

DYNAMICS OF QUANTUM ENTANGLEMENT IN RELATIVISTIC OPEN SYSTEMS*

Mădălin Calamanciuc[†] Aurelian Isar[‡]

Dedicated to the memory of Professor Mihail Megan

DOI 10.56082/annalsarscimath.2026.3.205

Abstract

We review the results obtained by investigating, in the framework of the theory of open systems based on completely positive quantum dynamical semigroups, the time evolution of quantum entanglement of two different bimodal systems, each one interacting with a thermal environment. The first system is formed of the free massless bosonic mode of a stationary observer in a region that is asymptotically flat (Kruskal observer) or freely falling in Schwarzschild black hole, and the mode of a uniformly accelerated observer hovering outside the event horizon of black hole. For initial bimodal Gaussian squeezed thermal states of the system, we show that under the influence of Hawking radiation the quantum entanglement of the two modes is destroyed in a finite time, for non-zero values of the temperature of the thermal environment, i.e. the phenomenon of entanglement sudden death takes place. The second system consists of two bosonic modes associated with a scalar quantum field in de Sitter space, characterized by two regions that are causally disconnected and described using open universe coordinates on two open charts. The survival time of the entanglement of this bimodal system strongly depends on the competition between the contrary effects provided by the squeezing of the initial bimodal state, the curvature of de Sitter space, the mass parameter and the

*Accepted for publication on July 30, 2026

[†]calamanciuc.madalin@theory.nipne.ro, Faculty of Physics, University of Bucharest; National Institute of Physics and Nuclear Engineering, Magurele, Romania

[‡]isar@theory.nipne.ro, Faculty of Physics, University of Bucharest; National Institute of Physics and Nuclear Engineering, Magurele, Romania; Academy of Romanian Scientists

thermal environment. It is shown that the entanglement is minimized for values $1/2$ and $3/2$ of the mass parameter, corresponding to the conformally coupled scalar field and, respectively, minimally coupled massless field.

Keywords: quantum entanglement, Schwarzschild black hole, de Sitter space, Gaussian states, open systems.

MSC: 81P42, 81V73, 81T20.

1 Introduction

Entanglement is a quantum correlation of major importance in the theory of quantum information, being a fundamental resource for tasks of quantum information processing and communications, like quantum teleportation, quantum computing, quantum cryptography [22]. The research on quantum correlations has traditionally focused on continuous variable systems within inertial frames of reference. However, the realistic quantum systems are essentially non-inertial and they manifest relativistic and gravitational characteristics, therefore relativistic quantum investigations are of basic importance, both for applications in quantum information protocols [3, 5, 16, 20, 23, 29], and for understanding better the features of the universe. In recent years, extensive studies on quantum correlations in various scenarios, such as non-inertial frames, curved spacetime, and an expanding universe have been performed [4, 6, 9, 13, 24, 30].

In the present work we review the results obtained in Refs. [11, 12] by investigating, in the framework of the theory of open systems based on completely positive quantum dynamical semigroups, the time evolution of quantum entanglement of two different bimodal systems, each one interacting with a thermal environment. The first system is formed of the free massless bosonic mode of a stationary observer in a region that is asymptotically flat (Kruskal observer) or freely falling in Schwarzschild black hole [13], and the mode of a uniformly accelerated observer hovering outside the event horizon of black hole. For initial bimodal Gaussian squeezed thermal states of the system, we show that under the influence of Hawking radiation the quantum entanglement of the two modes is destroyed in a finite time, for non-zero values of the temperature of the thermal environment, i.e. the phenomenon of entanglement sudden death takes place. For a zero temperature of the thermal bath the initial existing entanglement is decreasing over time, but it keeps for all finite times a non-zero value, and the logarithmic negativity tends to zero only in the limit of infinite time. The second sys-

tem consists of two bosonic modes associated with a scalar quantum field in de Sitter space, characterized by two regions that are causally disconnected and described using open universe coordinates on two open charts [21, 26]. The positive frequency mode functions of a free scalar field correspond to the Bunch-Davies vacuum [10], also known as the Euclidean vacuum, which has support on both regions. The survival time of the entanglement of this bimodal system strongly depends on the competition between the contrary effects provided by the squeezing of the initial bimodal state, the curvature of de Sitter space, the mass parameter and the thermal environment. It is shown that the entanglement is minimized for values $1/2$ and $3/2$ of the mass parameter, corresponding to the conformally coupled scalar field and, respectively, minimally coupled massless field.

The work is structured in the following way. In Sec. 2 it is described the dynamics of the scalar field for massless modes. There are quantized the fields in the region of the event horizon of Schwarzschild black hole and it is shown that Unruh-Hawking effect is described by a Gaussian bosonic amplification channel acting on a bipartite system [3, 7]. We also introduce the formalism for describing the curvature effects associated with de Sitter space. In Sec. 3 we present the framework of the theory of open quantum systems and introduce the logarithmic negativity as a measure of quantum entanglement, by using the symplectic formalism. Then we analyze the time evolution of the entanglement of the systems under scrutiny, in interaction with the thermal environment. In Sec. 5 we give a summary and the conclusions.

