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CONSISTENCY OF PORTFOLIO ALLOCATION STRATEGIES ACROSS INDEPENDENT ASSET UNIVERSES: EVIDENCE FROM DJIA BACKTESTING*

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Abstract

To determine whether portfolio strategy outcomes are dataset-dependent or reflect broader trends, this study examines two distinct asset sets. We evaluate five allocation models—Equal Weighting, Maximum Sharpe Ratio, Inverse Variance, Markowitz Mean-Variance, and Robust Optimization—using two 15-stock groups derived from the Dow Jones Industrial Average. Strategy effectiveness is measured through a standardized backtest incorporating a 40-day warm-up window and regular rebalancing schedules. Evaluation criteria include historical returns, risk metrics, turnover frequency, drawdowns, and execution fees. Rather than relying on a single backtest, this work specifically tests whether strategic performance remains uniform across

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different asset pools. The findings demonstrate broad structural consistency across both tested groups, though localized shifts occur in secondary strategy return rankings. Although the Equal Weighted model produces the top cumulative return, the Inverse Variance approach reliably delivers superior downside protection by minimizing volatility, curtailing drawdowns, and maintaining steady position weights. Conversely, optimization-driven techniques incur greater rebalancing activity and transaction expenses without yielding better out-of-sample performance. These results indicate that risk-focused, resilient allocation frameworks offer more dependable portfolio behavior than models dependent on historical return forecasts.

Keywords: portfolio allocation, backtesting, portfolio optimization, equal weighting, inverse variance strategy, maximum Sharpe ratio, robust optimization, risk management, diversification, Dow Jones Industrial Average (DJIA).

MSC: 91G10, 90C90, 62P05.

1 Introduction

Portfolio allocation is one of the most widely studied problems in finance because investment performance depends not only on the selection of assets but also on how capital is distributed among them. The foundations of modern portfolio theory were established by Markowitz [10], who formalized the trade-off between expected return and risk through mean-variance optimization. Subsequent developments by Sharpe [20,21] extended the analysis of risk-adjusted performance and provided a theoretical framework linking expected returns to systematic risk. These contributions continue to serve as the foundation for modern portfolio construction methods.

Over time, portfolio allocation research has expanded far beyond the original mean-variance framework. Important developments include improved covariance estimation techniques, robust optimization models, diversification-based allocation methods, and Bayesian approaches that incorporate investor views into the asset allocation process [1,5,7,8]. More recently, comprehensive reviews of portfolio optimization have documented the evolution of the field from classical financial models toward more sophisticated approaches that consider multiple objectives, uncertainty, and practical implementation constraints [16,17].

Despite their theoretical appeal, optimization-based portfolio models remain highly sensitive to estimation error. The quality of portfolio allocations depends heavily on expected return and covariance estimates, which

are inevitably subject to uncertainty. Michaud [14] argued that optimized portfolios often exhibit instability due to estimation error, while DeMiguel et al. [4] demonstrated that simple diversification rules can frequently outperform complex optimized portfolios in out-of-sample settings. These findings have raised important questions regarding whether the additional complexity of optimization models consistently translates into superior realized investment performance.

As a consequence, simple diversification methods have received increasing attention in both academic and practical portfolio management. Equal-weighted portfolios avoid estimation error entirely and provide broad diversification across securities. Previous research has shown that equal weighting often performs competitively relative to more sophisticated optimization techniques [4, 15]. Similarly, diversification and risk-allocation studies have shown that portfolio performance can often be improved by focusing on risk distribution rather than return forecasting [3, 9, 13]. These approaches have become increasingly attractive because of their transparency, simplicity, and implementation stability.

Among risk-based approaches, inverse variance and related risk-parity strategies allocate capital according to asset risk characteristics rather than expected returns. Such methods seek to reduce exposure to highly volatile assets while maintaining diversification benefits. Research on risk-based portfolio construction has shown that these approaches can improve downside-risk control and achieve attractive risk-adjusted performance, particularly when return forecasts are unreliable [3, 9]. In many practical settings, portfolios constructed using simple risk measures have exhibited performance that is comparable to, or better than, portfolios derived from more complex optimization procedures.

To address the limitations of classical optimization, robust portfolio optimization methods have been developed to explicitly account for uncertainty in model parameters. Rather than relying on a single point estimate of expected returns, robust approaches seek allocations that remain effective under a range of plausible market conditions [5]. Recent studies continue to demonstrate that robust optimization can improve allocation stability, mitigate estimation risk, and produce stronger out-of-sample performance. Examples include distributionally robust optimization frameworks, robust return prediction models, sparse robust portfolio selection approaches, and hybrid optimization techniques that combine robustness with decision-making algorithms [6, 23–25].

The rapid growth of computational finance has further expanded portfolio optimization research. Modern studies increasingly employ machine

learning, artificial intelligence, and reinforcement learning techniques to support dynamic asset allocation decisions [2, 22]. While these methods offer new opportunities for portfolio construction, recent literature reviews indicate that issues of robustness, diversification, transparency, and implementation costs remain central considerations in practical investment management [16, 17, 22]. Interestingly, many empirical studies continue to report that simple allocation strategies remain highly competitive despite advances in optimization technology [15].

Recent work has also highlighted the importance of understanding asset risk characteristics before portfolio construction. In [18], a volatility-based analysis of KSE-30 equities was conducted using a one-factor credit copula framework and portfolio optimization techniques. The study showed that highly volatile assets tended to be excluded from optimized portfolios, whereas assets with moderate volatility contributed more significantly to final allocations. In [19], portfolio optimization was investigated using the Black-Litterman framework for the Pakistan Stock Exchange KSE-30 index. By combining market equilibrium returns with analysts' views, the study demonstrated how forward-looking information can substantially alter portfolio composition and asset selection decisions.

While [18] and [19] examined optimized portfolio composition–demons trating that highly volatile assets are systematically excluded and showing how forward-looking methods can recalibrate diversified allocations—both studies relied exclusively on expected-return-based optimization. Consequently, they lacked a comprehensive performance evaluation and did not offer a direct comparison between expected-return optimization and explicit risk-based weighting schemes.

Although substantial research exists on portfolio optimization and portfolio allocation, the majority of comparative studies evaluate competing strategies within a single investment universe. Whether considering classical mean-variance optimization, equal weighting, risk-parity methods, robust optimization, or modern artificial intelligence approaches, conclusions are often drawn from a single set of assets and a single portfolio experiment. Consequently, it becomes difficult to determine whether observed performance differences reflect genuine characteristics of the allocation method or whether they are largely driven by the particular securities selected for analysis. A strategy that performs well in one portfolio universe may not necessarily exhibit similar performance when applied to a different set of assets.

The present study addresses this limitation by comparing five portfolio allocation strategies, namely Equal Weighted, Maximum Sharpe Ratio,

Inverse Variance, Markowitz Mean-Variance Optimization, and Robust Optimization, across two independent asset universes constructed from Dow Jones Industrial Average (DJIA) constituents. Using a unified backtesting framework, portfolio performance is evaluated through realized return, volatility, drawdown, turnover, and transaction-cost measures. Unlike many previous studies that rely on a single investment universe, the current research focuses on the consistency of strategy behaviour across two independent asset universes. The study therefore provides evidence on the robustness of strategy rankings and offers practical insight into which portfolio allocation methods provide the most reliable balance between return generation, risk control, diversification, and allocation stability. Throughout this paper, we use the term *consistency* to refer specifically to the preservation of the relative ranking of strategies on a given performance criterion—return, volatility, drawdown, or turnover—across the two independent asset universes, rather than to the numerical stability of the criterion values themselves.

2 Materials and Methods

2.1 Dataset and asset universes

The performance of investment strategies is evaluated and compared using historical data through backtesting. In this study, one year of asset price data from the Dow Jones Industrial Average (DJIA) market is considered. Specifically, we analyze the daily closing prices of the following DJIA constituent assets to examine the performance of different investment strategies:

Apple (AAPL), Amgen (AMGN), Amazon (AMZN), American Express (AXP), Boeing (BA), Caterpillar (CAT), Salesforce (CRM), Cisco (CSCO), Chevron (CVX), Walt Disney (DIS), Goldman Sachs (GS), Home Depot (HD), Honeywell International (HON), Johnson & Johnson (JNJ), International Business Machines (IBM), JPMorgan (JPM), Coca-Cola (KO), McDonald's (MCD), 3M (MMM), Merck (MRK), Microsoft (MSFT), NIKE (NKE), NVIDIA (NVDA), Procter & Gamble (PG), Travelers (TRV), UnitedHealth (UNH), Visa (V), Sherwin-Williams (SHW), Verizon (VZ), and Walmart (WMT).

The dataset comprises the DJIA index and 252 trading days of daily closing prices for these constituent assets.

