}{B!\,(B-1)!} > 0.$$

Finally, if $Q_r(n) = \alpha_r n^{B-1} + O(n^{B-2})$, then the discrete difference satisfies

$$Q_r(n+1) - Q_r(n) = (B-1)\alpha_r\, n^{B-2} + O(n^{B-3}),$$

by a one-term binomial expansion. This proves the claims. $\square$

The previous lemma is crucial for the proof of Theorem 1. The next little lemma establishes that the polynomials Q_r are not simple shifts of perfect powers.

Lemma 2. *Let $Q(x) \in \mathbb{Q}[x]$ be a polynomial of degree $d \geq 2$ and fix an integer $k \geq 2$. Define*

$$T_k(Q) \ := \ \Big\{ t \in \mathbb{Q} : \ \exists\, a \in \mathbb{Q}^\times, \ R(x) \in \mathbb{Q}[x] \text{ with } \deg R \geq 1 \\ \text{and } Q(x) - t = a\, R(x)^k \Big\}.$$

Then $T_k(Q)$ is finite. In fact, we have that

$$T_k(Q) \ \subseteq \ \{\, Q(\xi) \ : \ \xi \in \overline{\mathbb{Q}} \ \text{ and } \ Q'(\xi) = 0 \,\},$$

and so $|T_k(Q)| \leq d-1$.

Proof. Suppose $Q(x) - t = a\, R(x)^k$ with $a \in \mathbb{Q}^\times$, $\deg R \geq 1$, and $k \geq 2$. Differentiating gives

$$Q'(x) \ = \ a\, k\, R(x)^{k-1} R'(x).$$

Hence $Q(x) - t$ and $Q'(x)$ have the common nonconstant factor $R(x)^{k-1}$, so $Q(x) - t$ has a multiple root. Thus, there is $\xi \in \overline{\mathbb{Q}}$ with $Q(\xi) = t$ and $Q'(\xi) = 0$ (i.e. t is a critical value of Q). Since Q' has degree $d-1$, there are at most $d-1$ such values. $\square$

Corollary 1. *For any fixed $X \geq 0$, we have*

$$|T_k(Q) \ \cap \ \{\, t \in \mathbb{Q} : \ |t| \leq X \,\}| < +\infty.$$

To prove Theorem 1, we require the following notions about polynomials $Q \in \mathbb{Z}[x]$. We say Q is *generic (relative to $k \geq 2$)* if $Q(x) \neq a\, R(x)^k$ for every $a \in \mathbb{Q}^\times$ and $R \in \mathbb{Q}[x]$. Otherwise, we say that Q is *power-type (relative to k)*. The next lemma is a Diophantine inputs. The first gives finiteness of perfect and near-perfect kth powers along any progression whose quasipolynomial piece is generic. The second lemma gives a pointwise lower bound in the power-type case.

Lemma 3. *Let* $Q(x) \in \mathbb{Q}[x]$ *have degree* $d \geq 2$*, let* $k \geq 3$ *be an integer, and fix* $D \in \mathbb{Z}_{\geq 0}$*. Assume that for every* $t \in \mathbb{Q}$ *with* $|t| \leq D$ *the polynomial* $Q(x) - t$ *is not of the form* $a \cdot R(x)^k$ *with* $a \in \mathbb{Q}^\times$ *and nonconstant* $R \in \mathbb{Q}[x]$*. Then we have*

$$\#\{(n,m) \in \mathbb{Z}_{\geq 0} \times \mathbb{Z} : \ |Q(n) - m^k| \leq D\} < \infty.$$

Moreover, except possibly in the pair $(d,k) = (2,2)$*, this set is bounded in terms of* Q, k, D*.*

Proof. Write $Q = \frac{1}{M} Q_e$ with $M \in \mathbb{Z}_{\geq 1}$ and $Q_e \in \mathbb{Z}[x]$ primitive. Then

$$|Q(n) - m^k| \leq D \quad \Longleftrightarrow \quad \left| Q_e(n) - M\, m^k \right| \leq MD.$$

Hence, it suffices to prove finiteness, for each fixed integer t with $|t| \leq MD$, of the Diophantine equation

$$M\, m^k \ = \ Q_e(n) - t. \tag{5}$$

Fix such a t. Factor M as $M = u^k\, b_0$ with $u \in \mathbb{Z}_{\geq 1}$ and $b_0 \in \mathbb{Z}_{\geq 1}$ k-th-power-free. Then every integer solution (n,m) to (5) yields an *integral* solution $(X,Y) = (n,\, u\, m)$ of the superelliptic equation

$$b_0\, Y^k \ = \ Q_e(X) - t. \tag{6}$$

Conversely, any integral solution (X,Y) to (6) with $u \mid Y$ corresponds to a solution $(n,m) = (X,\, Y/u)$ of (5). Therefore, the set of solutions to (5) is finite if and only if the set of integral points $(X,Y) \in \mathbb{Z}^2$ on (6) is finite.

Set $d = \deg Q_e = \deg Q \geq 2$ and, for each fixed $t \in \mathbb{Z}$, define

$$r_t \ := \ \#\{\, x \in \overline{\mathbb{Q}} : \ Q_e(x) = t\,\}$$

(counted *without* multiplicity). Note that the set of t with $r_t < d$ is finite (the critical values of Q_e).

