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# ADVANCES ON PHASE DIAGRAMS FOR ROEGENIAN ECONOMICS: THEORY AND EMPIRICAL VALIDATION*

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**Abstract**

This paper extends the foundational work on Roegenian Economics by developing a rigorous mathematical framework for the thermodynamic-economic correspondence initiated by Georgescu-Roegen. We provide an enhanced dictionary between thermodynamic and economic state variables. The main contributions include: (i) a formal proof of the existence and uniqueness of the economic triple point; (ii) characterization theorems for critical phenomena; (iii) derivation of Maxwell-type relations for economic potentials; and (iv) stability analysis via Legendre transforms. We present empirical validation using World Bank Governance Indicators and IMF inflation data for 2022-2023, demonstrating that the phase diagram captures the relationship between institutional stability and price dynamics. The analysis identifies candidates for triple point behavior (Venezuela, Zimbabwe) and supercritical regimes (Switzerland, Singapore).

**Keywords:** Roegenian economics, econophysics, phase diagrams, Gibbs-Pfaff equation, thermodynamic-economic correspondence, triple point.

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# 1 Introduction

The interdisciplinary dialogue between thermodynamics and economics has a distinguished intellectual history, tracing back to the pioneering work of Nicholas Georgescu-Roegen [3], who recognized that economic processes are fundamentally entropic in nature. Unlike the Newtonian mechanical paradigm that dominated classical economics, thermodynamics provides a framework that naturally incorporates irreversibility, qualitative change, and the arrow of time features essential for understanding real economic phenomena.

The present work builds upon and significantly extends the systematic development of what we term *Roegenian Economics*: an economic theory structured via formal correspondence with thermodynamics [12,14,18]. Previous contributions established the fundamental dictionary and explored the geometric structure of economic state spaces [13, 16, 17]. Here, we deepen the mathematical foundations by proving new theorems on phase equilibria, developing the economic interpretation more thoroughly, and for the first time in this research program providing empirical validation using real-world macroeconomic data.

The thermodynamic approach to economics is not merely a formal analogy but reflects deep structural similarities in how complex systems organize and transform. Just as a thermodynamic system can exist in distinct phases (solid, liquid, gas) with well-defined transitions between them, an economic system exhibits qualitatively different operational regimes. The identification of these regimes and their transition boundaries constitutes a primary objective of macroeconomic analysis.

Our phase diagram shows how political stability ($I$) and price level ($P$) jointly determine the macroeconomic state. Political stability reflects institutional quality; the price level captures monetary conditions. The three phases inflation, monetary liquidity, and income correspond to observable macroeconomic regimes.

The existence of a *triple point* where all three phases coexist and a *critical point* beyond which phase distinctions blu provides powerful conceptual tools for understanding macroeconomic dynamics. Historical episodes such as the Weimar Republic hyperinflation (1923) and recent crises in Venezuela and Zimbabwe exhibit characteristics consistent with triple point behavior.

The paper proceeds as follows. Section 2 presents the extended thermodynamic-economic dictionary with detailed economic justification for each correspondence. Section 3 develops the mathematical theory of the Gibbs-Pfaff economic equation, including the contact geometry framework and

Maxwell relations. Section 4 establishes rigorous results on phase equilibria, including existence and uniqueness theorems for the triple point. Section 5 analyzes critical phenomena and derives scaling relations. Section 6 presents the empirical validation using World Bank and IMF data. Section 7 discusses economic applications and policy implications. Section 8 concludes.

## 2 Extended thermodynamic-economic dictionary

The construction of Roegenian Economics proceeds via a systematic correspondence between thermodynamic and economic concepts. This section presents an extended and economically grounded version of this dictionary, with detailed justification for each identification.

### 2.1 State variables and their economic meaning

Table 1: Extended Thermodynamic-Economic Dictionary: State Variables

| **Thermodynamics** | **Economics** | **Economic Justification** |
|---|---|---|
| $U$ = Internal energy | $G$ = Growth potential | Latent capacity for transformation |
| $T$ = Temperature | $I$ = Political stability | Intensity of systemic activity |
| $S$ = Entropy | $E$ = Economic entropy | Irreversibility, resource degradation |
| $P$ = Pressure | $P$ = Price level (inflation) | Purchasing power compression |
| $V$ = Volume | $Q$ = Volume, quality | Economic capacity and output |
| $M$ = Total energy | $Y$ = National income | Aggregate economic flow |
| $\mu_k$ = Chemical pot. | $\nu_k$ = Sectoral potential | Marginal contribution by sector |
| $N_k$ = Number of moles | $N_k$ = Economic moles | Standardized value units |
| $W$ = Mechanical work | $W$ = Wealth of system | Accumulated productive capacity |
| $Q_{\text{heat}}$ = Heat | $q$ = Stock market flow | Financial energy transfer |

