

STOCHASTIC MODELING OF STRATEGIC SUPPLY CHAIN DESIGNMarcel ILIE¹, Augustin SEMENESCU²DOI [10.56082/annalsarscieng.2024.1.23](https://doi.org/10.56082/annalsarscieng.2024.1.23)

Rezumat. Managementul riscului lanțului de aprovizionare joacă un rol critic în orice mediu de afaceri sau industrie și permite o bună coordonare a parametrilor de intrare și de ieșire care pot afecta dezvoltarea fără probleme a proceselor, cum ar fi un proces de fabricație, de exemplu. Cu toate acestea, managementul riscului lanțului de aprovizionare este adesea predispus la impactul diferitelor incertitudini asociate cu întreruperi ale lanțului de aprovizionare cauzate de meteorologie, pandemie, deficit de resurse etc. Prin urmare, o modalitate de a cuantifica aceste incertitudini sunt abordările de modelare stocastică ale managementului lanțului de aprovizionare. Modelarea stocastică este un instrument puternic care poate prezice cu o anumită probabilitate evenimentele care pot apărea în cadrul lanțului de aprovizionare, cum ar fi cel asociat proceselor de producție. În cadrul cercetării de față este dezvoltat și propus un model stocastic, bazat pe teoria probabilității, pentru analiza managementului riscului lanțului de aprovizionare, pentru procesele de fabricație. Prin urmare, studiile sunt efectuate pentru a investiga impactul numărului de procese de fabricație asupra evoluției corecte a lanțului de aprovizionare. Studiul actual arată faptul că o creștere a numărului de procese de producție are ca rezultat o creștere a incertitudinii în managementul lanțului de aprovizionare și, astfel, crește probabilitatea de apariție a întreruperii lanțului de aprovizionare. Prin urmare, se recomandă ca un lanț de aprovizionare să conțină un număr minim de procese de fabricație, dacă timpul de livrare și produsul final o permit.

Abstract. Supply chain risk management plays a critical role in the any business or industry environments, and it enables a good coordination of the input and outputs parameters that may affect the smooth processes development such as a manufacturing process for example. However, the supply chain risk management is often prone to the impact of various uncertainties associated with supply chain disruptions caused by meteorological, pandemic, resources shortage, etc. Therefore, one way to quantify these uncertainties are the stochastic modeling approaches of supply chain management. The stochastic modeling is a powerful tool that can predict with certain probability the events that may occur within the supply chain such as that associated with manufacturing processes. In the present research a stochastic model, based on probability theory, is developed and proposed for the analysis of supply chain risk management, for manufacturing processes. Therefore, the studies are performed to investigate the impact of the number of manufacturing processes on the supply chain proper evolution. The current study shows that the increase of the number of the manufacturing processes

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results in an increase of uncertainty in the supply chain management and thus, it increases the probability of supply chain disruption occurrences, within the supply chain. Therefore, it is recommended that a supply chain should contain a minimum number of manufacturing process, if the delivery time and final product allows.

Keywords: Supply chain, stochastic modeling, numerical modeling, dynamical systems

1. Introduction

Various science and engineering processes can be modeled using either deterministic or stochastic approaches. It is important to mention here that a deterministic system is a system in which there is no randomness and thus, it ensures the certainty of the process. However, it is well known that in business or industrial processes there are always uncertainties, and therefore, these uncertainties must be taken into account, and have a plan to mitigate their impact on the supply chain risk management. On the other hand, the stochastic systems are governed by randomness and thus, the outcomes of these systems are prone to uncertainty. It is worth mentioning here that in the stochastic/probabilistic models the uncertainty is explicitly considered and thus, in spite of their uncertainty these models can enhance the supply chain by providing feedback and thus, it allows for the development of alternatives to mitigate the disruptions of the supply chain.

The stochastic /probabilistic models are mathematical techniques which represent the uncertainty of demand by a set of possible outcomes (i.e. a probability distribution). In factory environment setup stochastic/probabilistic models enable the inventory management strategies under probabilistic demands. Stochastic optimization methods are optimization algorithms that incorporate probabilistic (random) elements, either in the problem data (objective function, constrains, etc.) or in both.

The growth of the supply chain management as a discipline, and due to the expanded nature of uncertainty, complexity, and risk, it is important to use different techniques that take into account the uncertainty, to manage global risk of the supply chain. Studies showed that the probabilistic methods/techniques can facilitate the risk management within complex global supply chain [1-15]. Supply chains can be regarded as a network with speeds and feeds, inputs, outputs and processing times that can be modeled as dynamic systems.

