THE INFLUENCE OF FIBER ORIENTATION ON ENGINEERING CONSTANTS OF LAMINATED COMPOSITES

Iuliana DUPIR (HUDIȘTEANU)[1], Nicolae ȚĂRANU[2]

Rezumat. *O structură stratificată este realizată, în general, prin suprapunerea mai multor lamele armate unidirecțional, având unghiuri de orientare a fibrelor diferite, impuse în raport cu cerințele necesare de rezistență și rigiditate. Unghiurile de orientare a fibrelor au o influență decisivă asupra caracteristicilor stratificatelor. În lucrare se prezintă un studiu în care se analizează influența unghiului de orientare a fibrelor asupra caracteristicilor elastice ale stratificatelor. În acest sens, sunt determinate valorile constantelor elastice inginerești, atât pentru lamele ortotrope generale, cât și pentru stratificate unghiulare simetrice, alcătuite din aceste tipuri de straturi elementare. Rezultatele analitice sunt ilustrate prin grafice reprezentând variația constantelor elastice inginerești, în funcție de orientarea fibrelor, pentru diferite valori ale fracțiunii volumetrice de fibră din straturile elementare.*

Abstract. *A laminated composite is generally formed by stacking unidirectional laminas, with different fiber orientation angles, in the direction of the laminate thickness, in order to satisfy the design requirements. Fiber orientation influence is crucial on the composite laminate characteristics. The paper presents a study which analyzes the fiber orientation angles influence on the elastic behavior of the laminates. Therefore, the elastic engineering constants are determined for a generally orthotropic lamina and for a symmetric angle-ply laminate. The analytical results are represented by graphical distribution of the elastic engineering constants with respect to fiber orientation, for different fiber volume fractions.*

Keywords: angle-ply laminate, generally orthotropic lamina, engineering constants, fiber orientation angles, fiber volume fractions

1. Introduction

The elastic behavior in the transverse direction for a unidirectional reinforced lamina is low compared to the longitudinal one, therefore the composite structures must have sequences of stacking plies, with different fiber orientation angles [1].

The engineering elastic constants have significant importance when designing a multi-layered composite, because it predicts the elastic behavior of the laminates. The most important parameters that influence the elastic engineering constants are

[1]PhD student, Eng., Faculty of Civil Engineering and Building Services, Technical University "Gh. Asachi" of Iași, Romania (e-mail: iulianahudisteanu@ce.tuiasi.ro).
[2]Prof., PhD, Faculty of Civil Engineering and Building Services, Technical University "Gh. Asachi" of Iași, Romania, full member of the Academy of Romanian Scientists (e-mail: taranu@ce.tuiasi.ro).

the constituent materials properties, the fiber orientation angles, the fiber volume fractions and the stacking sequence.

The analysis of a composite laminate is dependent on the behavior of an individual layer. The most commonly used theory for analyzing multi-layered composites is the so called Classical Lamination Theory (CLT), derived from the classical plate theory proposed by Kirchhoff-Love [2].

The ply stiffness analysis is based on the following assumptions: (a) small deformations occur; (b) the theory of elasticity is valid; (c) each lamina is in a plate stress; (d) the ply is assumed to be homogeneous, that means that the properties are the same in any point [3].

2. The generally orthotropic lamina

When designing a multi-layered composite, it is essential to predict firstly the elastic behavior and the properties of a single layer, such as the off-axis unidirectional lamina [4].

Therefore, a composite elementary layer with unidirectional fibers embedded in a matrix, having the fiber orientation angle θ, with respect to a general system of reference (x, y), is called a generally orthotropic lamina [5, 6].

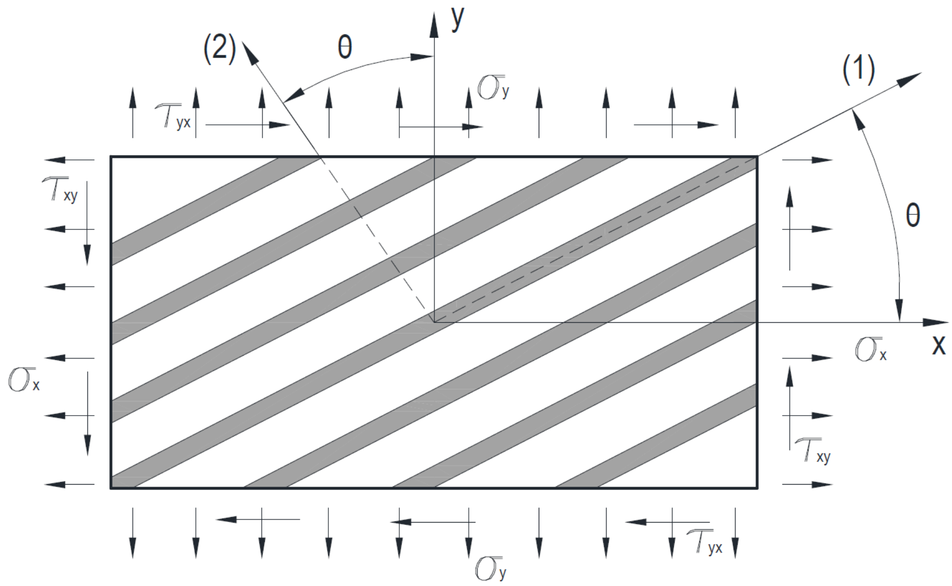


Fig. 1. Generally orthotropic lamina.