The comparison of investment strategies is conducted separately on two subsets of assets: the first 15 and the last 15 asset price series. This split is based purely on ticker order among the 30 DJIA constituents and is not

stratified by sector or market capitalization; consequently, the two subsets may differ somewhat in sector composition, which can influence covariance structure and the behavior of variance-based strategies. The analysis based on the assets AAPL, AMGN, AMZN, AXP, BA, CAT, CRM, CSCO, CVX, DIS, GS, HD, HON, IBM, and JNJ is referred to as *Analysis A*. The analysis based on the assets JPM, KO, MCD, MMM, MRK, MSFT, NKE, NVDA, PG, SHW, TRV, UNH, V, VZ, and WMT is referred to as *Analysis B*.

2.2 Portfolio allocation strategies

In this study, five distinct investment strategies are examined. Each strategy determines portfolio weights according to a specific allocation rule, enabling a comparative analysis of their performance under a common backtesting framework. Within this framework, each strategy is evaluated at predetermined points in time, where it is allowed to update its portfolio composition using information derived from recent market data.

This study evaluates five portfolio allocation strategies that translate historical information on asset prices into portfolio weights under a unified backtesting framework. Let N denote the number of assets, and let $\omega = (\omega_1, \omega_2, \ldots, \omega_N)^\top$ represent the vector of portfolio weights, where ω_i denotes the fraction of total capital allocated to asset i. All strategies satisfy the full-investment constraint $\sum_{i=1}^{N} \omega_i = 1$ and impose non-negativity constraints to prohibit short selling. These full-investment and non-negativity (long-only) constraints are enforced uniformly across all five strategies considered in this study—including the Maximum Sharpe Ratio and Markowitz Mean-Variance strategies—even though these two formulations are sometimes implemented with short selling permitted in the broader literature; no short positions were allowed in any of the backtests reported here.

2.2.1 Equal-weighted strategy

The equal-weighted strategy allocates capital uniformly across all assets in the investment universe. Each asset receives the same weight, independent of its expected return or risk characteristics. Formally, the portfolio weights are defined as [11, 12]

$$\omega_i = \frac{1}{N}, \quad i = 1, \ldots, N. \tag{1}$$

This strategy serves as a simple benchmark, as it avoids estimation error and does not rely on model-based assumptions regarding asset behaviour.

2.2.2 Maximum Sharpe ratio strategy

The maximum Sharpe ratio strategy constructs a portfolio by explicitly optimizing risk-adjusted performance. The Sharpe ratio measures the expected excess portfolio return per unit of total risk, where risk is quantified by the standard deviation of portfolio returns. Let $\mathbf{r}$ denote the vector of expected asset returns and Q denote the covariance matrix of asset returns estimated from historical data. The portfolio weights are obtained by solving the optimization problem [12]

$$\boldsymbol{\omega}^{SR} = \arg\max_{\boldsymbol{\omega}} \frac{\mathbf{r}^\top \boldsymbol{\omega}}{\sqrt{\boldsymbol{\omega}^\top Q \boldsymbol{\omega}}} \tag{2}$$

subject to the constraints $\boldsymbol{\omega} \geq 0$ and $\sum_{i=1}^{N} \omega_i = 1$, and upper bounds on individual weights to prevent excessive concentration in a single asset.

This formulation selects the portfolio that lies on the efficient frontier with the highest slope when measured relative to the risk-free asset. In practice, the strategy emphasizes assets with high expected returns relative to their contribution to overall portfolio volatility. While theoretically appealing, the maximum Sharpe ratio solution is known to be sensitive to estimation error, especially in expected returns, which may lead to unstable or highly concentrated allocations when implemented using finite samples.

2.2.3 Inverse variance strategy

The inverse variance strategy determines asset weights using individual risk measures rather than expected returns. Let σ_{ii} denote the variance of asset i, corresponding to the diagonal elements of the covariance matrix Q. Portfolio weights are assigned inversely proportional to these variances and normalized to satisfy the budget constraint [12]

$$\omega_i = \frac{\sigma_{ii}^{-1}}{\sum_{j=1}^{N} \sigma_{jj}^{-1}}, \quad i = 1, \ldots, N. \tag{3}$$

By emphasizing relatively low-variance assets and ignoring cross-asset covariances, this approach produces portfolios that favour stability and are less sensitive to estimation noise in expected returns.

2.2.4 Markowitz mean–variance optimization strategy

The Markowitz mean–variance strategy formalizes portfolio allocation as a trade-off between expected return and risk. Instead of maximizing a ratio,

this approach maximizes a quadratic objective function in which expected portfolio return is reduced by a penalty proportional to portfolio variance. Using the same notation for expected returns and covariance, the optimization problem is given by [12]

$$\boldsymbol{\omega}^{MV} = \arg\max_{\boldsymbol{\omega}} \left(\mathbf{r}^\top \boldsymbol{\omega} - \lambda \, \boldsymbol{\omega}^\top Q \boldsymbol{\omega} \right), \tag{4}$$

subject to non-negativity and full-investment constraints.

The parameter $\lambda > 0$ represents the investor's degree of risk aversion. Larger values of λ place greater emphasis on variance reduction, leading to more conservative portfolios, while smaller values allow higher exposure to risky assets in pursuit of increased expected return. Unlike the maximum Sharpe ratio strategy, which produces a single tangency portfolio, the Markowitz framework generates a family of optimal portfolios along the efficient frontier, each corresponding to a different level of risk tolerance.

Although mean–variance optimization provides a foundational framework for portfolio construction, its effectiveness depends critically on the quality of the estimated input parameters. In particular, inaccuracies in expected return estimates can significantly affect the resulting optimal weights, motivating the consideration of alternative methods that incorporate robustness to estimation uncertainty.

2.2.5 Robust optimization with uncertain expected returns

The robust optimization strategy extends the mean–variance framework by explicitly accounting for uncertainty in expected returns. Rather than assuming that the expected return vector is known exactly, it is assumed to lie within a predefined uncertainty set centered around a nominal estimate $\mathbf{r}_0$. In this formulation, portfolio weights are selected to maximize performance under the least favourable realization of expected returns within the uncertainty set [12]

$$\max_{\boldsymbol{\omega}} \min_{\mathbf{r} \in \mathcal{S}(\mathbf{r}_0)} \left(\mathbf{r}^\top \boldsymbol{\omega} - \lambda \boldsymbol{\omega}^\top Q \boldsymbol{\omega} \right). \tag{5}$$

In this study, the uncertainty set $\mathcal{S}(\mathbf{r}_0)$ is specified as an ellipsoid characterized by an uncertainty-aversion parameter κ and a matrix capturing estimation error in expected returns [12]

$$\mathcal{S}(\mathbf{r}_0) = \left\{ \mathbf{r} : (\mathbf{r} - \mathbf{r}_0)^\top \Sigma_r^{-1} (\mathbf{r} - \mathbf{r}_0) \leq \kappa^2 \right\}. \tag{6}$$

This formulation leads to a robust counterpart of the Markowitz optimization problem, in which an additional penalty term is introduced to control sensitivity to return uncertainty. The resulting allocations aim to improve stability and performance robustness when return estimates are imprecise.

2.3 Analysis A

For this part of the analysis, the closing prices of AAPL, AMGN, AMZN, AXP, BA, CAT, CRM, CSCO, CVX, DIS, GS, HD, HON, IBM, and JNJ are used to compute the portfolio weights corresponding to different investment strategies.

2.3.1 Backtesting framework and strategy implementation

The portfolio strategies considered in this study are dynamic, meaning that portfolio weights are periodically revised as new market information becomes available. At each rebalancing date, the existing portfolio composition is compared with the allocation suggested by the strategy's decision rule, and the portfolio is adjusted accordingly. This process allows the strategies to respond to changing market conditions while maintaining practical investment constraints such as full capital allocation and the absence of short selling. By distinguishing changes caused by market movements from those resulting from deliberate reallocation decisions, the framework provides a realistic setting for evaluating portfolio performance over time.

Before the evaluation period begins, an initial calibration stage is required to place each strategy in a meaningful starting state. Without this step, portfolios would start fully in cash and remain uninvested until the first scheduled rebalancing date, which could distort early performance results. To address this issue, the first 40 trading days of historical price data are used as a warm-up period. During this stage, each strategy estimates the quantities needed for portfolio construction, including return and risk measures. The resulting allocations approximate the positions the strategies would have held had they already been operating before the start of the evaluation window.