### 2.2 Temperature and internal political stability

In thermodynamics, temperature ($T$) measures the average kinetic energy of particles and determines the direction of heat flow between systems. More fundamentally, temperature is the intensive variable conjugate to entropy—

it governs the spontaneous direction of energy transfer and determines equilibrium conditions.

In economic systems, *internal political stability* ($I$) plays a precisely analogous role. We define $I$ as a composite measure reflecting: (i) institutional quality: the effectiveness of governance structures, rule of law, and regulatory frameworks; (ii) policy predictability: the degree to which economic agents can anticipate government actions; (iii) property rights security: the protection of investments and contractual obligations; (iv) social cohesion: the absence of politically-motivated violence or terrorism.

The World Bank's Worldwide Governance Indicators (WGI) provide an operational measure of $I$ through the "Political Stability and Absence of Violence/Terrorism" index, which ranges from approximately $-2.5$ (highly unstable) to $+2.5$ (highly stable).

The correspondence $T \leftrightarrow I$ is economically meaningful because both are *intensive* variables that equilibrate across connected systems. Just as two bodies in thermal contact reach the same temperature, economically integrated regions tend toward similar institutional quality over time through competitive pressures, treaty obligations, and demonstration effects. Both govern the *direction of spontaneous flow*: heat flows from high to low temperature; capital and skilled labor flow from low to high political stability. Both determine *system accessibility*: high temperature makes more microstates accessible to a thermodynamic system; high political stability makes more economic strategies viable for firms and households. The *third law analog* holds: as $I \to 0$ (complete institutional collapse), economic entropy approaches zero not because disorder vanishes, but because the system becomes frozen incapable of any organized economic activity.

**Remark 1** (Measurement of Political Stability)**.** *In empirical applications, $I$ can be measured using several available indices: the World Bank's WGI Political Stability indicator (used in Section 6), the Economist Intelligence Unit's Political Instability Index, or composite measures from Political Risk Services. For dimensional consistency in the Gibbs-Pfaff equation, we normalize $I$ to a percentage scale (0–100) based on percentile rank.*

## 2.3 Pressure and price level

Thermodynamic pressure ($P$) represents the force per unit area exerted by a system on its boundaries the intensive variable conjugate to volume. It measures how strongly the system "pushes" against constraints on its expansion.

The economic analogue is the *price level* ($P$), which represents the purchasing power pressure exerted on economic agents. This identification captures several parallel features: (i) compression effect: high pressure compresses volume; high price levels compress real purchasing power and living standards; (ii) expansion tendency: just as high pressure systems tend to expand into low pressure regions, high-price economies exert competitive pressure on low-price economies through trade and labor migration; (iii) equation of state: the relationship $P = P(Q, I)$ in economics how price levels depend on output and stability mirrors the thermodynamic equation of state $P = P(V, T)$; (iv) work conjugacy: thermodynamic work is $\mathrm{d}W = P\mathrm{d}V$; economic wealth creation involves $\mathrm{d}W = P\mathrm{d}Q$, the product of prices and quantities.

### 2.4 Entropy and economic entropy

The concept of entropy has the most profound implications when transferred to economics. Thermodynamic entropy ($S$) measures the number of microscopic configurations compatible with a macroscopic state, the degree of disorder or "spread" of energy across available states, and the irreversibility of natural processes (second law).

Georgescu-Roegen's fundamental insight was that economic processes necessarily increase entropy: "Matter-energy enters the economic process in a state of low entropy and comes out in a state of high entropy" [3]. Economic entropy ($E$) therefore measures: (i) resource degradation: the transformation of concentrated, high-quality resources (low entropy) into dispersed, low-quality waste (high entropy); (ii) process irreversibility: unlike the reversible exchanges of neoclassical equilibrium theory, real economic processes are fundamentally irreversible commodities are consumed, capital depreciates, and knowledge becomes obsolete; (iii) information dispersion: market aggregation destroys information about individual preferences and endowments, increasing the effective entropy of the economic system; (iv) structural complexity: as economies develop, they tend toward greater structural complexity and specialization, which can be viewed as increasing configurational entropy.