The determination of the elastic engineering constants with respect to the general system of reference (x, y) is related to the elastic constants with respect to the principal material axes (1, 2) and fiber orientation angles, as it follows [7-9]:

$$
\begin{aligned}
E_x &= \frac{1}{\frac{1}{E_1}c^4 + \left(\frac{1}{G_{12}} - 2\frac{\nu_{12}}{E_1}\right)s^2c^2 + \frac{1}{E_2}s^4} \\
E_y &= \frac{1}{\frac{1}{E_1}s^4 + \left(\frac{1}{G_{12}} - 2\frac{\nu_{12}}{E_1}\right)s^2c^2 + \frac{1}{E_2}c^4} \\
G_{xy} &= \frac{1}{2\left(\frac{2}{E_1} + \frac{2}{E_2} + \frac{4\nu_{12}}{E_1} - \frac{1}{G_{12}}\right)s^2c^2 + \frac{1}{G_{12}}\left(s^4 + c^4\right)} \\
\nu_{xy} &= -\frac{\left[c^2s^2\left(1 + \frac{E_1}{E_2} - \frac{E_1}{G_{12}}\right) - \left(c^4 + s^4\right)\nu_{12}\right]}{\left[c^4 + c^2s^2\left(-2\nu_{12} + \frac{E_1}{G_{12}}\right) + s^4\frac{E_1}{E_2}\right]} \\
\nu_{yx} &= -\frac{\left[c^2s^2\left(1 + \frac{E_1}{E_2} - \frac{E_1}{G_{12}}\right) - \left(c^4 + s^4\right)\nu_{12}\right]}{\left[s^4 + c^2s^2\left(-2\nu_{12} + \frac{E_1}{G_{12}}\right) + c^4\frac{E_1}{E_2}\right]}
\end{aligned}
\qquad (1)
$$

where: E_1, E_2 and G_{12} represent the axial and shear moduli with respect to the principal material axes; ν_{12} and ν_{21} are the Poisson's ratios, while θ is the fiber orientation angle; $c = \cos\theta$, $s = \sin\theta$.

The relations from the micromechanics that reveal the computation of the elastic engineering constants with respect to the principal material axes (1, 2), are the following [6, 9]:

$$
\begin{aligned}
E_1 &= E_f \cdot V_f + E_m \cdot V_m \\
E_2 &= \frac{E_f \cdot E_m}{V_f \cdot E_m + V_m \cdot E_f} \\
G_{12} &= \frac{G_f \cdot G_m}{V_f \cdot G_m + V_m \cdot G_f} \\
\nu_{12} &= \nu_f \cdot V_f + \nu_m \cdot V_m \\
\nu_{21} &= \nu_{12} \cdot \frac{E_2}{E_1}
\end{aligned}
\qquad (2)
$$

where: E_f and E_m represents the longitudinal Young's modulus of the fiber and matrix; G_f and G_m are the shear modulus of the fiber and matrix; ν_f and ν_m are the Poisson's ratios of the fiber and matrix; V_f and V_m represent the fiber and matrix volume fractions.

3. From lamina to laminates

The constitutive equation for an individual layer *k* of a generally orthotropic lamina in a multi-layered laminate is given by Equation (3a) or (3b) [1, 7]:

$$\{\sigma\}_k = [\overline{Q}]_k \{\varepsilon\}_k \tag{3a}$$

which can be expanded as:

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix}_k = \begin{bmatrix} \overline{Q}_{11} & \overline{Q}_{12} & \overline{Q}_{16} \\ \overline{Q}_{12} & \overline{Q}_{22} & \overline{Q}_{26} \\ \overline{Q}_{16} & \overline{Q}_{26} & \overline{Q}_{66} \end{bmatrix}_k \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix}_k \tag{3b}$$

where:

$[\overline{Q}]$ is the transformed reduced stiffness matrix;

$\sigma_x, \sigma_y, \tau_{xy}$ are the in-plane stress components along the global reference axes;

$\varepsilon_x, \varepsilon_y, \gamma_{xy}$ are the in-plane strain components along the global reference axes.

The elements $\overline{Q}_{ij}$ of the transformed reduced stiffness matrix can be evaluated with Equation (4) [6, 10]:

$$\begin{aligned}
\overline{Q}_{11} &= Q_{11}c^4 + 2(Q_{12} + 2Q_{66})c^2s^2 + Q_{22}s^4 \\
\overline{Q}_{12} &= (Q_{11} + Q_{22} - 4Q_{66})c^2s^2 + Q_{12}(s^4 + c^4) \\
\overline{Q}_{22} &= Q_{11}s^4 + 2(Q_{12} + 2Q_{66})c^2s^2 + Q_{22}c^4 \\
\overline{Q}_{16} &= (Q_{11} - Q_{12} - 2Q_{66})c^3s + (Q_{12} - Q_{22} + 2Q_{66})s^3c \\
\overline{Q}_{26} &= (Q_{11} - Q_{12} - 2Q_{66})cs^3 + (Q_{12} - Q_{22} + 2Q_{66})sc^3 \\
\overline{Q}_{66} &= (Q_{11} + Q_{22} - 2Q_{12} - 2Q_{66})c^2s^2 + Q_{66}(c^4 + s^4)
\end{aligned} \tag{4}$$

where: Q_{ij} are the elements of the stiffness matrix, written in terms of the elastic engineering constants of the off-axis unidirectional lamina [11]:

$$\begin{aligned}
Q_{11} &= \frac{E_1}{1 - \nu_{12} \cdot \nu_{21}} \\
Q_{12} &= \frac{\nu_{21} \cdot E_1}{1 - \nu_{12} \cdot \nu_{21}} = \frac{\nu_{12} \cdot E_2}{1 - \nu_{12} \cdot \nu_{21}} \\
Q_{22} &= \frac{E_2}{1 - \nu_{12} \cdot \nu_{21}} \\
Q_{66} &= G_{12}
\end{aligned} \tag{5}$$