To illustrate the effect of this initialization process and to highlight differences in initial capital allocation (Assets A) across strategies, Fig. 1 reports the portfolio weights obtained at the end of the 40-day warm-up period. Although all strategies use the same historical price data, they produce different starting allocations because of their distinct allocation rules. The equal-weighted strategy distributes capital evenly across all assets, whereas

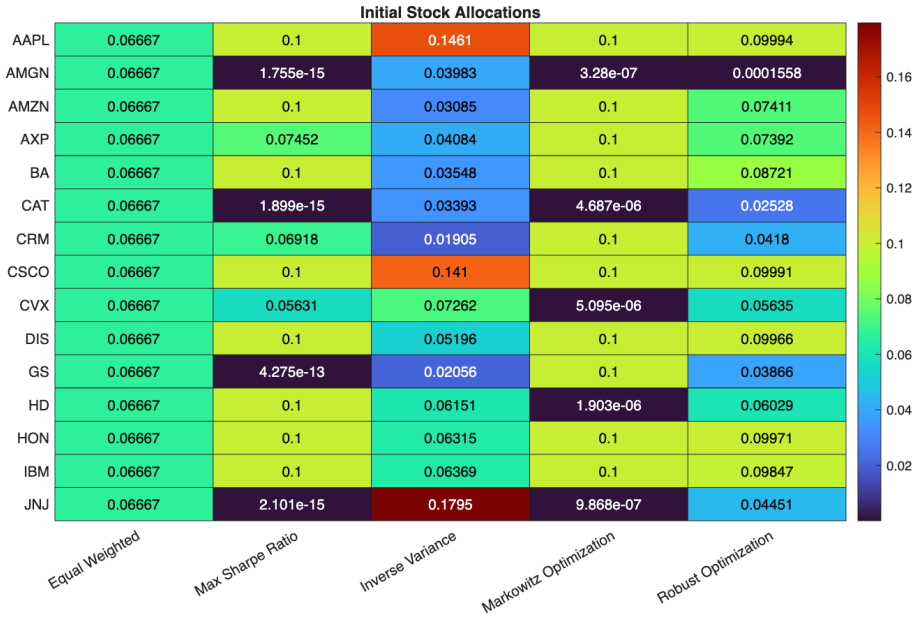


Figure 1: Initial portfolio weights (Assets A) after the 40-day warm-up period

the remaining strategies allocate capital according to their individual objectives. The inverse variance strategy places greater weight on assets with relatively lower estimated risk, such as JNJ, CSCO, and AAPL, while the optimization-based strategies produce more concentrated allocations based on their optimization criteria. These weights represent the initial portfolio positions used at the start of the backtest and should be viewed as calibration-based starting allocations rather than indicators of future performance.

After the warm-up period, the backtest is performed using the remaining data. Portfolio allocations are updated at discrete rebalancing dates rather than continuously, reflecting common portfolio management practice. In this study, portfolios are rebalanced every 21 trading days, approximately corresponding to a monthly adjustment cycle. Allocation decisions are based on a rolling look-back window of recent historical data, allowing the strategies to adapt to changing market conditions while avoiding reliance on outdated information.

All strategies are evaluated within a unified backtesting framework that applies the same market data, rebalancing schedule, and performance crite-

ria to every strategy. To ensure meaningful comparisons, the initial portfolio value and risk-free rate are specified at the framework level and remain identical across all simulations. The annualized risk-free rate is adjusted internally to match the daily frequency of the asset price series. By standardizing these settings, any differences in performance can be attributed to the allocation methodology rather than implementation choices.

The simulation begins only after the warm-up period has been completed so that every portfolio decision is based solely on information available at the time it is made. This prevents look-ahead bias and ensures that the reported results reflect realistic investment conditions. Throughout the evaluation period, the framework records portfolio values, asset holdings, realized returns, turnover, drawdowns, and trading costs. These outputs provide the basis for the subsequent comparison of portfolio performance across the competing investment strategies.

2.3.2 Evaluation of backtest performance outcomes (Assets A)

Following backtest execution, Table 1 outlines the empirical performance of each portfolio strategy across risk, return, and transaction metrics. Properly evaluating these outcomes requires accounting for the strictly out-of-sample nature of the decision process, as portfolio weights were computed using only historical data available at each rebalance date. For example, the Maximum Sharpe Ratio approach selects weights that maximize historical risk-adjusted returns over the preceding look-back window. Once fixed, these weights are evaluated against subsequent, unobserved price trajectories. Consequently, allocations optimized on historical windows can underperform during subsequent periods. This in-sample optimization drag is reflected in the lower realized returns and Sharpe ratios of the Maximum Sharpe Ratio strategy, reinforcing that optimized historical performance does not ensure superior out-of-sample results.

Table 1 compares the performance of the five portfolio strategies across return, risk, and trading characteristics. In terms of return, the Equal-Weighted strategy achieves the highest total (0.15462) and average return (0.00075), followed by Inverse Variance (total return 0.13567) and Robust Optimization (total return 0.12119). In contrast, the Maximum Sharpe Ratio (0.11600) and Markowitz Mean-Variance strategies (0.10239) yield comparatively lower realized performance. This demonstrates that strategies explicitly optimized on historical data do not necessarily deliver superior out-of-sample outcomes.

From a risk perspective, the Inverse Variance strategy exhibits the lowest

volatility (0.01162) and the smallest maximum drawdown (0.18125), proving its ability to stabilize fluctuations by allocating greater weight to lower-variance assets. Robust Optimization achieves a similarly low volatility (0.01194) and drawdown (0.19528). Conversely, the Markowitz strategy records the highest volatility (0.01439) and the largest maximum drawdown (0.23935), reflecting its sensitivity to parameter estimation error.

Trading activity and turnover further distinguish the performance across strategies. The Maximum Sharpe Ratio strategy records the highest average turnover (0.00776) and buy/sell costs (0.40795), driven by frequent position reallocations from unstable parameter estimates. The Markowitz strategy exhibits a similarly high average turnover (0.00726). In contrast, the Equal-Weighted (average turnover 0.00115) and Inverse Variance strategies (average turnover 0.00310) maintain substantially lower rebalancing activity, resulting in superior allocation stability over time.

In terms of portfolio structure, Equal Weighting guarantees uniform exposure across all constituent assets, whereas Inverse Variance achieves diversification through a risk-balanced allocation. Robust Optimization avoids extreme weight concentration while generating moderate returns. Overall, the Inverse Variance strategy provides the most balanced combination of risk control, reduced drawdown, and allocation stability, whereas the Equal-Weighted strategy stands out for its superior cumulative return and implementation simplicity.

Results	**Equal Weighted**	**Max Sharpe Ratio**	**Inverse Variance**	**Markowitz Opt.**	**Robust Opt.**
Total Return	0.15462	0.116	0.13567	0.10239	0.12119
Sharpe Ratio	0.057194	0.044587	0.053834	0.036221	0.04764
Volatility	0.012483	0.012399	0.011624	0.014396	0.011948
Average Turnover	0.0011468	0.0077634	0.0030982	0.0072563	0.0027509
Max Turnover	0.032336	0.30743	0.16096	0.30629	0.11499
Average Return	0.00075239	0.00059168	0.00066444	0.00056033	0.00060797
Max Drawdown	0.19851	0.2026	0.18125	0.23935	0.19528
Average Buy Cost	0.060207	0.40795	0.16084	0.36997	0.14165
Average Sell Cost	0.060207	0.40795	0.24126	0.36997	0.14165

Table 1: Strategy performance (Assets A)

2.3.3 Evolution of portfolio value (Assets A)

To illustrate the dynamic behavior of each allocation strategy over time, Fig. 2 displays the equity curves tracking portfolio value evolution across

the backtesting window. These curves capture the cumulative effect of periodic rebalancing and subsequent market movements, offering clear visual insight into wealth accumulation patterns, growth trajectories, and drawdowns. Specifically, the trajectories in Fig. 2 highlight that the Equal-Weighted portfolio achieves the steepest upward growth trajectory, while the Inverse Variance curve demonstrates the smoothest, most stable trajectory with minimal visible pullbacks. Conversely, the Markowitz and Maximum Sharpe Ratio curves exhibit noticeable structural dips and heightened fluctuations, visually confirming their higher drawdown vulnerability and lower out-of-sample stability compared to non-optimized approaches.

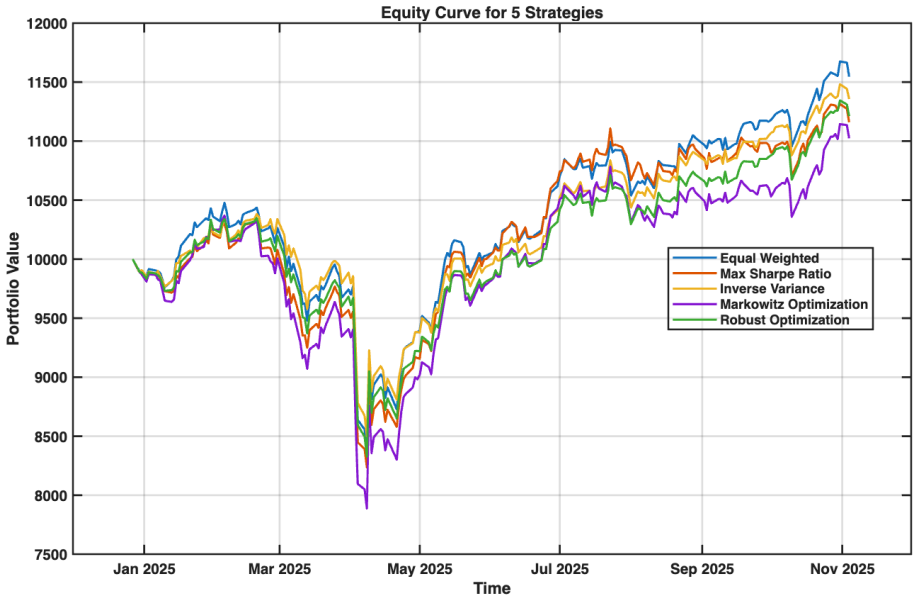


Figure 2: Evolution of portfolio value under competing investment strategies (Assets A)

The average turnover across strategies is shown in Fig. 3, providing insight into the frequency of portfolio rebalancing and associated trading activity. As illustrated, the Maximum Sharpe Ratio and Markowitz strategies exhibit the highest turnover levels, reflecting frequent weight reallocations driven by sensitivity to historical parameter estimates. Conversely, the Equal Weighted and Inverse Variance strategies maintain significantly lower turnover, demonstrating superior allocation stability over time. From a practical standpoint, lower rebalancing activity makes these passive and risk-balanced approaches more favorable because it minimizes execution drag and

preserves net returns. However, turnover alone is not the sole benchmark for evaluating portfolio performance; it must be assessed alongside net realized returns, maximum drawdowns, and volatility to capture the complete trade-off between implementation costs and risk-adjusted efficiency.