2 Dynamics of the quantum scalar field

2.1 Schwarzschild event horizon

The radiation produced by the black hole through the Unruh-Hawking effect can be formulated in terms of an amplification Gaussian bosonic channel [7, 27]. We take a metric characterizing the background of the spacetime in the region of an asymptotically flat and static Schwarzschild black hole of the form (we set the natural units ($\hbar = G = c = k_B = 1$)):

$$ds^2 = - \left(1 - \frac{2M}{r}\right) dt^2 + \left(1 - \frac{2M}{r}\right)^{-1} dr^2 + r^2 (d\theta^2 + \sin^2 \theta d\varphi^2), \quad (1)$$

where r is the radial coordinate, t denotes the time, angles (θ, φ) define the metric on the two-sphere and M denotes the black hole mass.

The massless bosonic field ϕ satisfies in the background of the black hole the following Klein-Gordon equation [8]:

$$\frac{1}{\sqrt{-g}} \frac{\partial}{\partial x^\mu} \left(\sqrt{-g} g^{\mu\nu} \frac{\partial \phi}{\partial x^\nu} \right) = 0, \quad (2)$$

where $x^\mu = (t, r, \theta, \varphi)$ denotes the general coordinate and $g = \det g^{\mu\nu}$.

We consider a bipartite system, with a stationary observer in asymptotically flat region and a Schwarzschild observer who hovers uniformly accelerated near the event horizon of the black hole. The Unruh vacuum state in single-mode approximation, when only bosons exist outside the event horizon (only particles can be detected as Hawking radiation) and antibosons are living inside event horizon, reduces to a two-mode squeezing state

$$|0_\Omega \rangle_H = \frac{1}{\cosh r_\Omega} \sum_{n=0}^{\infty} (\tanh r_\Omega)^n |n \rangle_{\text{out}} |n \rangle_{\text{in}}, \quad (3)$$

where $|n \rangle_{\text{out}}$ and $|n \rangle_{\text{in}}$ denote the bosonic and antibosonic states, outside and inside of the event horizon. Here $\sinh r_\Omega = \left(e^{\frac{\Omega}{T_H}} - 1 \right)^{-\frac{1}{2}}$ and T_H denotes the Hawking temperature. Hawking temperature parameter r_Ω is a function that monotonically increases with T_H .

The expression in Eq. (3) can be written as the action of a bimodal squeezing operator on associated vacuum states $|n \rangle_{\text{in}}$ and $|n \rangle_{\text{out}}$, inside and outside regions of event horizon, therefore, from Eq. (3) it follows that the Unruh-Hawking radiation can be expressed as an amplification bosonic channel [15, 30, 31]. The Gaussian operation expressed by this squeezing operator and preserving the Gaussianity of the input states has the following symplectic phase-space representation (we abbreviate $r_\Omega \equiv r$ for simplicity):

$$S(r) = \begin{pmatrix} \cosh r & 0 & \sinh r & 0 \\ 0 & \cosh r & 0 & -\sinh r \\ \sinh r & 0 & \cosh r & 0 \\ 0 & -\sinh r & 0 & \cosh r \end{pmatrix}. \quad (4)$$

We consider that the scalar massless field ϕ has two Unruh modes A and B , which share in the inertial frame a bimodal Gaussian state ρ_{AB} . We denote by $\mathbf{R} = \{x, p_x, y, p_y\}^T$ the operators of canonical quadratures of two bosonic modes and by σ_{AB} the two-mode covariance matrix, whose elements are given by the statistical moments of second order of quadrature operators, which completely characterize the two-mode Gaussian states:

$$\sigma_{ij} = \text{Tr}[(R_i R_j + R_j R_i) \rho_{AB}], i, j = 1, \dots, 4. \quad (5)$$

We neglect the moments of the first order, since they can be made zero by performing local displacements in the phase space. The covariance matrix fulfills the uncertainty relation $\sigma_{AB} + i\Omega_{AB} \geq 0$, where $\Omega_{AB} = \oplus_1^2 \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$ is the symplectic form.