The geometrical characteristics for a general composite laminate with *n* number of plies are shown in Figure 2:

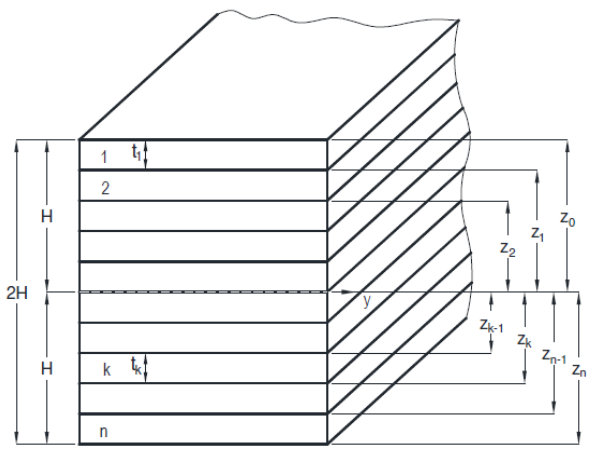


Fig. 2. Geometrical characteristics for the general n-layered laminate

The in-plane stiffness matrix *[A]* is defined as the sum of the product of the individual layers $\overline{Q}_{ij}$ and the laminas thicknesses, as shown in Equation (6a) or (6b) [7]. The matrix *[A]* relates the resultant in-plane forces to the in-plane strains [4, 9].

$$A_{ij} = \sum_{k=1}^{n} \left(\overline{Q}_{ij}\right)_k \cdot \left(z_k - z_{k-1}\right) \tag{6a}$$

$$[A] = \left[\sum_{k=1}^{n} \begin{bmatrix} \overline{Q}_{11} & \overline{Q}_{12} & \overline{Q}_{16} \\ \overline{Q}_{12} & \overline{Q}_{22} & \overline{Q}_{26} \\ \overline{Q}_{16} & \overline{Q}_{26} & \overline{Q}_{66} \end{bmatrix}_k \int_{z_{k-1}}^{z_k} dz \right] = \begin{bmatrix} A_{11} & A_{12} & A_{16} \\ A_{12} & A_{22} & A_{26} \\ A_{16} & A_{26} & A_{66} \end{bmatrix} \tag{6b}$$

where: z_k, z_{k-1} are the coordinates to the bottom and to the top of the *k* layer;

The bending-stretching coupling matrix *[B]* is determined with Equation (7a) or (7b) [7]. The matrix *[B]* is related to the coupling effect between the force and moment terms to the middle plane strains and middle plane curvatures [4].

$$B_{ij} = \frac{1}{2} \sum_{k=1}^{n} \left(\overline{Q}_{ij}\right)_k \cdot \left(z_k^2 - z_{k-1}^2\right) \tag{7a}$$

$$[B] = \left[\sum_{k=1}^{n} \begin{bmatrix} \overline{Q}_{11} & \overline{Q}_{12} & \overline{Q}_{16} \\ \overline{Q}_{12} & \overline{Q}_{22} & \overline{Q}_{26} \\ \overline{Q}_{16} & \overline{Q}_{26} & \overline{Q}_{66} \end{bmatrix}_k \int_{z_{k-1}}^{z_k} z dz \right] = \begin{bmatrix} B_{11} & B_{12} & B_{16} \\ B_{12} & B_{22} & B_{26} \\ B_{16} & B_{26} & B_{66} \end{bmatrix} \tag{7b}$$

The bending stiffness matrix *[D]* associates the resultant bending moments with the plate curvatures [4, 7] and it can be determined with the Equation (8a) or (8b), as follows:

$$D_{ij} = \frac{1}{3}\sum_{k=1}^{n}\left(\overline{Q}_{ij}\right)_k \cdot \left(z_k^3 - z_{k-1}^3\right) \tag{8a}$$

$$[D] = \left[\sum_{k=1}^{n}\begin{bmatrix}\overline{Q}_{11} & \overline{Q}_{12} & \overline{Q}_{16}\\ \overline{Q}_{12} & \overline{Q}_{22} & \overline{Q}_{26}\\ \overline{Q}_{16} & \overline{Q}_{26} & \overline{Q}_{66}\end{bmatrix}_k \int_{z_{k-1}}^{z_k} z^2\,dz\right] = \begin{bmatrix}D_{11} & D_{12} & D_{16}\\ D_{12} & D_{22} & D_{26}\\ D_{16} & D_{26} & D_{66}\end{bmatrix} \tag{8b}$$

The resultant in-plane forces (N_x, N_y, N_{xy}) and moments (M_x, M_y, M_{xy}) respectively, are shown in Figure 3.