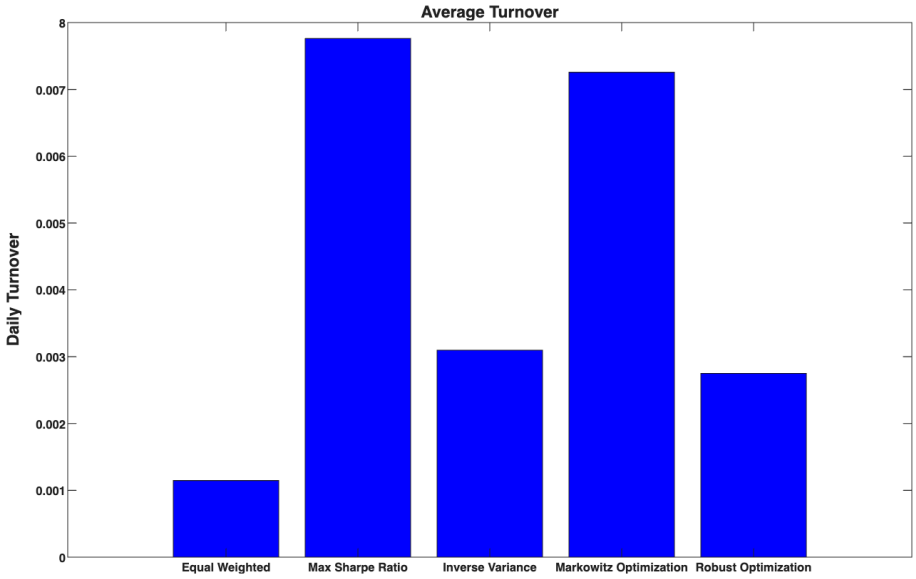


Figure 3: Average portfolio turnover across investment strategies (Assets A)

To examine how portfolio allocations evolve over time, the dynamic asset positions implied by each strategy are visualized using area charts of daily portfolio weights.

Fig. 4 through Fig. 8 present the evolution of asset-level portfolio weights over time for the five allocation strategies considered in Analysis A. Each figure is displayed as a stacked area plot, where individual components represent the proportion of total portfolio capital allocated to each asset at a given point in time, and the aggregate height reflects the full-investment constraint. The temporal variation in these stacked profiles captures both passive changes induced by asset price movements between rebalancing dates and active adjustments arising from periodic portfolio re-optimization. Across all figures, localized contractions in individual layers correspond to periods where the associated asset receives a reduced allocation, highlighting the dynamic redistribution of capital across the investment universe.

Equal-weighted strategy (A)—Asset allocation over time Fig. 4 presents the dynamic allocation path of the equal-weighted strategy, where asset weights evolve solely due to price fluctuations between rebalance dates. Capital remains evenly distributed across all constituent stocks, displaying smooth weight adjustments over time. This continuous stability accounts for the low trading turnover observed in Table 1 and confirms its role as a baseline benchmark for Analysis A.

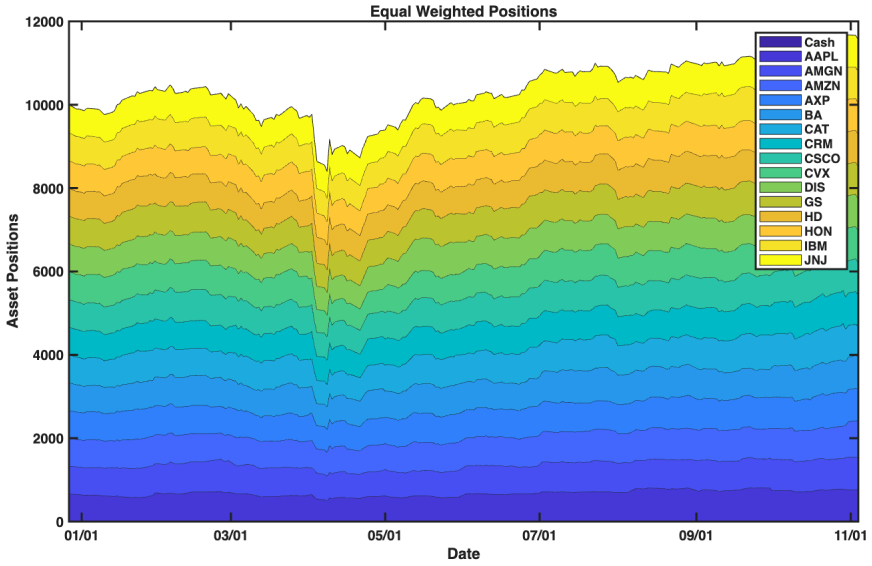


Figure 4: Asset-level portfolio positions over time (Equal-weighted strategy, A)

Maximum Sharpe ratio strategy (A)—Asset allocation over time Fig. 5 highlights the portfolio allocation dynamics under the maximum Sharpe ratio strategy, where weight adjustments are driven by periodic recalculations of risk-adjusted performance. The stacked area plot exhibits pronounced reallocation spikes, as capital is dynamically shifted toward top-performing assets while other position sizes contract or vanish completely. These notable weight swings align with the elevated turnover and higher trading costs reported in Table 1, reflecting the sensitivity of tangency optimization to input parameters.

Inverse variance strategy (A)—Asset allocation over time Fig. 6 illustrates the weight evolution for the inverse variance strategy, demonstrat-

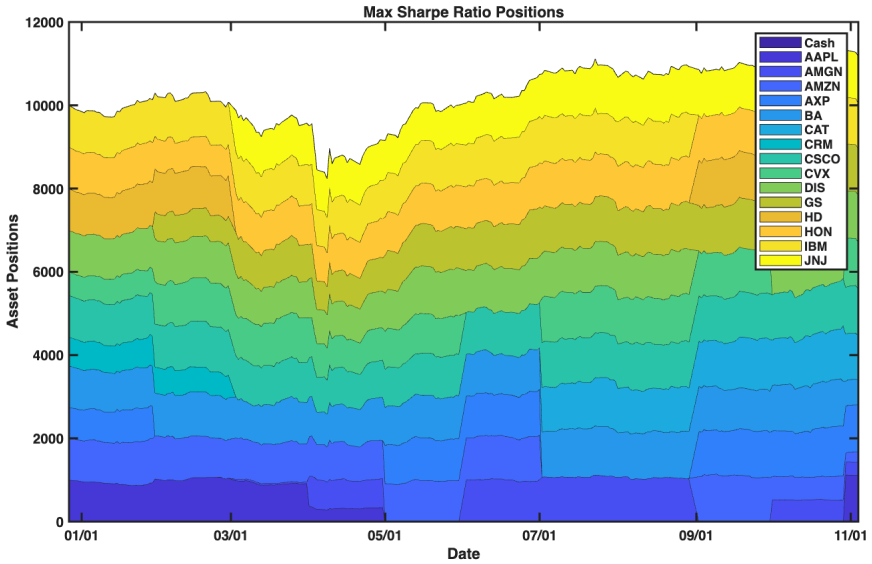


Figure 5: Asset-level portfolio positions over time (Maximum Sharpe ratio strategy, A)

ing the persistent allocation structure derived from risk-based weighting. The portfolio maintains stable position sizes in low-volatility constituents (such as JNJ, CSCO, and AAPL), with layer thickness adjusting gradually across rebalance points. This defensive asset distribution directly contributes to the lowest volatility and smallest maximum drawdown recorded in Table 1.

Markowitz optimization strategy (A)—Asset allocation over time Fig. 7 tracks the portfolio composition over time for the Markowitz mean–variance strategy, capturing the ongoing trade-off between return and risk. The allocation paths exhibit sharper position adjustments across assets as updated parameter estimates modify the optimal weighting decision. These variable allocation blocks help explain the strategy's increased turnover, higher trading expenditure, and larger drawdown in Table 1.