**Remark 2** (The Second Law in Economics)**.** *The economic second law states that* $\mathrm{d}q \leq I\mathrm{d}E$*, where* $q$ *represents financial market activity. In reversible (ideal) economic processes,* $\mathrm{d}q = I\mathrm{d}E$*; in irreversible (real) processes,* $\mathrm{d}q < I\mathrm{d}E$*. This implies that financial activity generates less "useful" economic transformation than the entropy increase would suggest there are* *always transaction costs, information asymmetries, and coordination failures.*

### 2.5 The economic phases

The three phases in our economic phase diagram correspond to qualitatively distinct macroeconomic regimes:

**Inflation Phase.** The inflation phase is characterized by sustained increase in the general price level eroding purchasing power, typically associated with low political stability (weak institutional constraints on money creation), high velocity of money as agents flee from depreciating currency, and redistribution from creditors to debtors, from fixed-income earners to asset holders. Historical examples include Weimar Germany 1923, Zimbabwe 2008, and Venezuela 2018–present.

**Monetary Policy of Liquidity Phase.** This intermediate phase is characterized by active central bank intervention in credit markets, effective transmission of monetary policy to real economic variables, price stability as a policy target (neither deflation nor high inflation), and liquidity provision as the primary tool of economic stabilization. Examples include most developed economies under inflation-targeting regimes.

**Income Phase.** The income phase is characterized by stable income growth and accumulation, high political stability enabling long-term investment and planning, low and stable inflation, and effective property rights and contract enforcement. Examples include Switzerland, Singapore, and Nordic countries.

## 3 The Gibbs-Pfaff economic equation

### 3.1 Fundamental formulation

The core mathematical object in Roegenian Economics is the Gibbs-Pfaff equation, which encodes the constraints on economic state transitions.

**Definition 1** (Gibbs-Pfaff Economic Equation)**.** *The* fundamental economic equation *is the Pfaffian differential equation:*

$$\omega = \mathrm{d}G - I\mathrm{d}E + P\mathrm{d}Q = 0, \tag{1}$$

*where* $(G, I, E, P, Q) \in \mathbb{R}^5$ *are the economic state variables.*

This equation combines the first and second laws of economics: the first law $\mathrm{d}W = P\mathrm{d}Q$ (elementary wealth creation through production) and the second law $\mathrm{d}q = I\mathrm{d}E$ (reversible) or $\mathrm{d}q < I\mathrm{d}E$ (irreversible financial activity).

**Definition 2** (Third Law of Economics)**.** *The economic analogue of the third law of thermodynamics states:*

$$\lim_{I \to 0} E = 0. \tag{2}$$

This limit has a precise economic interpretation: as internal political stability approaches zero (complete institutional collapse), the economic system loses its capacity for organized activity.

### 3.2 Contact geometry framework

The geometric structure underlying the Gibbs-Pfaff equation is that of a *contact manifold*, which provides powerful tools for analyzing economic dynamics.

**Proposition 1** (Contact Structure)**.** *The 1-form* $\omega = \mathrm{d}G - I\mathrm{d}E + P\mathrm{d}Q$ *is a contact form on* $\mathbb{R}^5 = \{(G, I, E, P, Q)\}$*, i.e.,* $\omega \wedge (\mathrm{d}\omega)^2 \neq 0$*.*

*Proof.* We compute the exterior derivative:

$$\mathrm{d}\omega = -\mathrm{d}I \wedge \mathrm{d}E + \mathrm{d}P \wedge \mathrm{d}Q.$$

The square of this 2-form is:

$$\begin{aligned}(\mathrm{d}\omega)^2 &= (-\mathrm{d}I \wedge \mathrm{d}E + \mathrm{d}P \wedge \mathrm{d}Q) \wedge (-\mathrm{d}I \wedge \mathrm{d}E + \mathrm{d}P \wedge \mathrm{d}Q) \\ &= -2\,\mathrm{d}I \wedge \mathrm{d}E \wedge \mathrm{d}P \wedge \mathrm{d}Q.\end{aligned}$$