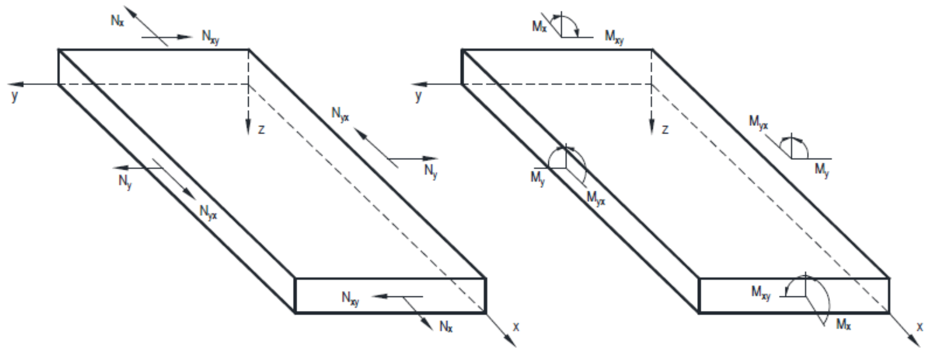


Fig. 3. In-plane forces and moments per unit width.

The six force and moment resultants form a system that is statically equivalent to the stress system on the laminate, with respect to the middle plane of the multi-layered composite [10].

$$\begin{Bmatrix}N_x\\ N_y\\ N_{xy}\end{Bmatrix} = \begin{bmatrix}A_{11} & A_{12} & A_{16}\\ A_{12} & A_{22} & A_{26}\\ A_{16} & A_{26} & A_{66}\end{bmatrix}\begin{Bmatrix}\varepsilon_x^0\\ \varepsilon_y^0\\ \gamma_{xy}^0\end{Bmatrix} + \begin{bmatrix}B_{11} & B_{12} & B_{16}\\ B_{12} & B_{22} & B_{26}\\ B_{16} & B_{26} & B_{66}\end{bmatrix}\begin{Bmatrix}k_x\\ k_y\\ k_{xy}\end{Bmatrix}$$
$$\begin{Bmatrix}M_x\\ M_y\\ M_{xy}\end{Bmatrix} = \begin{bmatrix}B_{11} & B_{12} & B_{16}\\ B_{12} & B_{22} & B_{26}\\ B_{16} & B_{26} & B_{66}\end{bmatrix}\begin{Bmatrix}\varepsilon_x^0\\ \varepsilon_y^0\\ \gamma_{xy}^0\end{Bmatrix} + \begin{bmatrix}D_{11} & D_{12} & D_{16}\\ D_{12} & D_{22} & D_{26}\\ D_{16} & D_{26} & D_{66}\end{bmatrix}\begin{Bmatrix}k_x\\ k_y\\ k_{xy}\end{Bmatrix} \tag{9}$$

where:

$(\varepsilon_x^0, \varepsilon_y^0, \gamma_{xy}^0)$ represents the laminate mid-plane strains;

(k_x, k_y, k_{xy}) represents the laminate curvatures.

In condensed form, the constitutive equation of the laminate can be written as:

$$\{N\} = [A]\{\varepsilon^0\} + [B]\{k\}$$
$$\{M\} = [B]\{\varepsilon^0\} + [D]\{k\} \tag{10}$$

or

$$\begin{Bmatrix} N \\ M \end{Bmatrix} = \begin{bmatrix} A & B \\ B & D \end{bmatrix} \begin{Bmatrix} \varepsilon^0 \\ k \end{Bmatrix} \tag{11}$$

The corresponding inverted force-deformation relationship for the constitutive equation (11) is given by Equation (12), such as [7]:

$$\begin{Bmatrix} \varepsilon^0 \\ k \end{Bmatrix} = \begin{bmatrix} A^{'} & B^{'} \\ C^{'} & D' \end{bmatrix} \begin{Bmatrix} N \\ M \end{Bmatrix} \tag{12}$$

where:

$$\begin{gathered} \left[A^{'}\right] = [A]^{-1} + [A]^{-1}[B]\left[D^{*}\right]^{-1}[B][A]^{-1} \\ \left[B^{'}\right] = -[A]^{-1}[B]\left[D^{*}\right]^{-1} \\ \left[C^{'}\right] = -\left[D^{*}\right]^{-1}[B][A]^{-1} \\ \left[D^{'}\right] = \left([D] - [B][A]^{-1}[B]\right)^{-1} \end{gathered} \tag{13}$$

4. Angle-ply laminates

The angle-ply laminates are classified as special orthotropic laminates, characterized by an equal number of equal thickness layers of +θ and –θ fiber orientations [7].

A laminate is considered symmetric if ply of the same thickness, fiber orientation angles and composite material constituents are symmetrically disposed with respect to the middle plane of the composite structure [12]. Figure 4 shows a symmetric angle-ply laminate with $[(\pm\theta)_2]_s$ codification.

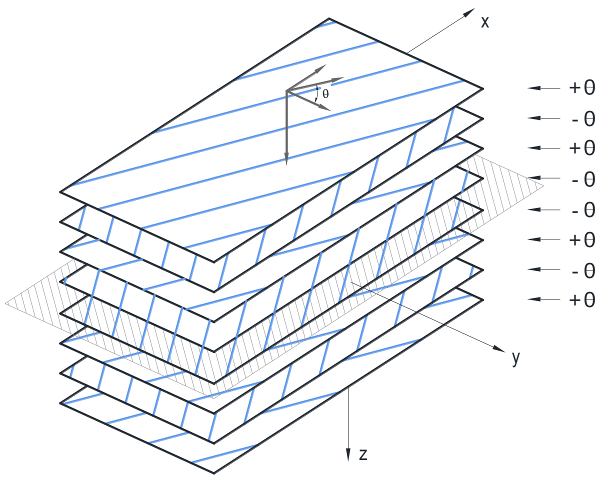


Fig. 4. Symmetric angle-ply laminate $[(\pm\theta)_2]_s$.