The previously introduced amplification channel is expressed by the bimodal squeezing operation associated with symplectic transformation (4). The amplification maps the mode B of the Schwarzschild observer into the modes B and $\bar{B}$ situated outside and inside of event horizon, respectively. Therefore, from inertial point of view the considered system has two parties, while, from the point of view of a Schwarzschild observer, becomes relevant an additional mode $\bar{B}$. Consequently, the initial bipartite state is transformed into a three-partite state, with the mode A of the stationary observer in asymptotically flat region, mode B of the Schwarzschild observer, and mode $\bar{B}$ of a hypothetical observer, situated inside event horizon of Schwarzschild black hole. Then the three-mode system can be described by the following covariance matrix [3]:

$$\sigma_{AB\bar{B}}(s, r) = [I_A \oplus S_{B,\bar{B}}(r)] [\sigma_{AB}^0(s) \oplus I_{\bar{B}}] [I_A \oplus S_{B,\bar{B}}(r)]^T. \quad (6)$$

We denote by $\sigma_{AB}^0(s)$ the covariance matrix of the bimodal Gaussian thermal squeezed state of bosonic fields:

$$\sigma_{AB}^0(s) = \begin{pmatrix} a_0 & 0 & c_0 & 0 \\ 0 & a_0 & 0 & -c_0 \\ c_0 & 0 & b_0 & 0 \\ 0 & -c_0 & 0 & b_0 \end{pmatrix}, \quad (7)$$

where

$$\begin{aligned} a_0 &= 2n_1 \cosh^2 s + 2n_2 \sinh^2 s + \cosh 2s, \\ b_0 &= 2n_2 \cosh^2 s + 2n_1 \sinh^2 s + \cosh 2s, \\ c_0 &= (n_1 + n_2 + 1) \sinh 2s. \end{aligned} \quad (8)$$

Here s denotes the squeezing parameter of the state and n_1, n_2 denote the associated average thermal photon numbers.

The covariance matrix for the outside region can be obtained by performing trace over mode $\bar{B}$ situated inside black hole, associated with the hypothetical observer. Then we obtain from Eq. (6) the following covariance matrix of modes A and B [3]:

$$\sigma_{AB}(s, r) = \begin{pmatrix} \mathcal{A} & \mathcal{C} \\ \mathcal{C}^T & \mathcal{B} \end{pmatrix}, \quad (9)$$

where:

$$\begin{aligned}\mathcal{A} &= a_0 I, \\ \mathcal{B} &= [b_0 \cosh^2 r + \sinh^2 r] I, \\ \mathcal{C} &= c_0 \cosh r Z,\end{aligned}\tag{10}$$

I is the identity matrix and Z is Z -Pauli matrix.

2.2 Euclidean de Sitter space

The 4D Euclidean de Sitter space can be embedded in the 5D Euclidean space. Therefore, the open charts in de Sitter space, defined by the Hubble radius H^{-1} , have coordinate frames that can be derived through analytic continuation of the Euclidean metric. These frames can be separated into three regions (R,C,L), each one with its own individual metric:

$$\begin{aligned}ds_R^2 &= H^{-2} [-dt_R^2 + \sinh^2 t_R (dr_R^2 + \sinh^2 r_R d\Omega^2)], \\ ds_C^2 &= H^{-2} [dt_C^2 + \cosh^2 t_C (-dr_C^2 + \cosh^2 r_C d\Omega^2)], \\ ds_L^2 &= H^{-2} [-dt_L^2 + \sinh^2 t_L (dr_L^2 + \sinh^2 r_L d\Omega^2)],\end{aligned}\tag{11}$$

where $d\Omega^2$ is the metric on the two-sphere. The regions R and L , described by the coordinates (t_R, r_R) and (t_L, r_L) , respectively, are causally disconnected. The region C is described by the coordinates (r_C, t_C) .

The Bunch-Davies vacuum [10] for a global observer in de Sitter space connects R vacuum and L vacuum through the Bogoliubov transformation and can be expressed as a two-mode squeezed state of the R and L vacua in the Fock space:

$$|0\rangle_{BD} = U_{R,L}(\gamma_p) |0\rangle_R |0\rangle_L = \sqrt{1 - |\gamma_p|^2} \sum_{n=0}^{\infty} \gamma_p^n |n\rangle_R |n\rangle_L, \tag{12}$$

where $U_{R,L}(\gamma_p)$ is the two-mode squeezing operator corresponding to a Gaussian channel and the squeezing parameter is given by

$$\gamma_p = i \frac{\sqrt{2}}{\sqrt{\cosh 2\pi p + \cos 2\pi\nu} + \sqrt{\cosh 2\pi p + \cos 2\pi\nu} + 2}. \tag{13}$$

Here ν is the mass parameter (m is the mass of the scalar field):

$$\nu = \sqrt{\frac{9}{4} - \frac{m^2}{H^2}}. \tag{14}$$

For $\nu = 1/2$ and $\nu = 3/2$ this parameter simplifies to $|\gamma_p| = e^{-\pi p}$, which tends to 1 in the limit of small p (large curvature) and takes small values in

the limit of large p . The value $\nu = 1/2$ corresponds to the conformal coupled scalar field and $\nu = 3/2$ to the minimally coupled massless scalar field. The parameter p is normalized by H and it represents the curvature parameter of de Sitter space, namely the effect of the curvature of the 3D hyperbolic space appears when p approaches to 1, and gets stronger by decreasing p , so that the limit of infinite curvature is reached for $p \rightarrow 0$.

The Bunch-Davies vacuum observed by the global observer of the mode A can be expressed as a two-mode squeezed state of the R vacuum observed by the observer of the mode B and the L vacuum observed by the observer of the mode $\bar{B}$.