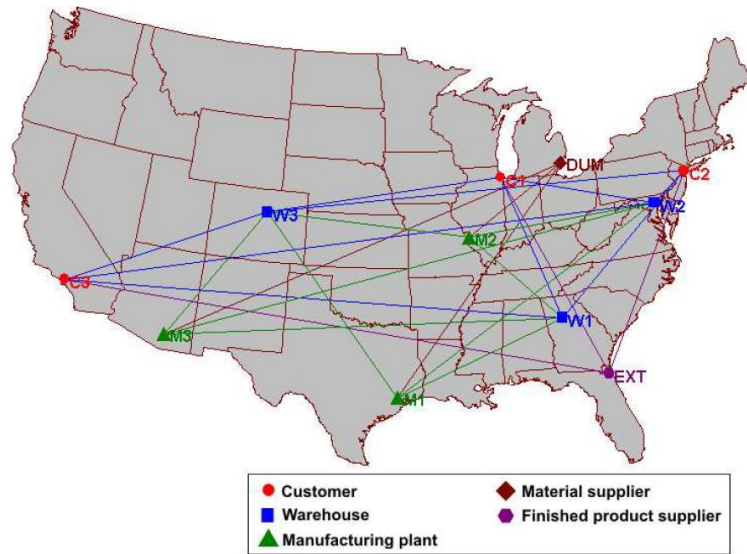


Fig. 1. Schematic of network of supply chain [1]

The probability methods, digital modeling, and discrete-event simulations coupled with risk assessments for a wide range of scenarios can enable better supply chain risk management [1, 2].

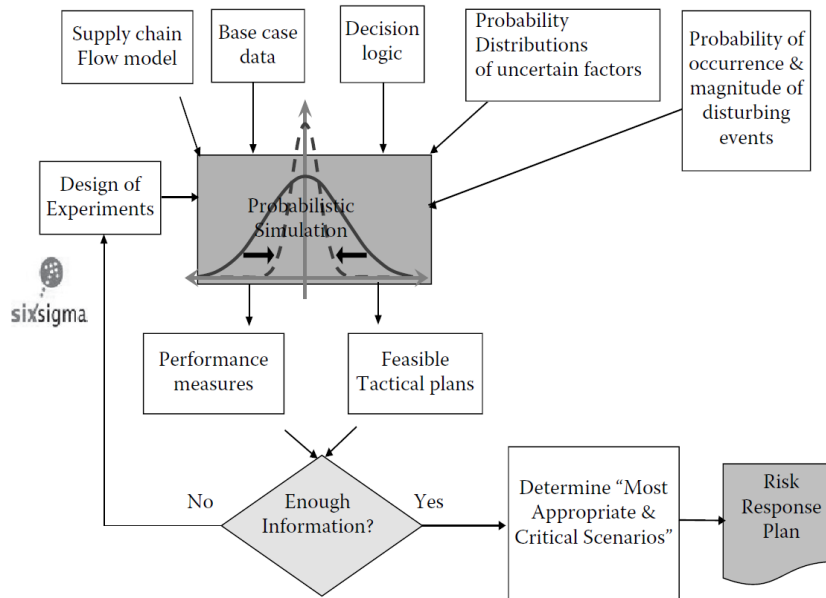


Fig. 2. Schematic of supply chain risk management [2]

Probability theory or the science of uncertainty is a powerful tool for the study of dynamical systems where uncertainty plays a critical role. Therefore, stochastic approaches for modeling the various processes encountered in science and engineering are based on the fundamentals of probability theory. The stochastic processes, also called random processes, represent a process in which the outcome of the process is uncertain. It is worth mentioning here that in general functions and differential equations are usually employed to describe the deterministic process, while random variables and probability are used to describe the stochastic systems. The stochastic approaches use the probability and random variables as are foundation for the description of the stochastic systems.

A stochastic model may try to answer questions related to the average behavior of the variable involved in the stochastic system, the probability related to the elements of the stochastic system or the distribution of the variable of the stochastic system. A schematic of stochastic supply chain management is shown in Figure 2.