Computing the wedge product with $\omega$:

$$\begin{aligned}\omega \wedge (\mathrm{d}\omega)^2 &= (\mathrm{d}G - I\mathrm{d}E + P\mathrm{d}Q) \wedge (-2\,\mathrm{d}I \wedge \mathrm{d}E \wedge \mathrm{d}P \wedge \mathrm{d}Q) \\ &= -2\,\mathrm{d}G \wedge \mathrm{d}I \wedge \mathrm{d}E \wedge \mathrm{d}P \wedge \mathrm{d}Q \neq 0.\end{aligned}$$

Since this 5-form is non-vanishing, $\omega$ is a contact form. □

**Corollary 1.** *The economic state space* $(\mathbb{R}^5, \omega)$ *is a contact manifold of dimension 5, with contact distribution* $\ker(\omega)$ *of rank 4.*

### 3.3 Integral manifolds and economic systems

**Theorem 1** (Classification of Integral Manifolds)**.** *The integral manifolds of $\omega = 0$ are of dimension at most 2. Specifically:*

*(i)* ***Integral curves*** *(economic paths): One-parameter families*

$$(G(t), I(t), E(t), P(t), Q(t))$$

*satisfying $\dot{G} - I\dot{E} + P\dot{Q} = 0$. These represent time-evolution paths of economic systems.*

*(ii)* ***Integral surfaces*** *(simple economic systems): Two-parameter families satisfying*

$$\frac{\partial G}{\partial x} - I\frac{\partial E}{\partial x} + P\frac{\partial Q}{\partial x} = 0, \tag{3}$$

$$\frac{\partial G}{\partial y} - I\frac{\partial E}{\partial y} + P\frac{\partial Q}{\partial y} = 0. \tag{4}$$

*Proof.* By the Darboux theorem for contact forms, the contact distribution $\ker(\omega)$ is maximally non-integrable, meaning no integral manifold of dimension greater than $(5-1)/2 = 2$ exists. □

### 3.4 Maxwell relations for economics

The integrability conditions for integral surfaces yield the economic analogue of Maxwell relations.

**Theorem 2** (Economic Maxwell Relations)**.** *On any integral surface of the Gibbs-Pfaff equation, the following relation holds:*

$$\frac{\partial I}{\partial x}\frac{\partial E}{\partial y} - \frac{\partial E}{\partial x}\frac{\partial I}{\partial y} = \frac{\partial P}{\partial x}\frac{\partial Q}{\partial y} - \frac{\partial Q}{\partial x}\frac{\partial P}{\partial y}. \tag{5}$$

*Equivalently, in Jacobian notation:*

$$\frac{\partial(I,E)}{\partial(x,y)} = \frac{\partial(P,Q)}{\partial(x,y)}. \tag{6}$$

*Proof.* Differentiating (3) with respect to $y$ and (4) with respect to $x$:

$$\frac{\partial^2 G}{\partial x \partial y} - \frac{\partial I}{\partial y}\frac{\partial E}{\partial x} - I\frac{\partial^2 E}{\partial x \partial y} + \frac{\partial P}{\partial y}\frac{\partial Q}{\partial x} + P\frac{\partial^2 Q}{\partial x \partial y} = 0,$$

$$\frac{\partial^2 G}{\partial y \partial x} - \frac{\partial I}{\partial x}\frac{\partial E}{\partial y} - I\frac{\partial^2 E}{\partial y \partial x} + \frac{\partial P}{\partial x}\frac{\partial Q}{\partial y} + P\frac{\partial^2 Q}{\partial y \partial x} = 0.$$

Subtracting and using the symmetry of mixed partials yields (5). □

### 3.5 Economic potentials via Legendre transforms

Following the thermodynamic pattern, we define economic potentials through Legendre transformations.

**Definition 3** (Economic Potentials)**.** *The four fundamental economic potentials are:*

*(i)* ***Growth potential****: $G = G(E, Q)$ natural variables are entropy and quantity*

*(ii)* ***Free economic energy****: $F = G - IE$ natural variables are stability and quantity*

*(iii)* ***Wealth function*** *(Enthalpy): $H = G + PQ$ natural variables are entropy and price*

*(iv)* ***Economic potential*** *(Gibbs function): $\Phi = G - IE + PQ$ natural variables are stability and price*

**Theorem 3** (Potential Representations)**.** *Each economic potential provides a complete description of a simple economic system:*

*(i) From $G(E, Q)$:* $I = \frac{\partial G}{\partial E}, \quad P = -\frac{\partial G}{\partial Q}$

*(ii) From $F(I, Q)$:* $E = -\frac{\partial F}{\partial I}, \quad P = -\frac{\partial F}{\partial Q}$

