

## METHOD FOR DETERMINING QUALITY INDICATORS OF ELECTRICAL POWER

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**Abstract.** *This paper proposes to introduce a small number of quality indicators for electrical power in systems with three-phased consumers which are disbalanced, asymmetrical and distorting. These indicators are required for measuring and pre-determining the effects of such consumers on the supply network in order to establish measures for improving the quality of power supply in the electrical networks.*

**Keywords:** Quality indicators, electrical power, energy quality

### 1. Introduction

The problem related to the power quality in electrical networks are the following:

- frequency variations;
- active and reactive power shocks;
- distorting, asymmetrical and disbalanced regime.

This last problem has some very important consequences:

- the increase of power losses and active power;
- the interlocking of installations transport and supply capacity;
- difficulties related to the voltage regulating;
- parasitic torques in electrical machines;
- errors done by measure and control apparatus.

### 2. Three-phased disbalanced, asymmetrical and distorting consumer

Let it be a three-phased asymmetrical consumer R, S, T, whose impedances  $Z_R \neq Z_S \neq Z_T$ ,

Supplied from a three-phased voltage system  $u_R, u_S, u_T \in u_{RST}$  with disbalanced currents  $i_R, i_S, i_T \in i_{RST}$  running under distorting conditions.

Instantaneous value expressions and the phase voltage  $u_{RST}$  and the  $i_{RST}$  currents are:

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$$\begin{aligned}
u_{c(RST)} &= \sum_1^{\infty} U_{k(RST)} \times \sqrt{2} \times \cos(k\omega t + \alpha_{k(RST)}) = U_{c(RST)} \times \cos \omega t - U_{s(RST)} \times \sin \omega t \\
u_{s(RST)} &= \sum_1^{\infty} U_{k(RST)} \times \sqrt{2} \times \sin(k\omega t + \alpha_{k(RST)}) = U_{s(RST)} \times \cos \omega t + U_{c(RST)} \times \sin \omega t \\
i_{c(RST)} &= \sum_1^{\infty} I_{k(RST)} \times \sqrt{2} \times \cos(k\omega t + \beta_{k(RST)}) = I_{c(RST)} \times \cos \omega t - I_{s(RST)} \times \sin \omega t \\
i_{s(RST)} &= \sum_1^{\infty} I_{k(RST)} \times \sqrt{2} \times \sin(k\omega t + \beta_{k(RST)}) = I_{s(RST)} \times \cos \omega t + I_{c(RST)} \times \sin \omega t
\end{aligned} \tag{1}$$

where:

$$\begin{aligned}
U_{c(RST)} &= \sum_1^{\infty} U_{k(RST)} \times \sqrt{2} \times \cos[\alpha_{k(RST)} + (k-1)\omega t] \\
U_{s(RST)} &= \sum_1^{\infty} U_{k(RST)} \times \sqrt{2} \times \sin[\alpha_{k(RST)} + (k-1)\omega t] \\
I_{c(RST)} &= \sum_1^{\infty} I_{k(RST)} \times \sqrt{2} \times \cos[\beta_{k(RST)} + (k-1)\omega t] \\
I_{s(RST)} &= \sum_1^{\infty} I_{k(RST)} \times \sqrt{2} \times \sin[\beta_{k(RST)} + (k-1)\omega t]
\end{aligned} \tag{2}$$

Current and voltage systems satisfy the following requirements:

$$\begin{aligned}
u_{CR} + u_{CS} + u_{CT} &= u_{CO} \\
u_{SR} + u_{SS} + u_{ST} &= u_{SO} \\
i_{CR} + i_{CS} + i_{CT} &= i_{CO} \\
i_{SR} + i_{SS} + i_{ST} &= i_{SO}
\end{aligned} \tag{3}$$

There has been demonstrated [1], [2], [3], that:

$$u_{RST}^2 = \frac{u_{C(RST)}^2 + u_{S(RST)}^2}{2}; \quad i_{RST}^2 = \frac{i_{C(RST)}^2 + i_{S(RST)}^2}{2} \quad (4)$$

$$p_{RST} = \frac{u_{C(RST)} \times i_{C(RST)} + u_{S(RST)} \times i_{S(RST)}}{2} \quad q_{RST} = \frac{u_{C(RST)} \times i_{S(RST)} - u_{S(RST)} \times i_{C(RST)}}{2} \quad (5)$$