2. Literature review

Studies showed that the supply chain management can be enhanced using various simulation techniques [4-15]. Therefore, in the past decade, the industry has oriented towards data-driven approaches for the supply chain risk management [8-12]. The supply chain is associated with uncertainty due to internal disruptions, inside uncertainty and demand uncertainty and all these factors need to be analyzed in an integrated framework. The supply chain risk management comprises five main categories namely: (1) risk monitoring, (2) risk identification, (3) risk assessment, (4) risk modeling, (5) risk mitigation. Nowadays, organizations focus on data-driven approaches toward supply chain risk management due to the increasing risks, technology advancements, and the overwhelmingly growing data provided to the supply chain [3]. In this research, we propose a supply chain risk management approach based on the stochastic mathematical modeling.

2.1 Modeling and algorithms

In this research we focus on the modeling of the supply chain risk management for in complex and global context of nowadays. A predictive method that can estimate the risks and disruptions of the supply chain would help to minimize its impact on the daily lifetime.

A stochastic process can be defined as a collection of random variables $\{X_i, i \in D\}$ where i usually represents the time and D is the index set of the process. The index set D can be represented in two different forms. Therefore, $D = \{0, 1, 2 \dots\}$

for discrete-time stochastic processes, while $D=[0,\infty)$ represents the continuous-time stochastic processes. It is important to mention here that the discrete-time stochastic processes represent sequences of random variables, while the continuous-time stochastic processes represent an uncountable collection of random variables.

Usually, we describe a discrete-time stochastic process as $\{X_i, i \geq 0\}$. It is also worth mentioning that a stochastic process is a collection of random variables $\{X(i), i \in D\}$ which is indexed by the parameter i taking values in the parameter set D . therefore, the random variable takes values in the set D , which is called the state-space of the stochastic process. It is worth mentioning here that in many applications i represents the time. This can be mathematically expressed as $x:D \rightarrow S$, where x is a function and $\{X(i), i \in D\}$ is a stochastic process with state-space D . also, the function can be written as $\{x(i), i \in D\}$ as being a possible evolution (trajectory) of $\{X(i), i \in D\}$. It is important to mention here that the stochastic process follows one of the sample paths with certain randomness, it usually called a random function. Moreover, it is also worth mentioning here that the set of all possible sample paths is often called the sample space of the stochastic process and it is uncountable. The discrete-time stochastic processes defined on a finite state-space are also uncountable.

The random variable X is usually described by its cumulative distribution function (cdf)

$$F(x) = P(X \leq x), -\infty < x < \infty \quad (1)$$

It is worth noting here that a multivariate random variable

$$(X_1, X_2, \dots, \dots, \dots, X_n) \quad (2)$$

is usually described by its joint cumulative distribution function.

$$F(x_1, x_2, \dots, \dots, \dots, x_n) = P(X_1 \leq x_1, X_2 \leq x_2, \dots, \dots, X_n \leq x_n) \quad (3)$$

for all $-\infty < x_i < \infty, i = 0, 1, 2, 3, \dots, \dots$.

The distribution function F of a random variable X is given by

$$F(\lambda) = \int_{-\lambda}^{\lambda} p(t) dt \quad (4)$$

where p is the probability density of X . if X discrete random variable, then its m^{th} moment is given by

$$E[X^m] = \sum_i \lambda_i^m P\{x = \lambda_i\} \quad (5)$$

if the series converges absolutely, where λ_i represents a finite set of distinct values such that

$$P\{X = \lambda_i\} > 0 \quad (6)$$

If X is a continuous random variable with probability density $P(\cdot)$, its m^{th} moment is given by

$$E[X^m] = \int_{-\infty}^{\infty} x^m p(x) dx \quad (7)$$

if the integral converges absolutely.

If X is a random variable and g is a function, then $Y=g(X)$ is also a random variable. If X is a discrete random variable with possible values $x_1, x_2, x_3, \dots, \dots, \dots$ then the expectations of $g(X)$ is given by

$$E[X^m] = \sum_{i=1}^{\infty} g(x_i) P\{X = x_i\} \quad (8)$$

if the sum converges absolutely.

If X is a discrete random variable with possible values $x_1, x_2, x_3, \dots, \dots, \dots$, then the expectation of $g(X)$ is given by

$$E[g(X)] = \int_{-\infty}^{\infty} g(x) P_x(x) dx \quad (9)$$

The general formula, covering both the discrete and continuous cases is

$$E[g(X)] = \int g(x) dF_x(x) \quad (10)$$

where F_x is the distribution function of the random variable X . It is worth mentioning here that the integral of equation 10 is the Lebesgue-Stieltjes integral.