Robust optimization strategy (A)—Asset allocation over time. Fig. 8 depicts the allocation path generated by the robust optimization strategy, showing how asset weights adapt over time while incorporating parameter uncertainty. Compared to the standard Markowitz approach, the robust formulation leads to smoother and more balanced position tran-

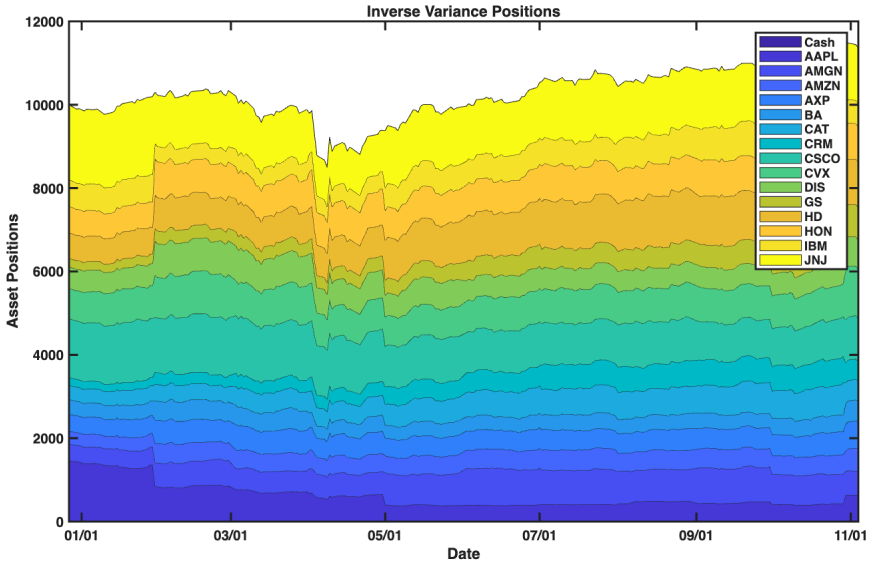


Figure 6: Asset-level portfolio positions over time (Inverse variance strategy, A)

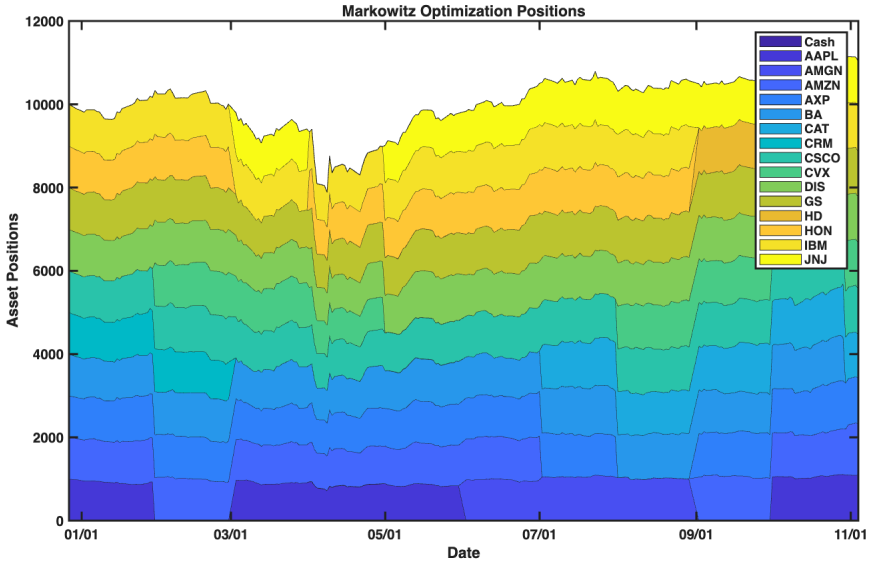


Figure 7: Asset-level portfolio positions over time (Markowitz optimization strategy, A)

sitions. By penalizing return estimation errors, it mitigates extreme capital shifts while preserving portfolio flexibility and maintaining controlled volatility (Table 1).

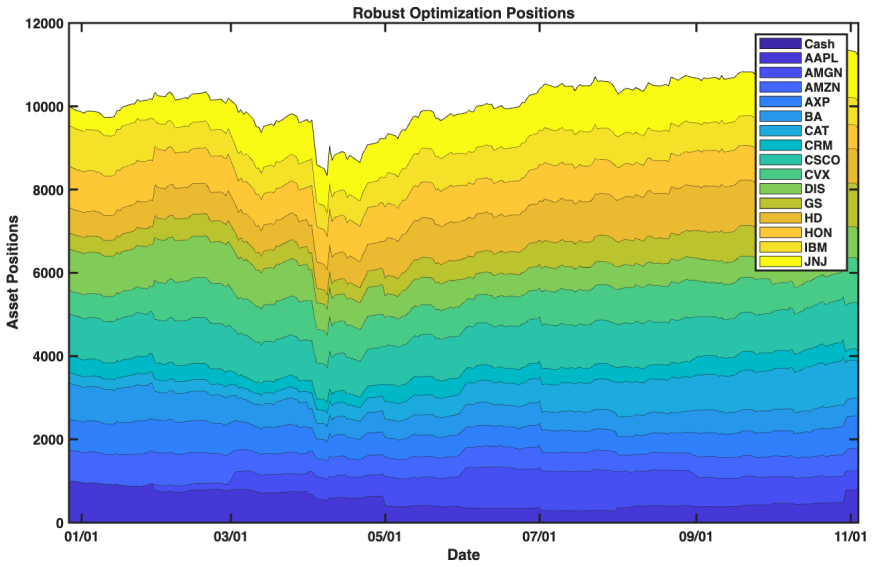


Figure 8: Asset-level portfolio positions over time (Robust optimization strategy, A)

2.4 Analysis B

For this part of the analysis, the closing prices of JPM, KO, MCD, MMM, MRK, MSFT, NKE, NVDA, PG, SHW, TRV, UNH, V, VZ, and WMT are used to compute the weights of the different investment strategies.

To highlight differences in initial capital allocation across strategies, Fig. 9 displays the portfolio weights computed at the end of the warm-up period, where each strategy translates the same historical price information into a starting allocation. The equal-weighted strategy gives uniform exposure to the full asset set and therefore serves as the broad diversification benchmark. Under the inverse variance strategy, assets such as KO, MCD, PG, V, VZ, and WMT receive comparatively larger weights, indicating that their estimated variances over the calibration window were relatively low compared to other assets. The maximum Sharpe ratio and Markowitz strategies are more selective, assigning substantial weight to a narrower set of assets while giving very small positions to others. The robust optimiza-

tion strategy keeps a wider spread of exposures than the more aggressive optimization rules, with visible emphasis on assets that combine moderate risk with stable historical performance. The strategies differ mainly in how they balance concentration, defensive exposure, and sensitivity to estimated returns. These initial weights should therefore be interpreted as calibration-period positions rather than evidence of long-term asset performance.

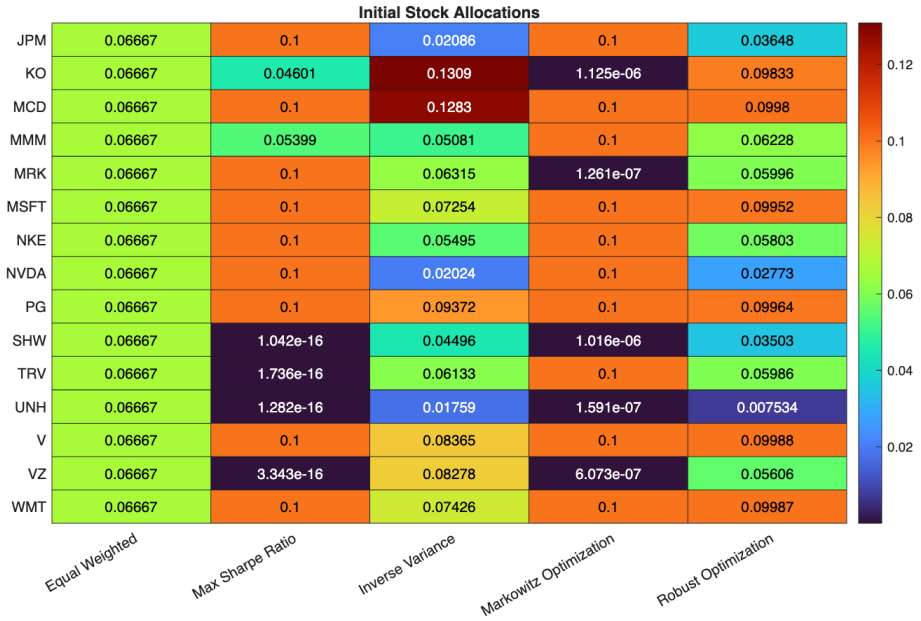


Figure 9: Initial portfolio weights (Assets B) after the 40-day warm-up period

2.4.1 Evaluation of backtest performance outcomes (Assets B)

After running the backtest, Table 2 summarizes the performance of each strategy for Assets B using standard evaluation metrics: realized return, volatility, drawdown, turnover, and trading cost. The market conditions and testing rules were kept the same for all methods.