$$s_{RST}^2 = u_{RST}^2 \times i_{RST}^2 \quad (6)$$

which represent characteristic functions of voltage  $u_{RST}$ , current  $i_{RST}$ , active power  $p_{RST}$ , complementary power  $q_{RST}$ , and apparent power  $s_{RST}$ . These functions of voltage and current are graphically represented in figure 1 and figure 2. Their property is that of being able to describe the variations of sizes characteristic to power quality as compared to the ideal conditions. The characteristic functions:

$$U_{RST}^2 = \sum_0^{\infty} U_k^2 = \frac{1}{T} \int_0^T u_{RST}^2 \times dt = f \int_0^{\frac{1}{f}} u_{RST}^2 \times dt \quad (7)$$

$$I_{RST}^2 = \sum_0^{\infty} I_k^2 = \frac{1}{T} \int_0^T i_{RST}^2 \times dt = f \int_0^{\frac{1}{f}} i_{RST}^2 \times dt \quad (8)$$

$$P_{RST} = \sum_0^{\infty} P_k = \frac{1}{T} \int_0^T p_{RST} \times dt = f \int_0^{\frac{1}{f}} p_{RST} \times dt \quad (9)$$

$$Q_{RST} = \sum_0^{\infty} Q_k = \frac{1}{T} \int_0^T q_{RST} \times dt = f \int_0^{\frac{1}{f}} q_{RST} \times dt \quad (10)$$

$$S_{RST}^2 = U_{RST}^2 \times I_{RST}^2 \quad (11)$$

enable us to obtain voltage, currents, active, inactive and apparent powers.

In them papers mentioned above these has been demonstrated that among the characteristic functions we have the relation:

$$s_{RST}^2 = p_{RST}^2 + q_{RST}^2 \quad (12)$$

which represents the principle of power separation introduced by C. I. Budeanu. It also may be rendered as:

$$S^2 = P^2 + Q^2 + D^2 \quad (13)$$

where D represents the distorting power.

### 3. Quality indicators

To define the power quality we do introduce two indicators (figure 1 and figure 2):

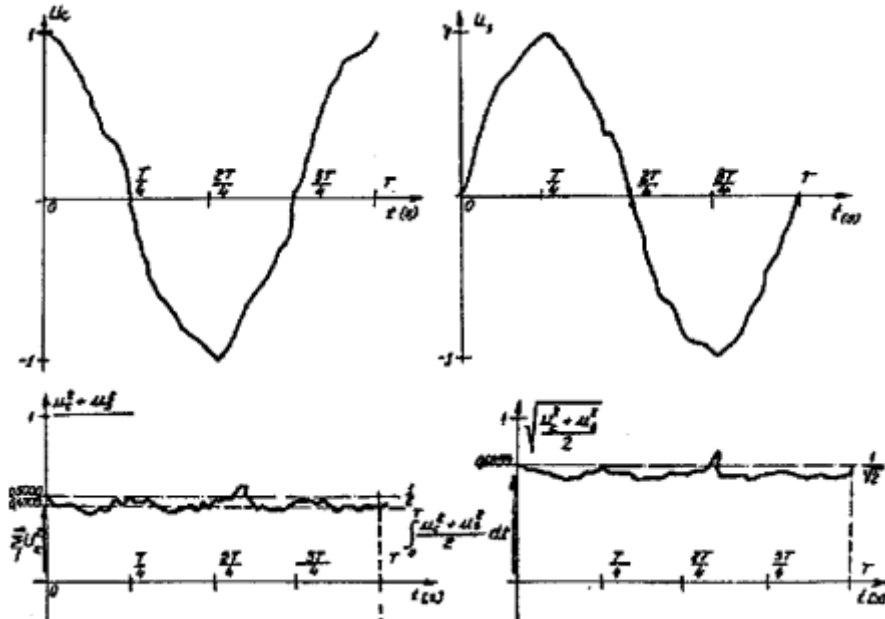


Fig. 1. The characteristic function of voltage.