3. Results and discussion

Let's consider a manufacturing process consisting of different stages from the raw material to the final product. Let's assume now that the manufacturing process

would involve processes such as metal cutting, lathe, milling, planning, grinding, drilling, boring, polishing, assembly, receiving, quality control and packing and shipping.

Due to the complex manufacturing process and the steps involved from the raw material to the final product, there are uncertainties that need to be accounted for such that the final product is delivered on time and there is a good correlation between the feed and demand processes. Therefore, to ensure that the supply chain management is well designed it requires the assessment of all the uncertainties throughout the process. In the following, we employ a stochastic model for the strategic design of the supply chain risk management.

One of the main objectives of this research is to assess the impact of the number of sites of the manufacturing process on the supply chain management. However, each manufacturing process involves different levels/stages of quality control which would require the return of the product to the previous manufacturing step of the flow, and therefore, delays and interruptions in the production flux may appear. However, any company would be interested in minimizing these interruptions and delays. Since these delays and disruptions within the manufacturing flux can occur randomly, the probability of completing the final stage of the manufacturing process and implicitly the final product would enable a better assessment of the timelines.

In the following, the impact of the number of stages of the manufacturing flux on the delivery of the final product is analyzed. Therefore, five different manufacturing scenarios are considered and analyzed in this study.

Therefore, Figure 3 presents the results of the stochastic modeling of the supply chain management for a manufacturing process consisting of 6 different steps which may include metal cutting, lathe, milling, receiving, quality control, and packing and shipping. The analysis shows that in the case of simple manufacturing processes, there is a quite smooth flow of the process. Based on the stochastic modeling of the manufacturing process it is expected that the supply chain would encounter fewer disruptions and thus, the on-time delivery of the final product.

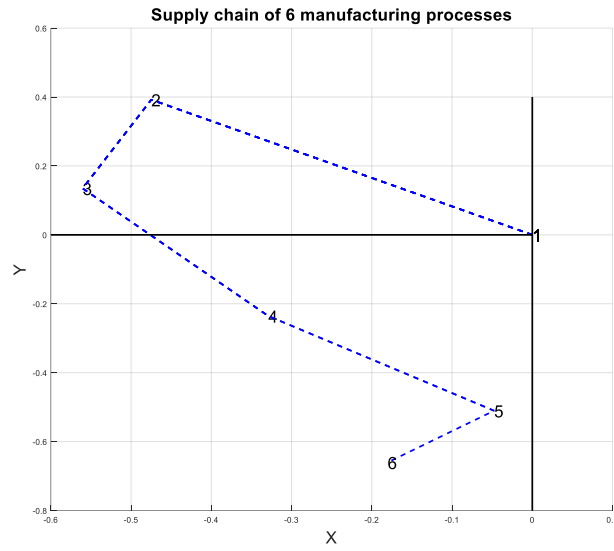


Fig. 3. Stochastic modeling of 6 manufacturing processes

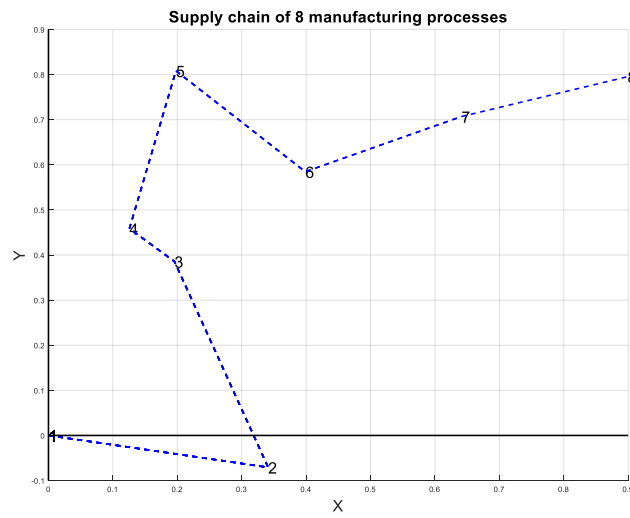


Fig. 4. Stochastic modeling of 8 manufacturing processes

Figure 4 presents the stochastic modeling of a manufacturing process consisting of 8 different steps. In this case it is assumed that the final product requires a more

refined surface and tighter tolerances and therefore, additional manufacturing processes are introduced in the manufacturing flux. Hence, the manufacturing flux includes the following processes, metal cutting, lathe, milling, polishing, assembly, receiving, quality control, and packing and shipping. The analysis of the stochastic modeling results shows that the flow of the process is also quite smooth and without complex trajectories. Based on the stochastic modeling it is also expected that this manufacturing flux would ensure a supply chain of minimum risk management.