The interpretation of Table 2 must account for the fact that portfolio decisions are made using only information available before each rebalance date. This point is especially relevant for Analysis B because the optimization-based strategies often form concentrated allocations from a limited estimation window. Their selected portfolios may appear efficient at the time of construction, yet their realized performance depends on price movements

Results	Equal Weighted	Max Sharpe Ratio	Inverse Variance	Markowitz Opt.	Robust Opt.
Total Return	0.07382	0.056951	0.046644	0.048307	0.052193
Sharpe Ratio	0.035303	0.028645	0.023888	0.023782	0.026636
Volatility	0.0096722	0.0091613	0.0089821	0.009571	0.0089964
Average Turnover	0.0011811	0.0056974	0.0027293	0.0066467	0.0022865
Max Turnover	0.032776	0.21149	0.10211	0.21104	0.090746
Average Return	0.00038078	0.00030194	0.00025419	0.00026722	0.0002792
Max Drawdown	0.11513	0.12427	0.10273	0.12852	0.10863
Average Buy Cost	0.061486	0.30048	0.14273	0.34909	0.1195
Average Sell Cost	0.061486	0.30048	0.21409	0.34909	0.1195

Table 2: Strategy performance (Assets B)

that occur after the weights are fixed. The empirical results show that greater optimization complexity can fail to translate into stronger realized outcomes in this asset universe.

In terms of return, the Equal-Weighted strategy achieves the highest total (0.07382) and average return (0.00038), demonstrating that simple, broad capital distribution effectively captures market recovery without relying on active selection or frequent reoptimization. The Maximum Sharpe Ratio (0.05695) and Robust Optimization strategies (0.05219) follow, whereas the Markowitz (0.04831) and Inverse Variance strategies (0.04664) yield weaker cumulative performance. Inverse Variance deliberately forfeits upside potential by concentrating on low-risk assets, while the Markowitz strategy is impacted by parameter instability and unfavorable reallocations.

From a risk perspective, the Inverse Variance strategy provides the strongest downside protection, recording the lowest volatility (0.00898) and the smallest maximum drawdown (0.10273). This confirms that the Assets B universe contains sufficient low-risk constituents for a variance-based rule to mitigate portfolio stress. Robust Optimization exhibits a similarly defensive profile with low volatility (0.00899) and a controlled maximum drawdown (0.10863). The Equal-Weighted strategy incurs slightly higher volatility (0.00967) but maintains a moderate drawdown (0.11513) due to its broad asset distribution, whereas the Markowitz strategy experiences the largest maximum drawdown (0.12852).

Trading activity and costs further separate the strategies. The Markowitz strategy records the highest average turnover (0.00665) and trading costs (0.34909), followed by the Maximum Sharpe Ratio strategy (average turnover 0.00570). These high turnover rates highlight the sensitivity of classical optimization to shifting historical estimates, where additional trading activity

fails to yield a performance advantage. Conversely, the Equal-Weighted (average turnover 0.00118) and Robust Optimization strategies (average turnover 0.00229) maintain high allocation stability and minimal transaction drag.

Overall, the results demonstrate a clear separation between broad capital distribution and risk-balanced diversification. The Equal-Weighted strategy offers the strongest total return and risk-adjusted efficiency through simple diversification, while the Inverse Variance strategy remains the most effective choice for capital preservation and downside control. Robust Optimization serves as a practical middle ground among optimized methods by maintaining allocation stability while capturing higher returns than purely defensive approaches.

2.4.2 Evolution of portfolio value (Assets B)

Fig. 10 traces the portfolio value trajectories for Assets B, illustrating how each strategy accumulates wealth over the backtest period. Consistent with the trends in Analysis A, the Equal-Weighted strategy maintains the highest equity curve throughout the horizon, confirming its superior cumulative return capability. The Inverse Variance curve displays the most consistent, low-volatility progression, maintaining a smooth upward slope even during market pullbacks. In contrast, the Markowitz and Maximum Sharpe Ratio curves lag behind, exhibiting sharp drawdowns and noticeable stagnation that visually illustrate their out-of-sample instability and sensitivity to estimation errors.

The average turnover across strategies is shown in Fig. 11, providing insight into the frequency of portfolio rebalancing and associated trading activity. Mirroring the behavior observed in Analysis A, the results for Assets B confirm that the Maximum Sharpe Ratio and Markowitz strategies generate the highest turnover, whereas the Equal Weighted and Inverse Variance models maintain the lowest rebalancing requirements. This consistency reinforces that parameter-dependent optimizations persistently incur higher trading frequency across different asset pools, while risk-weighted and static allocations offer greater operational stability. Consequently, while low turnover makes Equal Weighted and Inverse Variance allocations structurally more favorable for cost management, rebalancing frequency remains just one dimension of strategy selection and must be weighed against drawdown control and net realized returns.

To examine how portfolio allocations evolve over time, the dynamic asset positions implied by each strategy are visualized using area charts of daily

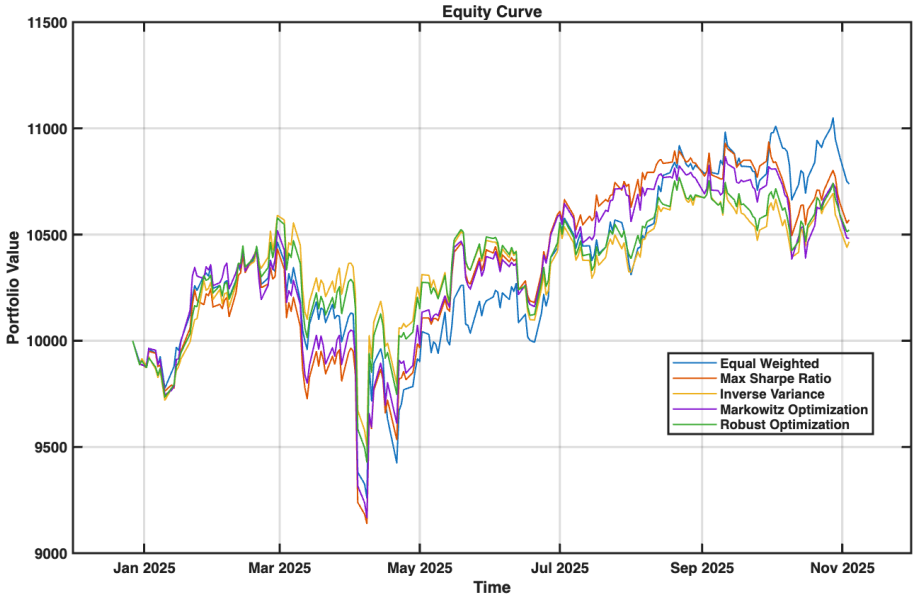


Figure 10: Evolution of portfolio value under competing investment strategies (Assets B)

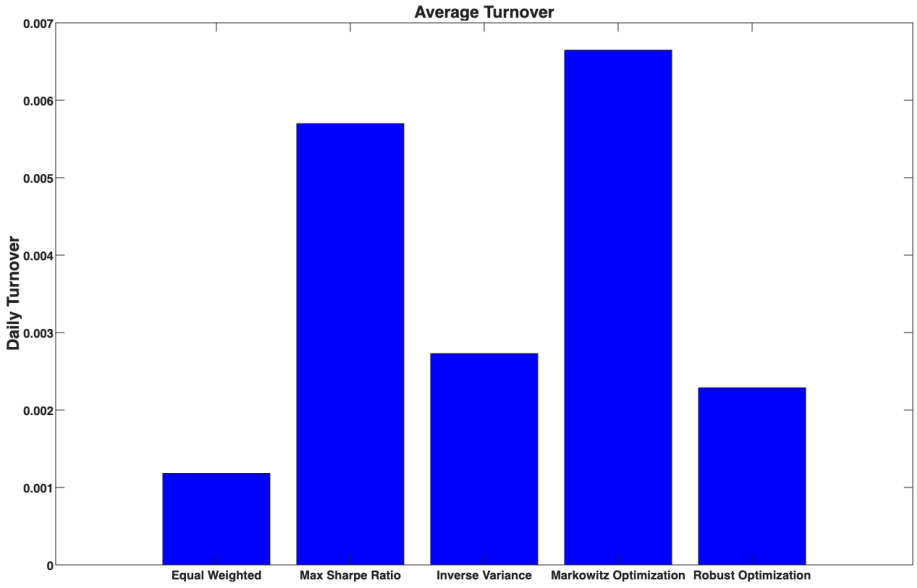


Figure 11: Average portfolio turnover across investment strategies (Assets B)

portfolio weights.

Fig. 12 through Fig. 16 present the evolution of asset-level portfolio weights over time for the allocation strategies considered in Analysis B. The stacked area plots show how each method distributes capital across the same asset universe and how those distributions change as prices move and rebalancing decisions occur.

Equal-weighted strategy (B)—Asset allocation over time Fig. 12 illustrates the time-varying portfolio composition of the equal-weighted strategy. The allocation remains broadly distributed, and changes in the stacked areas mainly reflect asset price movements between rebalancing dates. This pattern explains the strategy's low turnover and shows why it functions as a stable benchmark in Analysis B.