The squeezing operator $U_{R,L}(\gamma_p)$ is a Gaussian operation that preserves the Gaussianity of the state, and in the phase space it is represented by the following curvature-induced symplectic operator acting on the modes B and $\bar{B}$:

$$S_{B,\bar{B}}(\gamma_p) = \frac{1}{\sqrt{1-|\gamma_p|^2}} \begin{pmatrix} I & |\gamma_p|Z \\ |\gamma_p|Z & I \end{pmatrix}, \quad (15)$$

with I being the unity matrix in 2×2 space and Z the Z -Pauli matrix. Therefore, under the transformation (15) the single mode B is converted to two modes in two open charts.

We consider an initial entangled Gaussian two-mode squeezed vacuum state of the free scalar field shared by the modes A and B . In the Bunch-Davies vacuum, this state, with the squeezing parameter s , is described by the following covariance matrix:

$$\sigma_{AB}^0(s) = \begin{pmatrix} \cosh 2s & I & \sinh 2s & Z \\ \sinh 2s & Z & \cosh 2s & I \end{pmatrix}. \quad (16)$$

Considering that there are no initial correlations between the state described by the covariance matrix (16) and the subsystem observed by $\bar{B}$, the initial covariance matrix of the entire system is $\sigma_{AB}^0(s) \oplus I_{\bar{B}}$. Consequently, a full description of the system involves three modes: the mode A observed by a global observer, the mode B observed by observer in the region of open charts R and the mode $\bar{B}$ observed in the region L of open charts. The covariance matrix of the three-mode Gaussian state describing the complete system has the following form [3]:

$$\sigma_{AB\bar{B}}(s, \gamma_p) = [I_A \oplus S_{B,\bar{B}}(\gamma_p)] [\sigma_{AB}^0(s) \oplus I_{\bar{B}}] [I_A \oplus S_{B,\bar{B}}(\gamma_p)]^T. \quad (17)$$

Since an observer living in region L is causally disconnected from region R , the physically accessible information is encoded in the mode A and the

mode B . By performing partial trace over $\bar{B}$ in the tripartite covariance matrix (17) we will get rid of the mode $\bar{B}$, obtaining the initial covariance matrix for the modes A and B , associated with region R :

$$\sigma_{AB}(s, \gamma_p) = \begin{pmatrix} \mathcal{A} & \mathcal{C} \\ \mathcal{C}^T & \mathcal{B} \end{pmatrix}, \quad (18)$$

with

$$\begin{aligned} \mathcal{A} &= \cosh 2s \, I, \\ \mathcal{B} &= \frac{|\gamma_p|^2 + \cosh 2s}{1 - |\gamma_p|^2} I, \\ \mathcal{C} &= \frac{\sinh 2s}{\sqrt{1 - |\gamma_p|^2}} Z. \end{aligned} \quad (19)$$

3 Time evolution of quantum entanglement

In this Section we will investigate the time evolution of the entanglement between the two bosonic modes of the considered systems, each one immersed in its common thermal bath. To study the dynamics of the bimodal systems we use the axiomatic formalism of open systems, based on completely positive quantum dynamical semigroups. In this framework the Markovian irreversible time evolution of an open system is described by the following Gorini-Kossakowski-Sudarshan-Lindblad master equation for the density operator $\rho(t)$ (we set $\hbar = 1$) [17–19]

$$\frac{d\rho(t)}{dt} = -i[\mathcal{H}, \rho(t)] + \frac{1}{2} \sum_k \left(2L_k \rho(t) L_k^\dagger - \left\{ \rho(t), L_k^\dagger L_k \right\}_+ \right), \quad (20)$$

where

$$\mathcal{H} = \frac{1}{2}(x^2 + p_x^2 + y^2 + p_y^2) \quad (21)$$

is the Hamiltonian of the two bosonic modes (we take identical frequencies $\omega = 1$ for the two modes) and the operators $L_k, L_k^\dagger$ describe the interaction of the system with a general environment.

If the initial states are Gaussian and the operators L_k are chosen polynomials of first degree in the canonically conjugated quadrature operators x, p_x, y, p_y of the two bosonic modes, then, due to the linear character of the dynamics, the Gaussianity is preserved in time [14, 28].

From Eq. (20) it is obtained that the time evolution of the corresponding bimodal covariance matrix $\sigma(t)$ is given by the following Lyapunov equation of motion [25]:

$$\frac{d\sigma(t)}{dt} = Y\sigma(t) + \sigma(t)Y^T + 2D, \quad (22)$$

where

$$Y = \begin{pmatrix} -\lambda & 1 & 0 & 0 \\ -1 & -\lambda & 0 & 0 \\ 0 & 0 & -\lambda & 1 \\ 0 & 0 & -1 & -\lambda \end{pmatrix} \quad (23)$$

is the drift matrix, λ is the dissipation rate, and we assume that the diffusion matrix D has the form [25]

$$D = \text{diag}\left\{\lambda \coth \frac{1}{2k_B T}, \lambda \coth \frac{1}{2k_B T}, \lambda \coth \frac{1}{2k_B T}, \lambda \coth \frac{1}{2k_B T}\right\}, \quad (24)$$

where k_B is the Boltzmann constant and T is the temperature of the thermal bath.