Because of the symmetry of the laminate, the extension response is uncoupled from the bending response and therefore, the bending-stretching stiffness matrix *[B]* is identically zero [6]. The constitutive equation in case of symmetric laminates derived from Equation (10), can be written as:

$$\begin{aligned} \{N\} &= [A]\{\varepsilon^0\} \\ \{M\} &= [D]\{k\}, \end{aligned} \tag{14}$$

Since $[B]=0$, the Equation (12) is converted into Equation (15), as follows:

$$\begin{Bmatrix} \varepsilon^0 \\ k \end{Bmatrix} = \begin{bmatrix} A^{-1} & 0 \\ 0 & D^{-1} \end{bmatrix} \begin{Bmatrix} N \\ M \end{Bmatrix} \tag{15}$$

The extensional stiffness matrix *[A]* for a symmetric angle-ply laminate with $[(\pm\theta)_2]_s$ codification, with equal layer thickness *t* and total laminate thickness *2H*, is written according to [7]:

$$[A] = 2H \begin{bmatrix} \overline{Q}_{11}(\theta) & \overline{Q}_{12}(\theta) & 0 \\ \overline{Q}_{12}(\theta) & \overline{Q}_{22}(\theta) & 0 \\ 0 & 0 & \overline{Q}_{66}(\theta) \end{bmatrix} \tag{16}$$

Since the expressions of the elastic engineering constants for symmetric angle-ply laminates are required, it is needed to define the laminate compliance $[a^*]$[7]:

$$[a^*] = 2H[A]^{-1} \tag{17}$$

In general, matrix $[a^*]$ is fully populated, such as:

$$[a^*] = \begin{bmatrix} a_{11}^* & a_{12}^* & a_{16}^* \\ a_{12}^* & a_{22}^* & a_{26}^* \\ a_{16}^* & a_{26}^* & a_{66}^* \end{bmatrix} \tag{18}$$

where: a_{ij}^* are the elements of the laminate compliance.

According to the Equations (16) and (17), the laminate compliance $[a^*]$ for a symmetric angle-ply laminate becomes [7]:

$$[a^*] = \begin{bmatrix} \left(\dfrac{\overline{Q}_{22}}{\overline{Q}_{11}\overline{Q}_{22} - \overline{Q}_{12}\overline{Q}_{12}}\right) & \left(\dfrac{\overline{-Q_{12}}}{\overline{Q}_{11}\overline{Q}_{22} - \overline{Q}_{12}\overline{Q}_{12}}\right) & 0 \\ \left(\dfrac{\overline{-Q_{12}}}{\overline{Q}_{11}\overline{Q}_{22} - \overline{Q}_{12}\overline{Q}_{12}}\right) & \left(\dfrac{\overline{Q}_{11}}{\overline{Q}_{11}\overline{Q}_{22} - \overline{Q}_{12}\overline{Q}_{12}}\right) & 0 \\ 0 & 0 & \dfrac{1}{\overline{Q}_{66}} \end{bmatrix} \tag{19}$$

The elastic engineering constants for a symmetric angle-ply laminate can be determined using the elements of the laminate compliance $\left[a^*\right]$, as it follows [7]:

$$
\begin{aligned}
E_x &= \frac{1}{a_{11}^*} = \frac{\overline{Q}_{11}\overline{Q}_{22} - \overline{Q}_{12}\overline{Q}_{12}}{\overline{Q}_{22}}, \text{ for } \overline{\sigma}_x \neq 0,\ \overline{\sigma}_y = 0,\ \overline{\tau}_{xy} = 0 \\
E_y &= \frac{1}{a_{22}^*} = \frac{\overline{Q}_{11}\overline{Q}_{22} - \overline{Q}_{12}\overline{Q}_{12}}{\overline{Q}_{11}}, \text{ for } \overline{\sigma}_y \neq 0,\ \overline{\sigma}_x = 0,\ \overline{\tau}_{xy} = 0 \\
G_{xy} &= \frac{1}{a_{66}^*} = \overline{Q}_{66}, \text{ for } \overline{\tau}_{xy} \neq 0,\ \overline{\sigma}_x = 0,\ \overline{\sigma}_y = 0 \\
\nu_{xy} &= -\frac{a_{12}^*}{a_{11}^*} = \frac{\overline{Q}_{12}}{\overline{Q}_{22}}, \text{ for } \overline{\sigma}_x \neq 0,\ \overline{\sigma}_y = 0,\ \overline{\tau}_{xy} = 0 \\
\nu_{yx} &= -\frac{a_{12}^*}{a_{22}^*} = \frac{\overline{Q}_{12}}{\overline{Q}_{11}}, \text{ for } \overline{\sigma}_y \neq 0,\ \overline{\sigma}_x = 0,\ \overline{\tau}_{xy} = 0
\end{aligned} \tag{20}
$$

where: ($\overline{\sigma}_x$, $\overline{\sigma}_y$, $\overline{\tau}_{xy}$) are the average stresses of the matrix $\left\{\overline{\sigma}\right\}$, defined as:

$$\left\{\overline{\sigma}\right\} = \frac{1}{2H}\left\{N\right\} \tag{21}$$

These engineering constants are useful to express the elastic properties of a laminate for design purposes, but also for comparison with other materials.