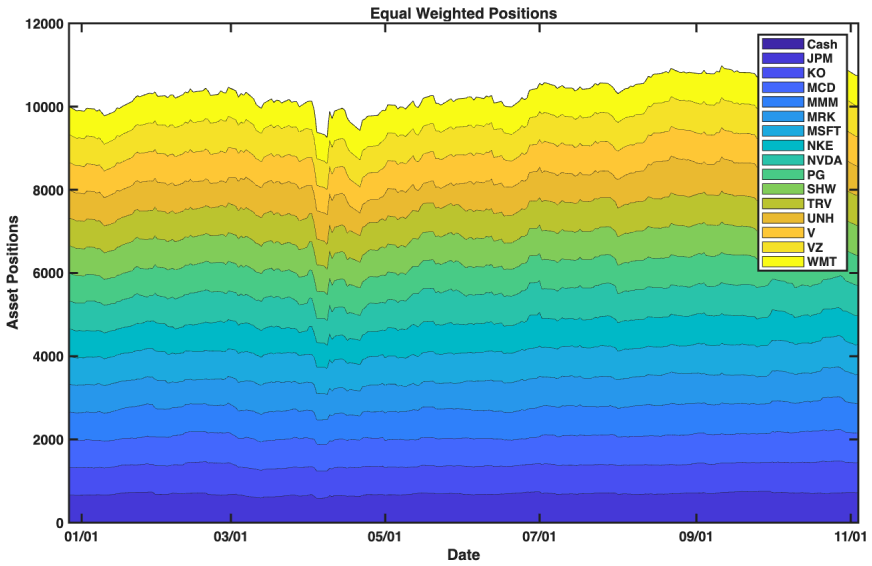


Figure 12: Asset-level portfolio positions over time (Equal-weighted strategy, B)

Maximum Sharpe ratio strategy (B)—Asset allocation over time Fig. 13 depicts the evolution of asset allocations under the maximum Sharpe ratio strategy. The area chart displays more abrupt changes than the equal-weighted and inverse variance strategies, with capital shifting toward selected assets as risk-adjusted return estimates are updated. Several layers

expand or contract sharply at rebalancing points, and some asset exposures become temporarily very small or nearly disappear from the stacked area plot. These visible reallocations are consistent with the higher turnover reported in Table 2 and with the sensitivity of Sharpe-ratio optimization to estimated returns and covariance inputs.

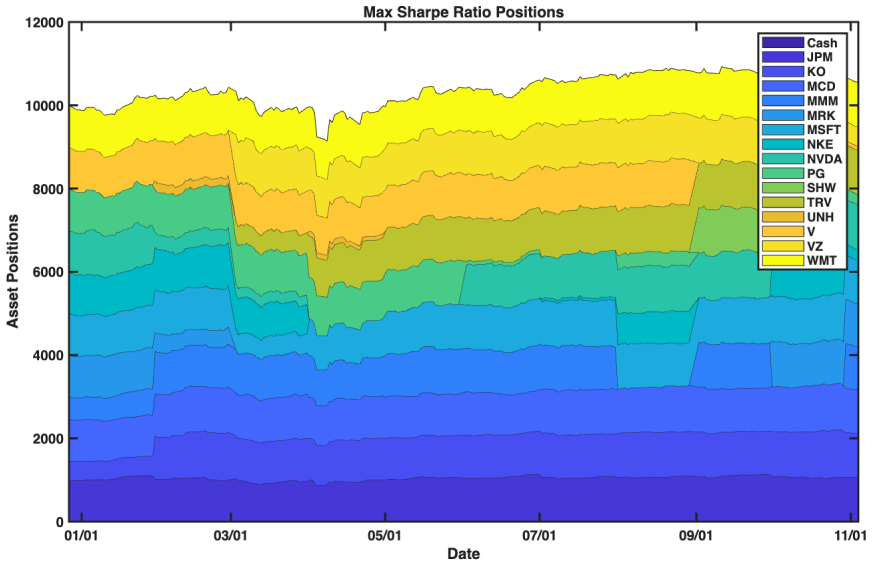


Figure 13: Asset-level portfolio positions over time (Maximum Sharpe ratio strategy, B)

Inverse variance strategy (B)—Asset allocation over time Fig. 14 presents the allocation dynamics of the inverse variance strategy. The portfolio keeps relatively persistent exposure to lower-volatility assets, and its weights adjust more gradually over time. This stable structure supports the low volatility and drawdown results observed in Table 2.

Markowitz optimization strategy (B)—Asset allocation over time Fig. 15 shows the evolution of portfolio weights for the Markowitz strategy. The allocation path contains sharper shifts across assets, reflecting the strategy's sensitivity to estimated expected returns and covariance relationships. This behaviour helps explain its higher turnover, larger trading cost, and weaker drawdown control.

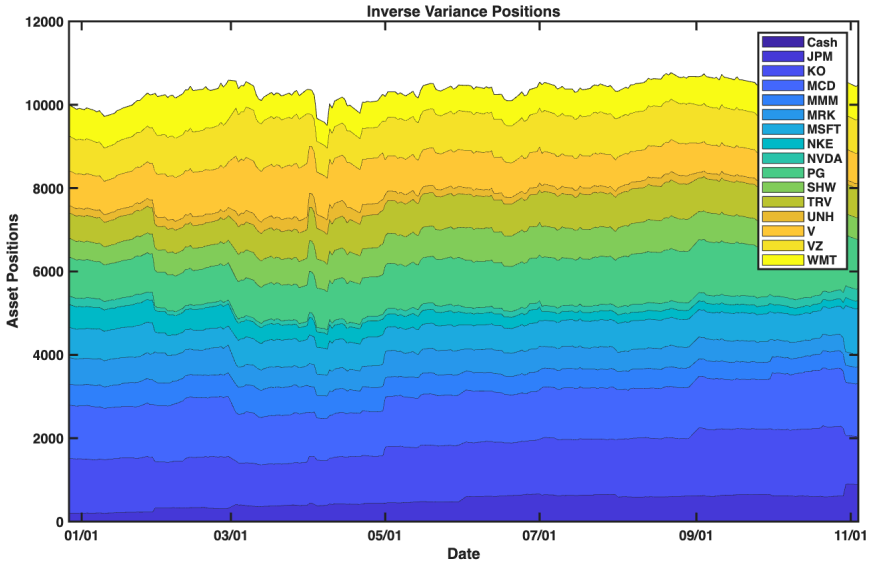


Figure 14: Asset-level portfolio positions over time (Inverse variance strategy, B)

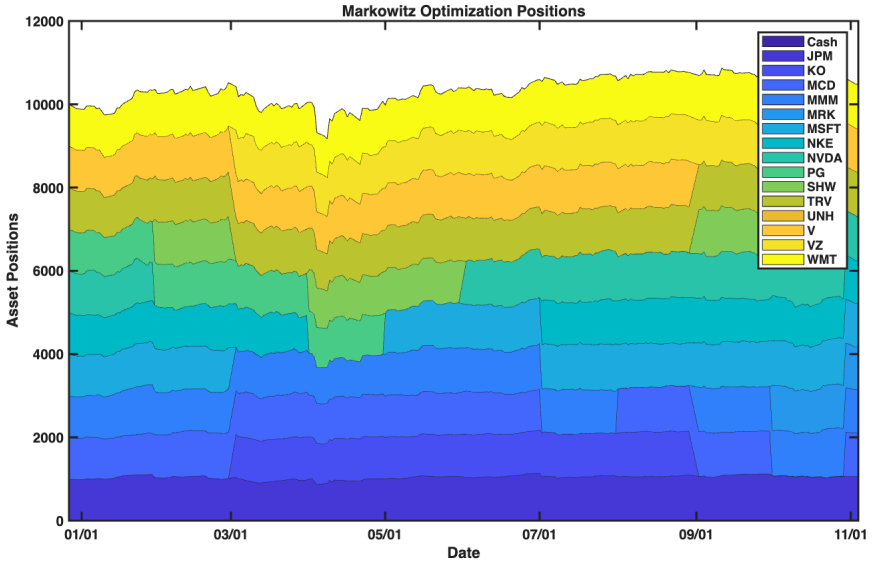


Figure 15: Asset-level portfolio positions over time (Markowitz optimization strategy, B)

Robust optimization strategy (B)—Asset allocation over time Fig. 16 illustrates the asset allocation path generated by the robust optimization strategy. Compared with the standard Markowitz allocation, the robust strategy produces smoother and more balanced position changes. Its allocation path reflects the effect of accounting for uncertainty in expected returns, which reduces extreme reallocations while preserving adaptability.