The solution of Eq. (22) is given by [25]:

$$\sigma(t) = \Gamma(t)[\sigma_{AB}(s) - \sigma_T]\Gamma^T(t) + \sigma_T, \quad (25)$$

where $\sigma_{AB}(s)$ is the initial covariance matrix (9) or (18) of the two modes, $\Gamma(t) = \exp(Yt)$, with $\Gamma(t) \rightarrow 0$ when $t \rightarrow \infty$. We notice that the evolution generated by the Gaussian completely positive map is determined by two 4×4 real matrices Γ and $A = \sigma_T - \Gamma\sigma_T\Gamma^T$, which satisfy $A + i\Omega_{AB} \geq i\Gamma\Omega_{AB}\Gamma^T$.

The covariance matrix corresponding to the asymptotic Gibbs state of the system of two bosonic modes, interacting with the thermal reservoir of temperature T , is given by (we set Boltzmann constant $k_B = 1$):

$$\sigma_T = \begin{pmatrix} \coth \frac{1}{2T} & 0 & 0 & 0 \\ 0 & \coth \frac{1}{2T} & 0 & 0 \\ 0 & 0 & \coth \frac{1}{2T} & 0 \\ 0 & 0 & 0 & \coth \frac{1}{2T} \end{pmatrix}.$$

To quantify the quantum entanglement we choose logarithmic negativity as a measure, defined in terms of the symplectic invariants of the covariance matrix $\sigma(t)$ written in block form

$$\sigma(t) = \begin{pmatrix} \mathcal{A}(t) & \mathcal{C}(t) \\ \mathcal{C}^T(t) & \mathcal{B}(t) \end{pmatrix}, \quad (26)$$

as follows [1, 2]:

$$E_N = -\log_2 g(\sigma(t)), \quad (27)$$

$$g(\sigma(t)) = \frac{1}{\sqrt{2}} \sqrt{\Delta(t) - \sqrt{\Delta^2(t) - 4\det\sigma(t)}}, \quad (28)$$

where we introduced the serialian

$$\Delta(t) \equiv \det\mathcal{A}(t) + \det\mathcal{B}(t) - 2\det\mathcal{C}(t). \quad (29)$$

For $E_N \leq 0$ the state is separable and $E_N > 0$ determines the strength of the entanglement.

3.1 Time evolution of quantum entanglement in Schwarzschild background

To describe the time evolution of the logarithmic negativity as a measure of the Gaussian quantum entanglement between the considered observers, we suppose that they share initially a bimodal symmetric thermal squeezed state, with the covariance matrix given by Eq. (7) in which we denote $n \equiv n_1 = n_2 = 1$.

From Figs. 1 and 2 we notice that, due to decoherence and dissipation induced by the environment, the entanglement is destroyed in a finite time for all values of the Hawking parameter r , squeezing parameter of the initial bimodal state s and for non-zero values of the temperature T of the thermal environment. We witness the phenomenon of entanglement sudden death, with a single notable exception. Namely, for a zero temperature of the thermal bath, the initial existing entanglement is decreasing over time, but it keeps for all finite times a non-zero value and the logarithmic negativity asymptotically tends to zero only in the limit of infinite time. In Figs. 1 and 2 it is also illustrated that, as expected, the logarithmic negativity is decreasing by increasing the temperature of the environment and that the survival time of the entanglement is decreasing by increasing the temperature and the Hawking parameter r . Conversely, we see that the survival time of the entanglement increases with the squeezing parameter s . We can also see that the logarithmic negativity decreases by increasing the Hawking parameter r , which is a function that monotonically increases with the Hawking temperature T_H , so that for a relatively large values of the Hawking parameter the entanglement is destroyed. It is also illustrated that the logarithmic negativity increases with the squeezing parameter s of the initial state [11].

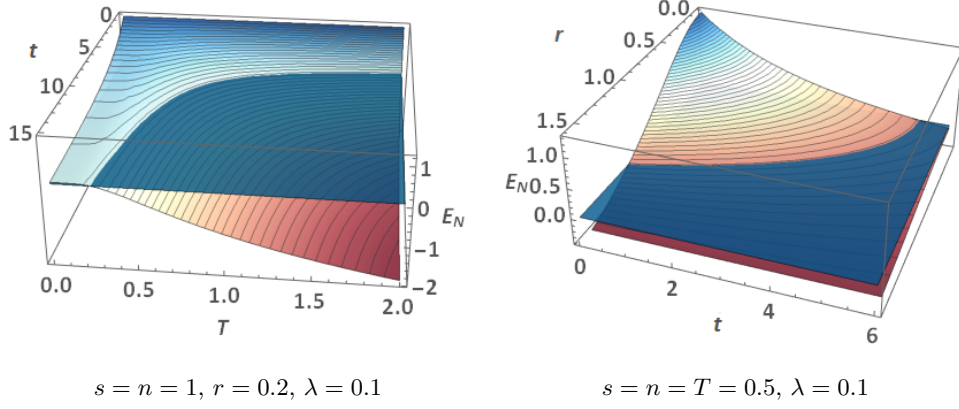


Figure 1: Logarithmic negativity E_N versus bath temperature T and time t (left), and versus Hawking parameter r and time t (right).