5. Case study

In order to analyze the elastic engineering constants prediction for multi-layered composites, a case study is carried out according to the theory of laminates. The analysis is done for unidirectional off-axis laminas and symmetric angle-ply laminates $\left[(\pm\theta)_2\right]_s$.

The composite material properties are given in Table 1, the laminas and laminates being made of aramid fibers kevlar 149/epoxy resin and high stiffness carbon fibers/vinylester resin.

Table 1) The constituent materials properties

Composite materials		*Longitudinal Young's modulus* [GPa]		*Poisson's ratios*	
Fibers	Matrix	E_f	E_m	ν_f	ν_m
kevlar 149	epoxy	175	4.1	0.35	0.40
carbon	vinylester	380	3.5	0.20	0.39

The case study analyzes the fiber orientation influence on the elastic characteristics of the laminates and also the stiffening effect when stacking two or more laminas.

The graphical distributions of the elastic engineering constants with respect to the fiber orientation angles are illustrated in Figures 5-7. Representative values for the fiber volume fractions are chosen to show the different variation curves.

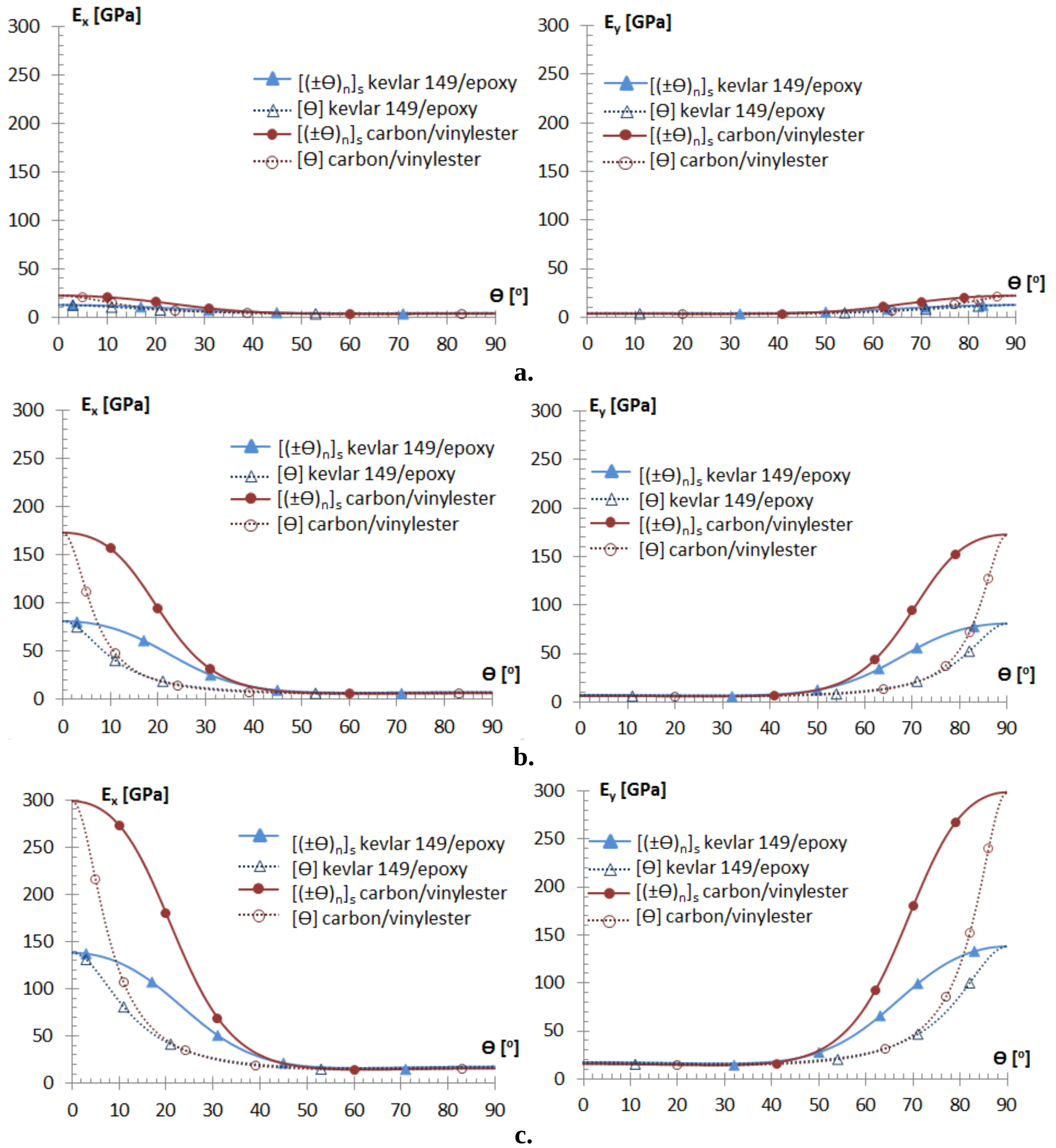


Fig. 5. Longitudinal and transverse moduli E_x and E_y variations with respect to fiber orientation angles, for different fiber volume fractions

a. V_f = 5%. **b.** V_f = 45%. **c.** V_f = 78.5%.

The variation curves of the longitudinal and transverse moduli E_x and E_y in terms of fiber orientation ($0^o<\theta<90^o$) are presented in Figure 5, for some particular fiber volume fractions of 5%, 45% and 78.5%. The stacking stiffening effect is proven for fiber orientation angles of 0^o to 50^o for longitudinal modulus E_x and for 50^o to 90^o for transverse modulus E_y. The angle-ply laminates have higher axial modulus than the generally orthotropic lamina for these intervals, for every fiber volume fractions. With increasing the fiber volume fractions, the difference between laminas and laminates and the stacking stiffening effect is more clearly illustrated.