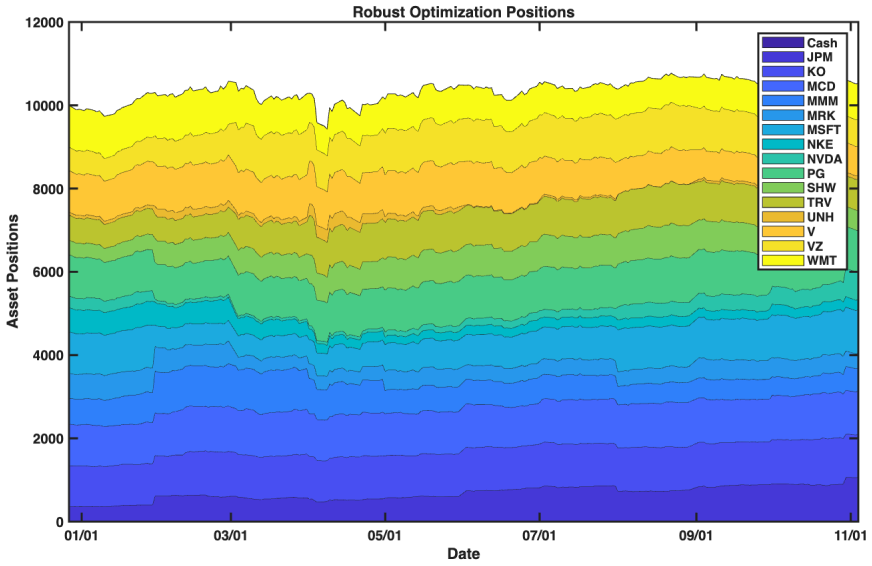


Figure 16: Asset-level portfolio positions over time (Robust optimization strategy, B)

Across these figures, the defining feature of Analysis B is the contrast between stable broad exposure and concentrated optimization. Equal weighting produces the smoothest benchmark allocation, inverse variance delivers a defensive risk-based profile, and robust optimization moderates the instability associated with standard mean–variance optimization. The maximum Sharpe ratio and Markowitz strategies show more visible reallocation blocks, which match their higher turnover and less stable realized performance. For Assets B, allocation stability and controlled risk exposure contribute more reliably to long-term performance than frequent re-optimization driven by unstable historical parameter estimates.

Criterion	**Equal Weighted (EW)**	**Inverse Variance (IV)**	**Robust Optimization (RO)**	**Max Sharpe Ratio (MSR)**	**Markowitz Mean-Variance (MV)**
Return performance	Highest in A and B	Second in A, lowest in B	Moderate	Moderate-low	Lowest in A, second lowest in B
Volatility control	Moderate	Lowest	Near-lowest	Moderate	Highest
Maximum drawdown	Moderate	Lowest	Second lowest	Higher	Highest
Turnover	Lowest	Low-moderate	Low	Highest	Highest
Transaction costs	Lowest	Low	Low	Highest	Highest
Allocation stability	Highest	High	High	Low	Low
Diversification	Maximum	Risk-based	Balanced	Concentrated	Concentrated
Sensitivity to estimation error	None	Low	Controlled	High	Very high
Practical portfolio construction	Excellent	Excellent	Good	Weak	Weak

Table 3: Comparative backtesting summary from Analysis A and Analysis B

3 Discussion

The results in Table 3 demonstrate broad structural consistency across both asset universes A and B. While the overall hierarchy between heuristic and optimization-driven models holds, specific return rankings vary slightly between the two pools (e.g., Inverse Variance yields the second-highest return in Analysis A but ranks lower in Analysis B, whereas Maximum Sharpe Ratio improves its return rank in Analysis B). Nevertheless, key performance characteristics—such as volatility control and turnover behavior—remain stable. While the Equal Weighting method yielded the top cumulative return in both groups, the Inverse Variance framework systematically achieved the lowest volatility, minimal maximum drawdowns, and modest turnover while preserving competitive return levels, making it the most balanced allocation overall. Conversely, despite their theoretical foundation, the Markowitz and Maximum Sharpe Ratio strategies exhibited elevated turnover rates, greater transaction drag, and lower realized returns. Robust Optimization enhanced weighting stability relative to classical mean–variance models, yet its performance remained less dependable than Inverse Variance.

While the Inverse Variance strategy appears to offer the most balanced overall performance, the results also demonstrate that portfolio allocation decisions depend strongly on the investor's objective. The Equal Weighted strategy consistently generated the highest realized returns in both analyses, making it attractive when return maximization is the primary goal. In contrast, the Inverse Variance strategy delivered the lowest volatility and smallest maximum drawdowns, indicating superior downside-risk control. It also maintained relatively low turnover and transaction costs compared with the Maximum Sharpe Ratio and Markowitz strategies, whose frequent reallocations increased implementation costs without producing superior realized outcomes. These findings suggest that investors seeking growth may prefer Equal Weighting, whereas investors emphasizing risk management, capital preservation, and allocation stability would likely favor the Inverse Variance approach. Overall, the latter provides the strongest combination of risk control, turnover management, and drawdown reduction, making it a compelling choice for practical portfolio allocation.

An additional implication of the comparative analysis concerns the identification of assets suitable for subsequent portfolio optimization. The volatility-based investigation [18] of KSE-30 equities showed that highly volatile stocks were generally excluded from optimized portfolios, whereas assets with lower or moderate sensitivity to market movements contributed more

meaningfully to portfolio construction. Similarly, the Black-Litterman analysis [19] retained only a limited set of assets with favorable return and risk characteristics after blending market equilibrium returns with analyst views. The appearance of assets such as HBL, LUCK, and POL—constituents of the Pakistan Stock Exchange KSE-30 index examined in [19]—within the final Black-Litterman allocation suggests that securities exhibiting both acceptable volatility characteristics and favorable expected-return profiles may represent promising candidates for future portfolio construction. Although a formal combined optimization framework would be required to confirm this observation, the current findings provide a preliminary indication that information from both analyses may assist in screening equities for subsequent optimal portfolio development and future Black-Litterman applications.

Furthermore, the comparison highlights an important limitation of historically popular optimization-based investment methods. In portfolio theory, the Maximum Sharpe Ratio strategy is frequently regarded as an attractive allocation rule because it seeks the highest risk-adjusted return on the efficient frontier. However, the backtesting results obtained in both analyses indicate that this theoretical advantage did not translate into consistently superior realized performance. The strategy exhibited substantially higher turnover and transaction costs than simpler allocation rules, while its realized returns and risk-adjusted outcomes remained comparatively modest. Similar behavior was observed for the Markowitz strategy. These findings emphasize that allocations derived from historical optimization may be highly sensitive to estimation error and changing market conditions. Consequently, relying exclusively on historically estimated inputs may not always be sufficient for index investment decisions, particularly when implementation costs and allocation stability are important considerations.

Ultimately, this analysis demonstrates that the Inverse Variance strategy provides the most balanced overall profile, while revealing broadly consistent structural performance patterns across two independent asset universes. Although asset-universe composition introduces slight variations in total return rankings for specific strategies, both evaluations yield identical overarching conclusions regarding return generation, risk control, turnover behavior, and allocation stability. Specifically, simpler diversification and risk-aware approaches consistently match or outperform complex optimization methods on realized portfolio outcomes. This indicates that the structural stability of the allocation mechanism is more vital to performance than aggressive optimization based on historical estimates. Because this pattern holds across multiple criteria and independent datasets, the observed behavior reflects a robust portfolio allocation principle rather than a dataset-specific artifact.

4 Conclusion

This study was conducted to examine whether different portfolio allocation strategies produce consistent investment outcomes when applied to different groups of equities within the same market environment. Using one year of daily closing price data from the Dow Jones Industrial Average (DJIA) constituents, two independent analyses were performed on separate sets of fifteen assets. Five portfolio allocation strategies, namely Equal Weighted, Maximum Sharpe Ratio, Inverse Variance, Markowitz Mean-Variance Optimization, and Robust Optimization, were evaluated under an identical backtesting framework with periodic rebalancing.

The main contribution of this work is the direct comparison of simple diversification approaches, risk-based allocation methods, and optimization-based portfolio strategies across two independent asset universes using the same evaluation methodology. This allows the robustness of strategy performance to be assessed rather than relying on results from a single asset set.

The evidence obtained from both analyses shows that strategy performance is highly consistent across the two asset universes. The Equal Weighted strategy produced the highest realized returns, while the Inverse Variance strategy consistently achieved the lowest volatility, the smallest drawdowns, and relatively low turnover. In contrast, the Maximum Sharpe Ratio and Markowitz strategies exhibited higher turnover, higher transaction costs, and weaker realized performance despite their theoretical optimization advantages. The Robust Optimization strategy provided a compromise between return generation and risk control but did not consistently outperform the Inverse Variance approach.

The principal conclusion supported by the results is that stable and risk-aware allocation methods can be more effective in practice than conventional strategies that depend heavily on estimates of expected returns. Across both analyses, simple diversification and variance-based risk control delivered more consistent portfolio outcomes than more complex optimization procedures. These findings suggest that portfolio allocation decisions may benefit more from robustness, stability, and risk management than from aggressive optimization of historical return estimates.

5 Limitations of the study

The analysis is based on a single year of historical data and two subsets of DJIA constituents. Consequently, the findings may be influenced by the specific market conditions observed during the study period. Although the consistency of results across both asset universes strengthens the conclusions, the performance of the strategies may differ under alternative market regimes or longer investment horizons. In addition, the two asset subsets were formed by ticker order rather than by a sector-stratified procedure; a sector-balanced split of the DJIA constituents would help confirm whether the observed consistency in strategy rankings is robust to differences in sector composition across the two universes.

6 Future research directions

Future work may extend the analysis to multiple years of data and additional equity markets to examine the robustness of the observed results across different market environments. The findings of this study may also be combined with volatility-based screening and Black-Litterman portfolio optimization frameworks to investigate whether assets consistently favored by risk-aware allocation strategies can improve portfolio construction and asset selection decisions.

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