Case $r_t = 1$. If $Q_e(X) - t = c\,(X-a)^d$ with $c \in \mathbb{Z} \setminus \{0\}$ and $d = \deg Q_e \geq 2$, then (6) becomes

$$b_0\, Y^k \ = \ c\,(X-a)^d.$$

Write $u = \gcd(k,d)$ and $k = uk_1$, $d = ud_1$ with $\gcd(k_1, d_1) = 1$. If $k \mid d$ and c/b_0 is a kth power, then $Q_e(X) - t$ would be a constant times a kth power, contrary to the hypothesis. Otherwise, by standard finiteness results for Thue/Thue-Mahler type equations (for example, see Chapter 9 of [1]), this Diophantine equation has only finitely many integer solutions.

Case $r_t \geq 3$. Then the smooth projective model of (6) has genus $g \geq 1$ (see, e.g., the standard genus formula for superelliptic curves in §IV of [2]). By Siegel's theorem (see Chapter 8 of [3]), (6) has only finitely many integral points.

Case $r_t = 2$. Write $Q_e(X) - t = c(X-a)^2(X-b)$ with $a \neq b$. If $k \geq 3$, (6) is a Thue/Thue-Mahler-type equation, and standard results (for example, see Chapter 9 of [1]) imply finiteness of integral solutions.

Case $(d,k) = (2,2)$. This is the classical conic case, which may have infinitely many integral points (Pell-type).

When $g \geq 1$, Siegel's theorem on integral points (for example, see Chapter 8 of [3]) implies that the set of integral solutions $(X,Y) \in \mathbb{Z}^2$ to (6) is finite. Because there are only finitely many t with $|t| \leq MD$, taking the union over these t shows that there are only finitely many (n,m) with $|Q(n) - m^k| \leq D$. $\square$

2.2 Proof of Theorem 1

By Lemma 1, there is a period L and polynomials $Q_0, \ldots, Q_{L-1} \in \mathbb{Q}[X]$ of degree $B-1$ with positive leading coefficients such that $p_B(Ln+r) = Q_r(n)$ for $0 \leq r < L$. It suffices to prove that for each fixed residue class r and each fixed $d \geq 0$, the set

$$\mathcal{S}_{r,d} \;:=\; \left\{(n,m) \in \mathbb{Z}_{\geq 0} \times \mathbb{Z} : \; |Q_r(n) - m^k| \leq d\right\}$$

is finite; then a finite union over r gives (ii), and taking $d = 0$ gives (i).

Fix r and $d \geq 0$. For each integer t with $|t| \leq d$ consider the Diophantine equation

$$Q_r(n) - m^k \;=\; t, \qquad n \in \mathbb{Z}_{\geq 0},\; m \in \mathbb{Z}. \tag{7}$$

Let

$$T_r(d) \;:=\; \Big\{ t \in \mathbb{Z} : \; |t| \leq d \;\text{ and }\; \exists\, a \in \mathbb{Q}^\times,\, R \in \mathbb{Q}[X] \;\text{ nonconstant } \text{ with } Q_r(X) - t = a\, R(X)^k \Big\}.$$

By Lemma 2 and Corollary 1, $T_r(d)$ is finite.

Case of Non-exceptional shifts. For every $t \in \{-d, -d+1, \ldots, d\} \setminus T_r(d)$, the polynomial $Q_r(X) - t$ is *not* a constant times a kth power. Choose $b_0 \in \mathbb{Z}_{>0}$ so that $Q_e(X) := b_0 Q_r(X) \in \mathbb{Z}[X]$. Then any solution of (7) gives an integer solution of

$$b_0\, m^k \;=\; Q_e(n) - b_0 t.$$

By Lemma 3 (applied with the fixed polynomial $Q_e(X) - b_0 t$), there are only finitely many such integer solutions. Hence, $\#\{(n,m) : Q_r(n) - m^k = t\} < \infty$ for all non-exceptional t with $|t| \leq d$.

Case of exceptional shifts. Now fix $t \in T_r(d)$. As above, pass to $Q_e(X) = b_0 Q_r(X) \in \mathbb{Z}[X]$ and consider

$$b_0\, m^k \;=\; Q_e(n) - b_0 t.$$

Let r_t denote the number of distinct roots of $Q_e(X) - b_0 t$ in $\overline{\mathbb{Q}}$.

- If $r_t \geq 3$, then the affine curve $b_0 Y^k = Q_e(X) - b_0 t$ has the genus ≥ 1, so by Siegel's theorem (for example, see Chapter 8 of [3]) there are only finitely many integer points.
- If $r_t = 2$, standard Thue-Mahler finiteness applies (for example, see Chapter 9 of [1]) so there are only finitely many integer solutions.
- If $r_t = 1$, then $Q_e(X) - b_0 t = c\,(X-a)^d$ for some $a, c \in \mathbb{Z}$, $c \neq 0$, with $d = \deg Q_e = B - 1 \geq 3$. Dividing by b_0 gives

$$Q_r(X) - t \;=\; \frac{c}{b_0}\,(X-a)^d.$$

Since $k \nmid d = B - 1$ by hypothesis, the Diophantine equation $b_0\, m^k = c\,(n-a)^d$ is of Thue/Thue-Mahler type with coprime exponents and therefore has only finitely many integer solutions (for example, see Chapter 9 of [1]). Therefore, the case $r_t = 1$ also yields at most finitely many solutions.

Combining the three subcases, for each $t \in T_r(d)$ the equation (7) has only finitely many integer solutions. Therefore, $\mathcal{S}_{r,d}$ is the finite union, over the finitely many $t \in \{-d, \ldots, d\}$, of finite sets. This proves that $\mathcal{S}_{r,d}$ is finite. As noted at the start, summing over r establishes (ii), and taking $d = 0$ gives (i). $\square$

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