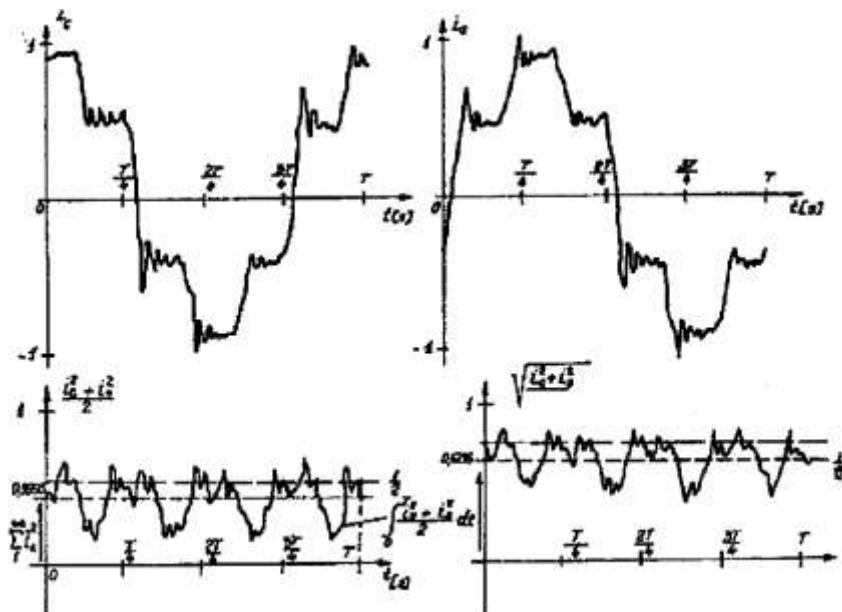


Fig. 2. The characteristic function of current.

- load of the isolation:  $S_u$

$$S_u = \int_0^T u_{RST}^2 \times dt = U_{RST}^2 \times T \quad (14)$$

- thermal load:  $S_i$

$$S_i = \int_0^T i_{RST}^2 \times dt = I_{RST}^2 \times T \quad (15)$$

These indicators are independent upon frequency variation and kind of sinusoidal energetical regime.

For sinusoidal energetical regime (S) there are two situations:

- a) for frequency  $f = 50\text{Hz}$ ,  $T = 1/f$ ;

$$S_u(S, T) = \int_0^T u_{RS(RST)}^2 \times dt = U_{RS(RST)}^2 \times T \quad (16)$$

$$Si(S, T) = \int_0^T i_{RS(RST)}^2 \times dt = I_{RS(RST)}^2 \times T \quad (17)$$

- b) for frequency  $f' \neq 50\text{Hz}$ ,  $T = 1/f'$

$$S_u(S, T') = \int_0^T u_{RST}^2 \times dt = U_{RS(RST)}^2 \times T' \quad (18)$$

$$Si(S, T') = \int_0^T i_{RS(RST)}^2 \times dt = I_{RS(RST)}^2 \times T' \quad (19)$$

For non-sinusoidal energetical regime (N) there are also situations:

- a) for frequency  $f = 50\text{Hz}$ ,  $T = 1/f$ ;

$$S_u(N, T) = \int_0^T u_{RN(RST)}^2 \times dt = U_{RN(RST)}^2 \times T \quad (20)$$

$$Si(N, T) = \int_0^T i_{RN(RST)}^2 \times dt = I_{RN(RST)}^2 \times T \quad (21)$$

- b) for frequency  $f' \neq 50\text{Hz}$ ,  $T = 1/f'$

$$S_u(N, T') = \int_0^T u_{RST}^2 \times dt = U_{RN(RST)}^2 \times T' \quad (22)$$

$$Si(N, T') = \int_0^T i_{RN(RST)}^2 \times dt = I_{RN(RST)}^2 \times T' \quad (23)$$

By using the representations in relative units of functions  $u_{RST}^2(t)$  and  $i_{RST}^2(t)$  in figure 1 and figure 2, we can define these function such as:

$$S_u(S, T) = U^2 T = \frac{1}{2} \times T \quad (24)$$

$$S_i(S, T) = I^2 T = \frac{1}{2} \times T$$

$$S_u(S, T') = U^2 T' = \frac{1}{2} \times T' \quad (25)$$

$$S_i(S, T') = I^2 T' = \frac{1}{2} \times T'$$

$$S_u(N, T) = \sum_1^n U_k^2 T = 0,4709T \quad (26)$$

$$S_i(N, T) = \sum_1^n I_k^2 T = 0,3995T$$

$$S_u(N, T') = \sum_1^n U_k^2 T' = 0,4709T' \quad (27)$$

$$S_i(N, T') = \sum_1^n I_k^2 T' = 0,3995T'$$

Having as reference basis the functions  $S_u(S, T)$  and  $S_i(S, T)$  corresponding to the distorting energetical regime at frequency  $f = \text{Hz}$ , any non-sinusoidal energetical regime at frequency  $f' \neq f = 50\text{Hz}$  is characterized by functions  $S_u$  and  $S_i$ .