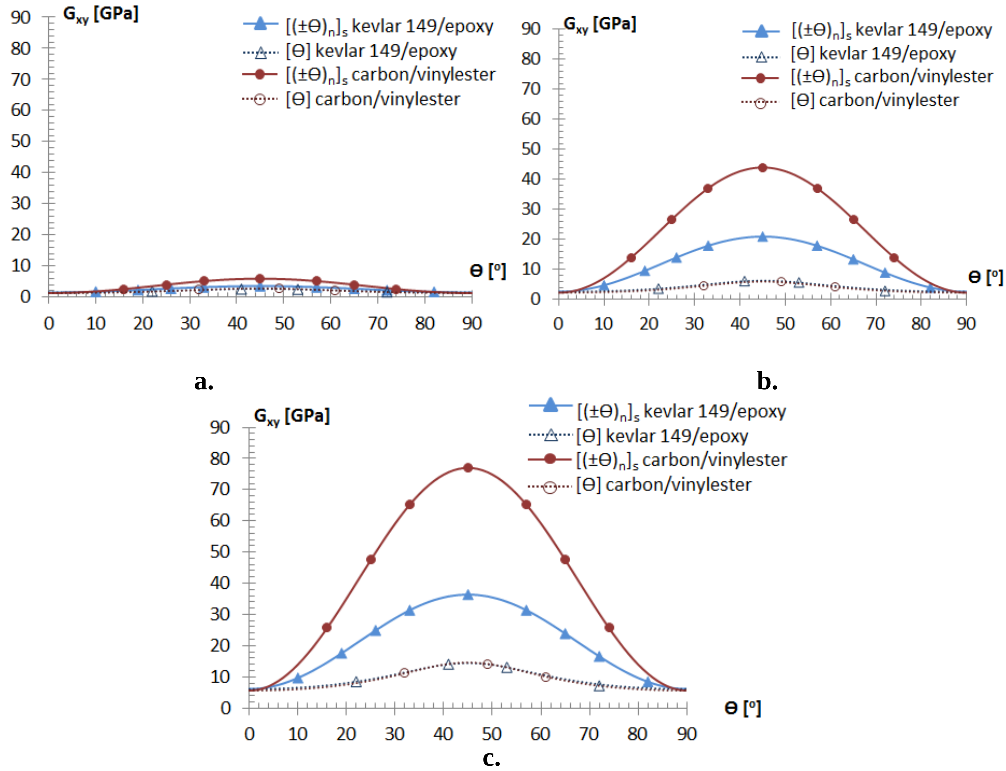
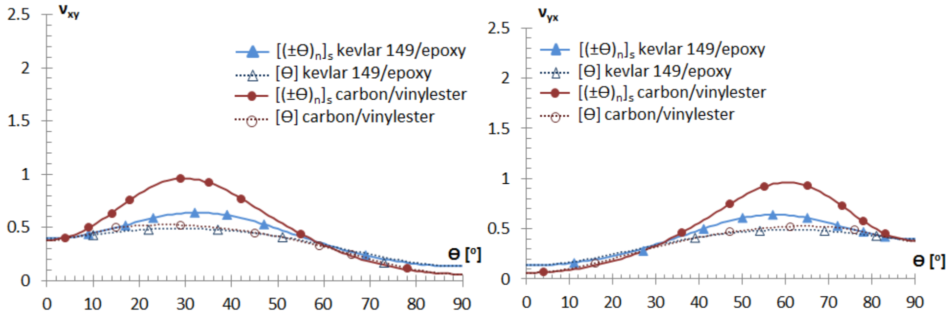


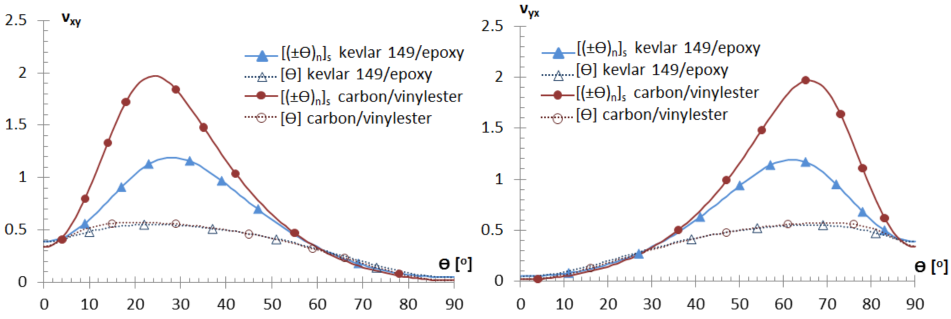
Fig. 6. Shear modulus G_{xy} variation with respect to fiber orientation angles, for different fiber volume fractions

a. V_f = 5%. **b.** V_f = 45%. **c.** V_f = 78.5%.