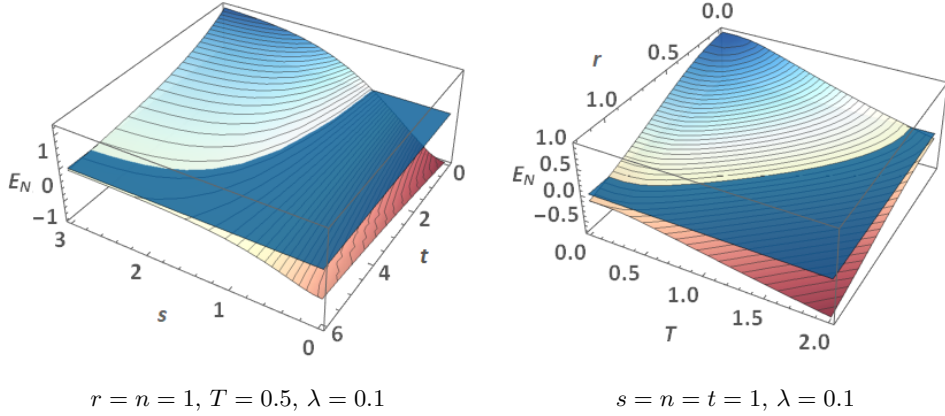


Figure 2: Logarithmic negativity E_N versus squeezing parameter s and time t (left), and versus bath temperature T and Hawking parameter r at a given moment of time (right).

3.2 Time evolution of quantum entanglement in de Sitter space

The time evolution of Gaussian quantum entanglement as a function of curvature parameter p is plotted in Fig. 3, for a given value of the mass parameter ν . We see that the entanglement increases with the curvature parameter p , and, due to the interaction with the thermal bath, the entanglement

vanishes at a finite moment of time. The survival time of entanglement increases by increasing the curvature parameter p . The same phenomenon of entanglement sudden death can be observed for the logarithmic negativity plotted as a function of the mass parameter ν and time, for a fixed value of the curvature parameter p .

We notice that entanglement has an oscillatory behavior with respect to the mass parameter ν , with period 1, which comes from $\cos 2\pi\nu$. We can also see that the entanglement is minimized when $\nu = 1/2$ and $\nu = 3/2$, corresponding to the conformal coupled scalar field, respectively minimally coupled massless field. Therefore, we can assert that a massive field preserves more entanglement than a massless field, i.e. the entanglement of a massive scalar field is more robust than that of a massless field in de Sitter space. We notice that the entanglement decreases over time and for a nonzero time the entanglement vanishes for definite values of the curvature parameter p . The entanglement can survive for definite values of times and in the presence of the curvature of the space, even in the limit of infinite curvature with $p \rightarrow 0$, but only for a mass parameter of a scalar field other than $\nu = 1/2$ and $\nu = 3/2$.

In Fig. 4 we notice that the amount of entanglement decreases as T is increasing, while it increases with p and there is a critical value of p beyond which the amount of entanglement saturates. At a given moment of time and for a given temperature of the thermal environment, the Gaussian quantum entanglement can survive only for a sufficiently large value of the parameter p , that is for a sufficiently small curvature. Entanglement is an increasing function of the squeezing s and the curvature parameter p [12].

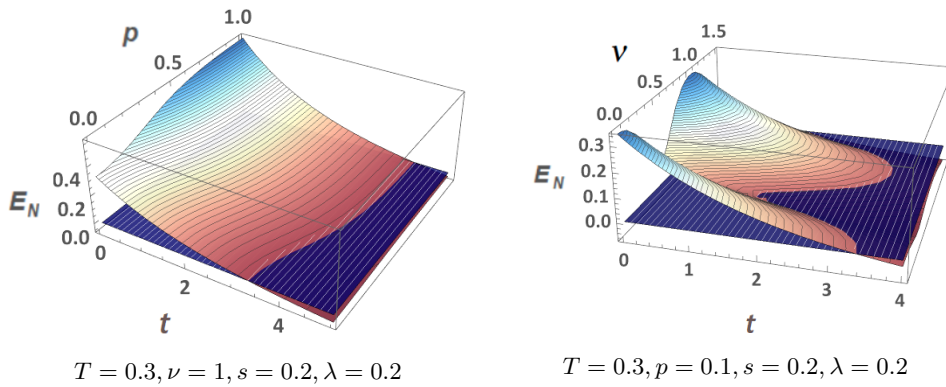


Figure 3: Logarithmic negativity E_N versus curvature parameter p and time t (left), and versus mass parameter ν and time t (right).