The deviation from energy quality (voltage and current) can be defined by following functions:

$$\Delta S_u(N, T') = \frac{S_u(N, T') - S_u(S, T)}{S_u(N, T')} = 1 - \frac{U^2 T}{\sum_1^n U_k^2 T'} \quad (28)$$

$$\Delta S_i(N, T') = \frac{S_i(N, T') - S_i(S, T)}{S_i(N, T')} = 1 - \frac{I^2 T}{\sum_1^n I_k^2 T'} \quad (29)$$

The quality deviation due to the network frequency variation is defined by the functions:

$$\Delta S_u(S, T') = \frac{S_u(S, T') - S_u(S, T)}{S_u(S, T')} = 1 - \frac{T}{T'} \quad (30)$$

$$\Delta S_i(S, T') = \frac{S_i(S, T') - S_i(S, T)}{S_i(S, T')} = 1 - \frac{T}{T'} \quad (31)$$

and then we have the relations:

$$\Delta S(f) = 1 - \frac{T}{T'} = 1 - \frac{f'}{f} = \Delta S_u(S, T') = \Delta S_i(S, T') \quad (32)$$

The quality deviation due to non-sinusoidal energetical regime is defined by the following functions:

$$\Delta S_u(D, T) = \frac{S_u(N, T) - S_u(S, T)}{S_u(N, T)} = 1 - \frac{U^2}{\sum_1^n U_k^2} \quad (33)$$

$$\Delta S_u(D, T') = \frac{S_u(N, T) - S_u(S, T')}{S_u(N, T')} = 1 - \frac{U^2}{\sum_1^n U_k^2} \quad (34)$$

$$\Delta S_i(D, T) = \frac{S_i(N, T) - S_i(S, T)}{S_u(N, T)} = 1 - \frac{I^2}{\sum_1^n I_k^2} \quad (35)$$

$$\Delta S_i(D, T') = \frac{S_i(N, T') - S_i(S, T')}{S_u(N, T')} = 1 - \frac{I^2}{\sum_1^n I_k^2} \quad (36)$$

$$\Delta S_u(D) = S_u(D, T) = \Delta S_u(D, T'); \quad (37)$$

$$\Delta S_i(D) = S_i(D, T) = \Delta S_i(D, T'); \quad (38)$$

Under these conditions, the deviation from energy quality (voltage and current) can be defined as follows:

$$\Delta S_u(N, T') = 1 - [1 - \Delta S_u(D)] \times [1 - \Delta S(f)] \quad (39)$$

$$\Delta S_i(N, T') = 1 - [1 - \Delta S_i(D)] \times [1 - \Delta S(f)] \quad (40)$$

These should be noted that from the representations of figure 1 and 2, when it is operated at a frequency,  $f = 47\text{Hz}$ , it results a deviation from energy quality  $\Delta S_u(N, T') = 12\%$  and  $\Delta S_i(N, T') = 24,8\%$ .

A unique energy quality indicator would be that which is including both insulation load and thermal load.

Laying down the characteristic  $S_{ui(RST)} = \left(\frac{u_{c(RST)}^2 + u_{s(RST)}^2}{2}\right)\left(\frac{i_{c(RST)}^2 + i_{s(RST)}^2}{2}\right)$  for an electrical period  $T$  or  $T'$ , we obtain the characteristic of insulations load and thermal load, SIT.

For sinusoidal energetical regime at frequency  $f = 50\text{Hz}$ ,  $T = 1/f$ , we have:

$$SIT(S, T) = \int_0^T \left(\frac{u_{c(RST)}^2 + u_{s(RST)}^2}{2}\right)\left(\frac{i_{c(RST)}^2 + i_{s(RST)}^2}{2}\right) dt = U^2 I^2 T \quad (41)$$

While for non-sinusoidal energetical regime at frequency  $f' \neq f = 50\text{Hz}$ ,  $T' = 1/f'$ , we have:

$$SIT(N, T') = \int_0^{T'} \left(\frac{u_{c(RST)}^2 + u_{s(RST)}^2}{2}\right)\left(\frac{i_{c(RST)}^2 + i_{s(RST)}^2}{2}\right) dt = \sum_1^n U_k^2 \sum_1^n I_k^2 T' \quad (42)$$

The deviation of isolation and thermal load characteristic is:

$$\Delta SIT(N, T') = \frac{SIT(N, T') - SIT(S, T)}{SIT(N, T')} = 1 - [1 - \Delta S_u(N, T')][1 - \Delta S_i(N, T')] \frac{T'}{T} \quad (43)$$

## Conclusions

There have been suggested the indications for isolation load and thermal load to characterize the derivation from the electric power quality. With their help we can define a general energy quality indicator by means of the isolation and thermal load deviation. The main advantage is that these indicators can be determined without performing any harmonic expression.

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