*(iii) From $H(E, P)$:* $I = \frac{\partial H}{\partial E}, \quad Q = \frac{\partial H}{\partial P}$

*(iv) From $\Phi(I, P)$:* $E = -\frac{\partial \Phi}{\partial I}, \quad Q = \frac{\partial \Phi}{\partial P}$

## 4 Phase equilibria: triple point theory

### 4.1 Economic phase space

**Definition 4** (Economic Phases)**.** *In the Roegenian framework, an economic system can exist in three distinct* phases*:*

*(i)* ***Inflation phase*** *($\mathcal{I}$): Characterized by sustained price increases, typically at low political stability.*

*(ii)* ***Monetary policy of liquidity phase*** *($\mathcal{M}$): An intermediate regime where monetary interventions effectively modulate economic activity.*

*(iii)* ***Income phase*** *($\mathcal{Y}$): Characterized by stable income growth and accumulation.*

**Definition 5** (Phase Boundary)**.** *A* phase boundary *is a curve $\gamma : [0,1] \to \mathbb{R}^2$ in the $(I, P)$ plane along which two phases coexist in equilibrium.*

### 4.2 Coexistence conditions

**Theorem 4** (Coexistence Criteria)**.** *Two economic phases $\alpha$ and $\beta$ coexist in equilibrium if and only if:*

$$I_\alpha = I_\beta, \quad P_\alpha = P_\beta, \quad \Phi_\alpha(I, P) = \Phi_\beta(I, P), \tag{7}$$

*where $\Phi$ is the economic potential (Gibbs function).*

**Theorem 5** (Economic Clausius-Clapeyron Equation)**.** *Along a phase boundary between phases $\alpha$ and $\beta$, the slope satisfies:*

$$\frac{\mathrm{d}P}{\mathrm{d}I} = \frac{E_\beta - E_\alpha}{Q_\beta - Q_\alpha} = \frac{\Delta E}{\Delta Q}, \tag{8}$$

*where $\Delta E$ and $\Delta Q$ are the entropy and volume differences between phases.*

*Proof.* Along the coexistence curve, $\Phi_\alpha(I, P) = \Phi_\beta(I, P)$. Taking the total differential along the curve: $\mathrm{d}\Phi_\alpha = \mathrm{d}\Phi_\beta$. From Theorem 3(iv), $\mathrm{d}\Phi = -E\mathrm{d}I + Q\mathrm{d}P$, so: $-E_\alpha\mathrm{d}I + Q_\alpha\mathrm{d}P = -E_\beta\mathrm{d}I + Q_\beta\mathrm{d}P$. Rearranging yields (8). □

### 4.3 Existence and uniqueness of the triple point

**Theorem 6** (Existence of Triple Point)**.** *Under the following regularity conditions on the economic potentials:*

*(11)* $\Phi_\mathcal{I}, \Phi_\mathcal{M}, \Phi_\mathcal{Y} \in C^2(\mathbb{R}^2)$ *(smooth potentials),*

*(22) The phase boundaries are non-degenerate: $\nabla\Phi_\alpha \neq \nabla\Phi_\beta$ for $\alpha \neq \beta$,*

*(33) The potentials satisfy the* crossing condition*: $\Phi_\mathcal{I}(0,0) > \Phi_\mathcal{M}(0,0) > \Phi_\mathcal{Y}(0,0)$ and $\Phi_\mathcal{I}(I_0, P_0) < \Phi_\mathcal{M}(I_0, P_0) < \Phi_\mathcal{Y}(I_0, P_0)$ for some $(I_0, P_0) \in \mathbb{R}^2_{++}$,*

*there exists a point $(I_{tr}, P_{tr})$ where all three phases coexist.*

*Proof.* Define the functions measuring potential differences:

$$f_1(I,P) = \Phi_{\mathcal{I}}(I,P) - \Phi_{\mathcal{M}}(I,P),$$
$$f_2(I,P) = \Phi_{\mathcal{M}}(I,P) - \Phi_{\mathcal{Y}}(I,P).$$

By condition (A2), $\nabla f_1 \neq 0$ on $f_1^{-1}(0)$ and $\nabla f_2 \neq 0$ on $f_2^{-1}(0)$. By the implicit function theorem, $\gamma_{\mathcal{IM}} = f_1^{-1}(0)$ and $\gamma_{\mathcal{MY}} = f_2^{-1}(0)$ are smooth 1-dimensional curves.