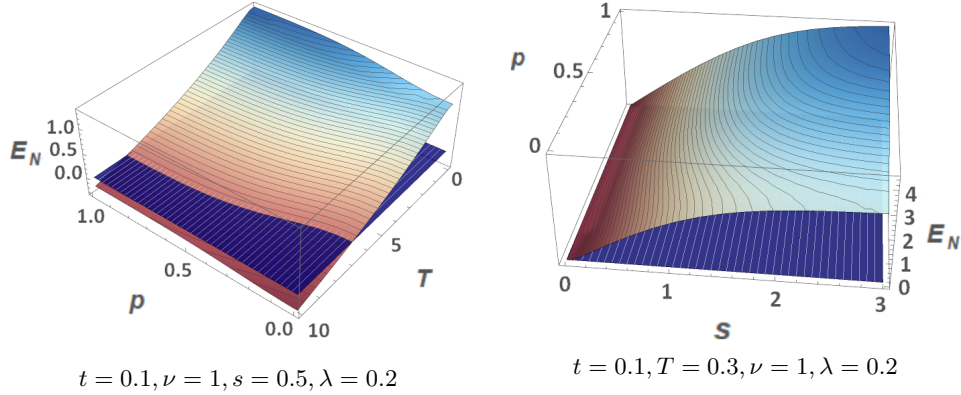


Figure 4: Logarithmic negativity E_N versus curvature parameter p and temperature T at a given moment of time (left), and versus squeezing s and curvature parameter p at a given moment of time (right).

4 Summary

In the framework of the theory of open quantum systems we investigated the evolution of Gaussian quantum entanglement, quantified by the logarithmic negativity, for two different systems in interaction with a thermal reservoir:

- two bosonic modes of a massless scalar quantum field in the presence of a Schwarzschild black hole, under the influence of the Hawking effect;
- two bosonic modes of a massive scalar quantum field in de Sitter space.

For both scenarios of the considered observers, namely a Kruskal observer and an accelerated observer hovering outside the event horizon of black hole and, respectively, a global observer situated in the Bunch-Davies vacuum and an observer who stays in the region R of open charts of de Sitter space, we have obtained the following results:

- due to decoherence and dissipation induced by the thermal environment, for non-zero values of the temperature, the initially existing quantum entanglement decreases over time and is destroyed in a finite time (phenomenon of entanglement sudden death);
- logarithmic negativity and the survival time of entanglement are decreasing by increasing the temperature of the environment, while they increase with the squeezing parameter of the bimodal state.

Therefore, the thermal environment has a destructive influence on the entanglement, whose survival time depends on the competition between the

contrary effects provided by the squeezing of the bimodal state and the thermal bath.

We have also shown that:

- the survival time of entanglement decreases by increasing the Hawking parameter, while it increases with de Sitter curvature parameter;
- as de Sitter parameter decreases, the amount of entanglement also decreases, which means that the effect of space curvature reduces the entanglement of the state, that is, the state becomes less entangled as the curvature of the open chart gets larger; at a given moment of time and for a given temperature of the thermal environment, the Gaussian quantum entanglement can survive only for a sufficiently large value of the curvature parameter, that is for a sufficiently small curvature;
- the entanglement is minimized for values $1/2$ and $3/2$ of the mass parameter, corresponding to the conformally coupled scalar field, respectively, minimally coupled massless field; therefore, the entanglement of a massive scalar field is more robust than that of a massless field in de Sitter space; the entanglement can survive for definite values of times and in the presence of the curvature of the space, even in the limit of infinite curvature, but only for a mass parameter of a scalar field different from $1/2$ and $3/2$.

The obtained results illustrate the role that the Hawking, mass and curvature of space parameters and the parameters describing the state squeezing and thermal environment play in order to ensure the preservation of the entanglement over time for practical implementation of quantum communication protocols that rely on quantum entanglement.

References

- [1] G. Adesso, A. Serafini and F. Illuminati, Extremal entanglement and mixedness in continuous variable systems, *Phys. Rev. A* 70 (2004), 022318.
- [2] G. Adesso, A. Serafini and F. Illuminati, Gaussian measures of entanglement versus negativities, *Phys. Rev. A* 73 (2006), 032345.
- [3] G. Adesso, I. Fuentes-Schuller and M. Ericsson, Continuous-variable entanglement sharing in noninertial frames, *Phys. Rev. A* 76 (2007), 062112.
- [4] G. Adesso, S. Ragy and D. Girolami, Continuous variable methods in relativistic quantum information, *Class. Quantum Grav.* 29 (2012), 224002.