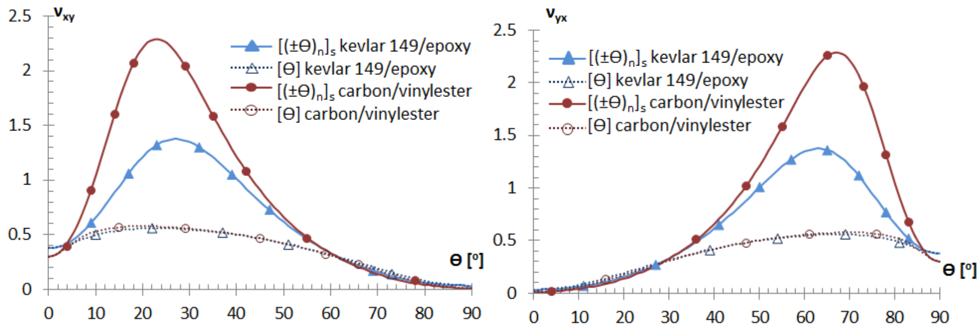
The variation of the shear modulus G_{xy} shown in Figure 6 illustrates that the angle-ply laminates have higher elastic characteristics than the off-axis unidirectional laminas, for all fiber orientations. All graphics reveal that the shear modulus is increased when greater values for the fiber volume fraction are used, so G_{xy} reaches maximum values for V_f=78.5%, but always at θ=45°. It shows that when composite laminates require high shear stiffness, the ±45° fiber orientation angles are recommended. Moreover, comparison of those two composite material used in the analysis can be noted. In case of laminates, the shear stiffness of the carbon/vinylester is visibly 2 times greater than the kevlar 149/epoxy angle-ply, while for laminas the differences are quite insignificant.



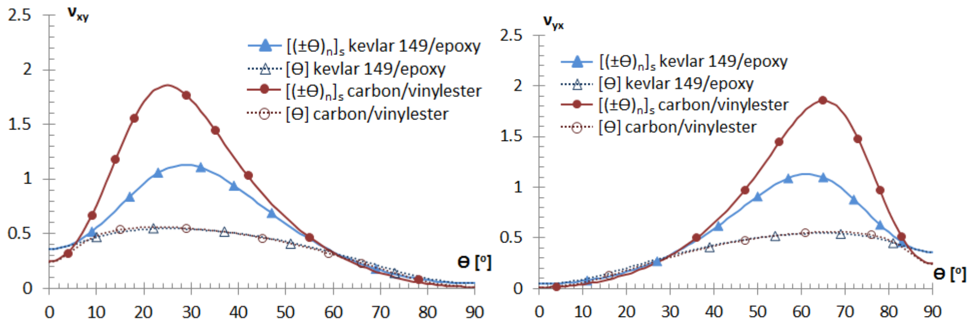
a.



b.



c.



d.

Fig. 7. Poisson's ratios ν_{xy} and ν_{yx} variations with respect to fiber orientation angles, for different fiber volume fractions

a. V_f = 5%. **b.** V_f = 25%. **c.** V_f = 45%. **d.** V_f = 78.5%.

The variations of the Poisson's ratios ν_{xy} and ν_{yx} show a continuous increase until V_f=40-50%, when both ν_{xy} and ν_{yx} reaches maximum values, then up to V_f=78.5% the values of the Poisson's ratios are decreasing until the graphical distributions are similar with those when V_f=25%. In all situations, the Poisson's ratios are the greatest when $\theta=25^o$ in case of ν_{xy} and, as expected, ν_{yx} reaches maximum value when $\theta=65^o$.

6. Conclusions

The influence of fiber orientation angles on the elastic properties of composite laminates is demonstrated by graphical distributions for every analyzed engineering constant. The importance of selecting the right fiber orientation angles is crucial when it is needed to fulfill the stiffness requirements of a composite structure design.

The variation curves illustrate improved elastic behavior of laminates compared with laminas, because of the stacking plies effect. As expected, carbon/vinylester composite material laminate reaches the maximum values of the elastic engineering constants, for specified intervals of fiber orientation angles.

R E F E R E N C E S

[1] I. Daniel and M. Ishai, *Engineering mechanics of composite materials* (Oxford University Press, Oxford, **1994**).

[2] A.E.H. Love, *On the small free vibrations and deformations of elastic shells* (Philosophical transactions, The Royal Society, London, **1888**) Vol. serie A, No. 7, pp. 491-549.

[3] M.H. Datoo, *Mechanics of fibrous composites* (Elsevier LTD, New York, **1991**).

[4] A.K. Kaw, *Mechanics of Composite Materials*, Second Edition (CRC Press, Taylor & Francis Group, New York, **2006**).

[5] N. Ţăranu and D. Isopescu, *Structures made of composite materials* (Ed. Vesper, Iaşi, **1996**).

[6] B.D. Agarwal and L.J. Broutman, *Analysis and performance of fibre composites*, Second Edition (Wiley-Interscience, New-York, **1990**).

[7] C.T. Herakovich, *Mechanics of Fibrous Composites* (University of Virginia, John Wiley & Sons, Inc., United States of America, **1998**).

[8] N. Ţăranu and L. Bejan, *Mecanica mediilor compozite armate cu fibre* (Ed. Cermi, Iaşi, **2005**).

[9] R.M. Jones, *Mechanics of composite materials*, Second Edition (Taylor & Francis, Inc., Philadelphia, **1999**).

[10] R.F. Gibson, *Principles of composite material mechanics*, Third Edition (CRC Press, London, **2012**).

[11] N. Ţăranu et al, *Materiale şi elemente compozite I. Prelegeri şi aplicaţii* (Ed. Politehnium, Iaşi, **2013**).

[12] E.J. Barbero, *Introduction to composite materials design*, Second Edition (CRC Press, Taylor & Francis Group, New York, **2011**).