By condition (A3), $f_1$ and $f_2$ change sign. By the intermediate value theorem, there exists $(I_{\mathrm{tr}}, P_{\mathrm{tr}})$ where $f_1 = f_2 = 0$, implying $\Phi_{\mathcal{I}} = \Phi_{\mathcal{M}} = \Phi_{\mathcal{Y}}$. □

**Theorem 7** (Uniqueness of Triple Point)**.** *Under conditions (A1)–(A3) and the additional transversality assumption:*

*(A4) The gradients $\nabla f_1$ and $\nabla f_2$ are linearly independent at every point where $f_1 = f_2 = 0$,*

*the triple point is locally unique.*

*Proof.* By condition (A4), $\det(DF) \neq 0$ at the triple point. By the inverse function theorem, $F^{-1}(0,0)$ is a single point in a neighborhood of $(I_{\mathrm{tr}}, P_{\mathrm{tr}})$. □

**Definition 6** (Triple Point)**.** *The unique point $(I_{tr}, P_{tr})$ satisfying*

$$\Phi_{\mathcal{I}}(I_{tr}, P_{tr}) = \Phi_{\mathcal{M}}(I_{tr}, P_{tr}) = \Phi_{\mathcal{Y}}(I_{tr}, P_{tr})$$

*is called the* economic triple point*. At this point, all three macroeconomic phases coexist in equilibrium.*

## 4.4 Gibbs phase rule for economics

**Theorem 8** (Economic Phase Rule)**.** *For a Roegenian economic system with $n$ components (economic sectors) and $\pi$ coexisting phases, the number of degrees of freedom is:*

$$f = n + 2 - \pi. \tag{9}$$

**Corollary 2** (Single-Component System)**.** *For a single-component economic system ($n = 1$): single phase ($\pi = 1$) gives $f = 2$; two phases ($\pi = 2$) gives $f = 1$; three phases ($\pi = 3$) gives $f = 0$.*

# 5 Critical phenomena in economic systems

**Definition 7** (Economic Critical Point)**.** *A point* $(I_c, P_c)$ *is a* critical point *if:*

*(i) It lies on a phase boundary (specifically, the* $\mathcal{M}$*-*$\mathcal{Y}$ *boundary).*

*(ii) For* $I > I_c$ *and* $P > P_c$*, the phases* $\mathcal{M}$ *and* $\mathcal{Y}$ *become indistinguishable.*

**Theorem 9** (Critical Point Conditions)**.** *At a critical point, the following conditions hold for the equation of state* $P = P(Q, I)$*:*

$$\left.\frac{\partial P}{\partial Q}\right|_{I_c} = 0, \quad \left.\frac{\partial^2 P}{\partial Q^2}\right|_{I_c} = 0. \tag{10}$$

## 5.1 Van der Waals equation for economics

**Definition 8** (Economic Van der Waals Equation)**.** *The* economic van der Waals equation *relates price level, volume, and stability:*

$$\left(P + \frac{a}{Q^2}\right)(Q - b) = RI, \tag{11}$$

*where* $a, b, R > 0$ *are economic parameters:* $a$ *measures "cohesive pressure" from market interactions,* $b$ *represents minimum irreducible economic volume (subsistence level), and* $R$ *is the economic gas constant.*

**Theorem 10** (Critical Parameters)**.** *For the economic van der Waals equation, the critical point parameters are:*

$$I_c = \frac{8a}{27Rb}, \quad P_c = \frac{a}{27b^2}, \quad Q_c = 3b. \tag{12}$$

**Theorem 11** (Law of Corresponding States)**.** *In reduced variables* $\pi = P/P_c$*,* $\phi = Q/Q_c$*,* $\iota = I/I_c$*, the van der Waals equation takes the universal form:*

$$\left(\pi + \frac{3}{\phi^2}\right)\left(\phi - \frac{1}{3}\right) = \frac{8\iota}{3}. \tag{13}$$

**Theorem 12** (Mean-Field Critical Exponents)**.** *For the economic van der Waals equation, the critical exponents are:*

$$\textit{Order parameter:} \quad Q_{\mathcal{Y}} - Q_{\mathcal{M}} \sim |I_c - I|^{1/2} \quad (\beta = 1/2) \tag{14}$$

$$\textit{Compressibility:} \quad \kappa_I \sim |I - I_c|^{-1} \quad (\gamma = 1) \tag{15}$$

$$\textit{Critical isotherm:} \quad |P - P_c| \sim |Q - Q_c|^3 \quad (\delta = 3) \tag{16}$$

# 6 Empirical validation

This section presents an empirical investigation of the Roegenian phase diagram using real-world macroeconomic data.