- [5] P.M. Alsing and G.J. Milburn, Teleportation with a uniformly accelerated partner, *Phys. Rev. Lett.* 91 (2003), 180404.
- [6] P.M. Alsing, I. Fuentes-Schuller, R.B. Mann and T.E. Tessier, Entanglement of Dirac fields in noninertial frames, *Phys. Rev. A* 74 (2006), 032326.
- [7] M. Aspachs, G. Adesso and I. Fuentes, Optimal quantum estimation of the Unruh–Hawking effect, *Phys. Rev. Lett.* 105 (2010), 151301.
- [8] N.D. Birrell and P.C.W. Davies, *Quantum Fields in Curved Space*, Cambridge University Press, Cambridge, 1982.
- [9] D.E. Bruschi, A. Dragan, I. Fuentes and J. Louko, Particle and antiparticle bosonic entanglement in noninertial frames, *Phys. Rev. D* 86 (2012), 025026.
- [10] T.S. Bunch and P.C.W. Davies, Quantum field theory in de Sitter space: Renormalization by point-splitting, *Proc. R. Soc. Lond. A* 360 (1978), 117-134.
- [11] M. Calamanciuc and A. Isar, Gaussian quantum entanglement in curved spacetime, *Results Phys.* 55 (2023), 107167.
- [12] M. Calamanciuc and A. Isar, Quantum entanglement of two bosonic modes in de Sitter space, *Eur. Phys. J. Plus* 140 (2025), 824.
- [13] J. Doukas, E.G. Brown, A. Dragan and R.B. Mann, Entanglement and discord: Accelerated observations of local and global modes, *Phys. Rev. A* 87 (2013), 012306.
- [14] A. Ferraro, S. Olivares and M.G.A. Paris, *Gaussian States in Quantum Information*, Bibliopolis, Napoli, 2005.
- [15] N. Friis, A.R. Lee, K. Truong, C. Sabín, E. Solano, G. Johansson and I. Fuentes, Relativistic quantum teleportation with superconducting circuits, *Phys. Rev. Lett.* 110 (2013), 113602.
- [16] I. Fuentes-Schuller and R.B. Mann, Alice falls into a black hole: Entanglement in noninertial frames, *Phys. Rev. Lett.* 95 (2005), 120404.
- [17] V. Gorini, A. Kossakowski and E.C.G. Sudarshan, Completely positive dynamical semigroups of N -level systems, *J. Math. Phys.* 17 (1976), 821-825.

- [18] A. Isar, A. Sandulescu, H. Scutaru, E. Stefanescu and W. Scheid, Open quantum systems, *Int. J. Mod. Phys. E* 3 (1994), 635-714.
- [19] G. Lindblad, On the generators of quantum dynamical semigroups, *Commun. Math. Phys.* 48 (1976), 119-130.
- [20] T. Liu, S. Cao and S. Wu, Characterization of quantum and classical correlations in the Earth curved space-time, *Sci. Rep.* 10 (2020), 14697.
- [21] J. Maldacena and G.L. Pimentel, Entanglement entropy in de Sitter space, *J. High Energy Phys.* 2013 (2013), 038.
- [22] M.A. Nielsen and I.L. Chuang, *Quantum Computation and Quantum Information*, Cambridge University Press, Cambridge, 2000.
- [23] A. Peres and D.R. Terno, Quantum information and relativity theory, *Rev. Mod. Phys.* 76 (2004), 93-123.
- [24] B. Richter and Y. Omar, Degradation of entanglement between two accelerated parties, *Phys. Rev. A* 92 (2015), 022334.
- [25] A. Sandulescu, H. Scutaru and W. Scheid, Open quantum system of two coupled harmonic oscillators, *J. Phys. A: Math. Gen.* 20 (1987), 2121-2133.
- [26] M. Sasaki, T. Tanaka and K. Yamamoto, Euclidean vacuum mode functions for a scalar field on open de Sitter space, *Phys. Rev. D* 51 (1995), 2979-2995.
- [27] J. Wang, H. Cao, J. Jing and H. Fan, Gaussian quantum steering and its asymmetry in curved spacetime, *Phys. Rev. D* 93 (2016), 125011.
- [28] C. Weedbrook, S. Pirandola, R. García-Patrón, N.J. Cerf, T.C. Ralph, J.H. Shapiro and S. Lloyd, Gaussian quantum information, *Rev. Mod. Phys.* 84 (2012), 621-669.
- [29] S.-M. Wu, H.-S. Zeng and T.-H. Liu, Quantum coherence of Gaussian states in curved spacetime, *Results Phys.* 14 (2019), 102398.
- [30] S.-M. Wu and H.-S. Zeng, Multipartite quantum coherence and monogamy for Dirac fields subject to Hawking radiation, *Quantum Inf. Process.* 18 (2019), 305.

- [31] S.-M. Wu and H.-S. Zeng, Quantum decoherence of Gaussian steering and entanglement in de Sitter space, *Int. J. Theory Phys.* 59 (2020), 861-874.