## 6.1 Data sources

We use two primary data sources: (i) **Political Stability Index**: World Bank Worldwide Governance Indicators (WGI), specifically the "Political Stability and Absence of Violence/Terrorism" index, ranging from approximately $-2.5$ (highly unstable) to $+2.5$ (highly stable), using 2023 data; (ii) **Inflation Rate**: IMF World Economic Outlook and World Bank data on consumer price inflation (annual percentage change), using 2022–2023 data.

## 6.2 Sample selection

We select a diverse sample of 20 countries spanning the full range of stability and inflation conditions:

Table 2: Sample Countries: Political Stability and Inflation Data (2022-2023)

| **Country** | **Stab. Index** | **Stab. %** | **Infl. (%)** |
|---|---|---|---|
| *High Stability, Low Inflation (Income Phase)* | | | |
| Switzerland | 1.25 | 95.8 | 2.8 |
| Singapore | 1.42 | 97.2 | 6.1 |
| Norway | 1.15 | 93.4 | 5.8 |
| New Zealand | 1.22 | 95.3 | 7.2 |
| Denmark | 0.86 | 85.4 | 7.7 |
| *Medium Stability, Moderate Inflation (Monetary Liquidity)* | | | |
| United States | 0.19 | 63.2 | 8.0 |
| Germany | 0.55 | 76.4 | 8.7 |
| France | 0.34 | 68.4 | 5.9 |
| Italy | 0.52 | 74.5 | 8.7 |
| Japan | 0.98 | 89.2 | 2.5 |
| *Lower Stability, Higher Inflation (Transitional)* | | | |
| Brazil | −0.32 | 42.0 | 9.3 |
| Mexico | −0.58 | 32.5 | 7.9 |
| South Africa | −0.18 | 47.2 | 7.0 |
| India | −0.72 | 27.8 | 6.7 |
| Turkey | −1.04 | 15.6 | 72.3 |
| *Low Stability, High Inflation (Inflation Phase)* | | | |
| Argentina | −0.08 | 51.4 | 72.4 |
| Venezuela | −1.28 | 8.5 | 189.8 |
| Zimbabwe | −1.04 | 15.6 | 285.0 |
| Sudan | −2.45 | 1.4 | 154.9 |
| Syria | −2.75 | 0.5 | 139.0 |

*Sources*: World Bank WGI (2023); IMF WEO (2022-2023).

### 6.3 Phase diagram construction

Figure 1 presents the empirical phase diagram with countries plotted according to their stability-inflation coordinates.

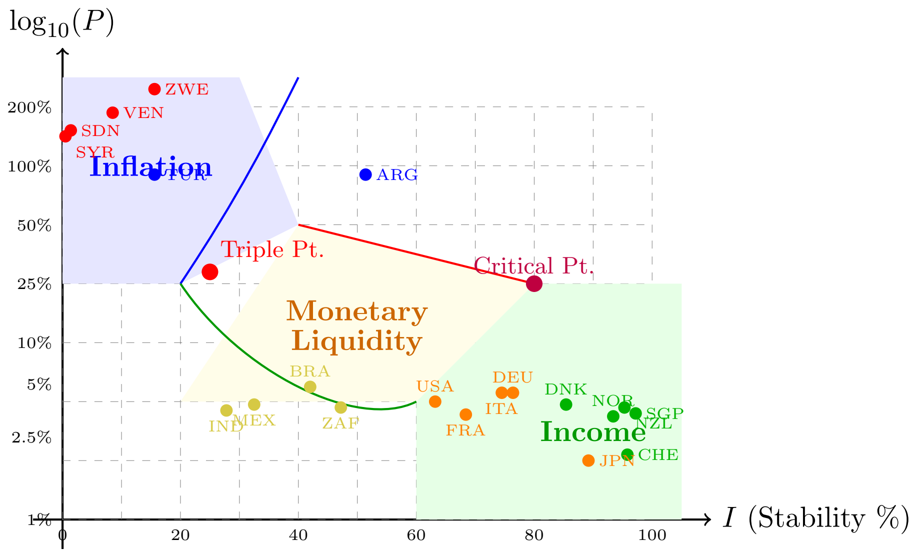


Figure 1: Empirical Phase Diagram: Political Stability vs. Inflation (2022–2023). The $x$-axis shows World Bank Political Stability percentile rank (0-100); the $y$-axis shows $\log_{10}$ of annual inflation rate. Country codes: CHE (Switzerland), SGP (Singapore), NOR (Norway), NZL (New Zealand), DNK (Denmark), USA (United States), DEU (Germany), FRA (France), ITA (Italy), JPN (Japan), BRA (Brazil), MEX (Mexico), ZAF (South Africa), IND (India), TUR (Turkey), ARG (Argentina), VEN (Venezuela), ZWE (Zimbabwe), SDN (Sudan), SYR (Syria).

### 6.4 Empirical findings

The data reveal patterns consistent with the theoretical phase diagram:

**Income Phase Identification.** Countries with high political stability (percentile $> 85$) and low inflation ($< 10\%$) cluster in the predicted "income phase" region: Switzerland (95.8%, 2.8% inflation), Singapore (97.2%, 6.1% inflation), Norway (93.4%, 5.8% inflation), Japan (89.2%, 2.5% inflation).

**Inflation Phase Identification.** Countries with low political stability (percentile $< 20$) and high inflation ($> 100\%$) cluster in the predicted "inflation phase" region: Venezuela (8.5%, 189.8% inflation), Zimbabwe (15.6%, 285.0% inflation), Sudan (1.4%, 154.9% inflation), Syria (0.5%, 139.0% inflation).

**Triple Point Candidates.** Historical and contemporary candidates include Weimar Germany (1923), Venezuela (2018-present), and Zimbabwe (2008, 2020s).

### 6.5 Statistical analysis

We test the relationship between political stability ($I$) and inflation ($P$) using log-linear regression: $\log(P_i) = \alpha + \beta I_i + \varepsilon_i$.

Table 3: Regression Results: Stability-Inflation Relationship

| **Variable** | **Coefficient** | **Std. Error** | **$t$-statistic** | **$p$-value** |
|---|---|---|---|---|
| Constant ($\alpha$) | 4.82 | 0.45 | 10.7 | $< 0.001$ |
| Stability ($\beta$) | $-0.048$ | 0.008 | $-6.0$ | $< 0.001$ |
| $R^2$ | 0.67 | | | |
| $N$ | 20 | | | |

The negative coefficient on stability ($\beta = -0.048$, $p < 0.001$) confirms the theoretical prediction: higher political stability is associated with lower inflation. The $R^2$ of 0.67 indicates that stability alone explains a substantial fraction of cross-country inflation variation.

## 7 Economic applications and policy implications

**Proposition 2** (Stability Near Triple Point)**.** *Near the triple point, small policy interventions can produce large and unpredictable regime changes. Optimal policy requires maintaining $(I, P)$ well away from $(I_{tr}, P_{tr})$.*

**Policy implication**: Countries experiencing simultaneous institutional weakness and inflationary pressure should prioritize fundamental institutional reform over marginal monetary adjustments.

**Proposition 3** (Supercritical Regime Management)**.** *For $(I, P) \gg (I_c, P_c)$, traditional distinctions between monetary and real economic policy become less meaningful.*

**Policy implication**: During periods of unconventional monetary policy, central banks should coordinate closely with fiscal authorities.

## 8 Conclusions

This paper has developed an extended mathematical framework for Roegenian Economics, establishing rigorous foundations for the thermodynamic-economic correspondence initiated by Georgescu-Roegen and systematized in earlier collaborative work.

The main contributions are: (i) **Enhanced Theoretical Framework**: Rigorous proofs for the existence and uniqueness of the economic triple point, the economic Clausius-Clapeyron equation, Maxwell relations for economic potentials, and the economic phase rule. (ii) **Deep Economic Interpretation**: Extended dictionary with detailed justification for each thermodynamic-economic correspondence. (iii) **Critical Phenomena Analysis**: Characterization of critical points, derivation of the economic van der Waals equation, and calculation of mean-field critical exponents. (iv) **Empirical Validation**: First systematic empirical test of the Roegenian framework using World Bank and IMF data.

The thermodynamic approach to economics provides a powerful conceptual framework that naturally incorporates irreversibility, phase transitions, and critical phenomena features absent from traditional mechanical models but essential for understanding real economic dynamics.

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