Let's assume now that the customer requires additional intermediate manufacturing steps in the regular flux for the increase of final product quality. Therefore, the stochastic modeling of a manufacturing process consisting of 10 different steps is analyzed and presented in Figure 5.

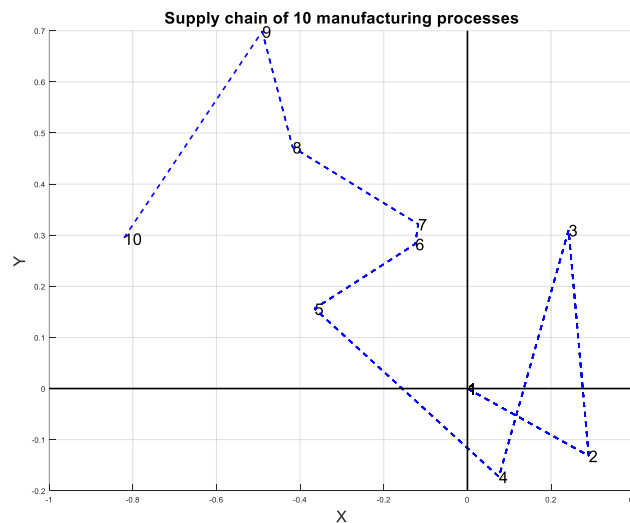


Fig. 5. Stochastic modeling of 10 manufacturing processes

The analysis of the stochastic modeling of the manufacturing process consisting of 10 different steps shows that the complexity of the supply chain management increases and thus, it is expected that it may pose challenges associated with disruption within the supply chain.

The increase of the number of time steps within the flux of the manufacturing processes increases farther the complexity of the supply chain and thus, it is expected to induce higher disruption within the supply chain. The analysis of the results of the stochastic modeling of a 12 manufacturing processes, Figure 6,

shows the complex interactions between the manufacturing processes as the manufacturing process increases in complexity.

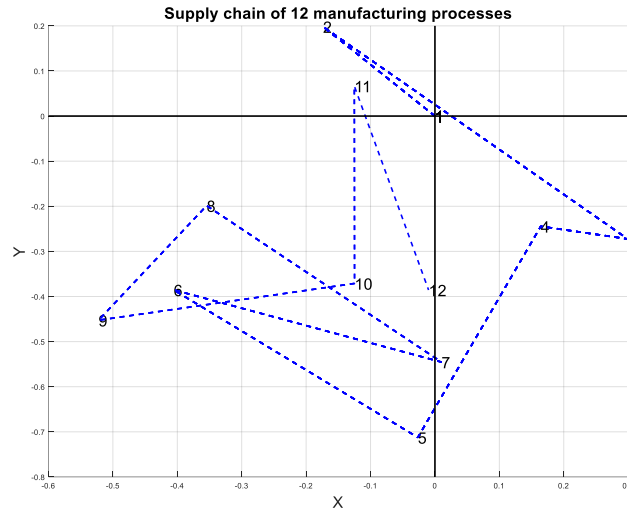


Figure 6. Stochastic modeling of 12 manufacturing processes

A better insight into the supply chain management, and the risk of the supply chain management, can be drawn from the analysis of the probability function of the supply chain management. Therefore, the analysis is performed for four different cases based on the number of manufacturing processes included in the supply chain. Figure 7 presents the probability of the supply chain management for a manufacturing process consisting of 6 different processes.

The analysis of the stochastic modeling of the supply chain reveals that the probability of supply chain disruption is $P=0.65$. However, to better quantify the effect of the number of manufacturing processes on the supply chain management a comparison of the probability distribution among the four different manufacturing processes consisting of 6, 8, 10, and 12 processes, respectively would provide a better insight into the supply chain risk management.

Thus, Figure 8 presents the probability of the supply chain disruptions associated with the manufacturing process consisting of 8 different manufacturing processes. The analysis reveals that there is an increase of the probability of the supply chain disruptions, $P=0.675$, and this is due to the enhanced complexity and networking of the manufacturing processes. This also suggests that the increase in the number of manufacturing processes poses more challenges to the supply chain and thus, it can delay the delivery of the final product to the customer.

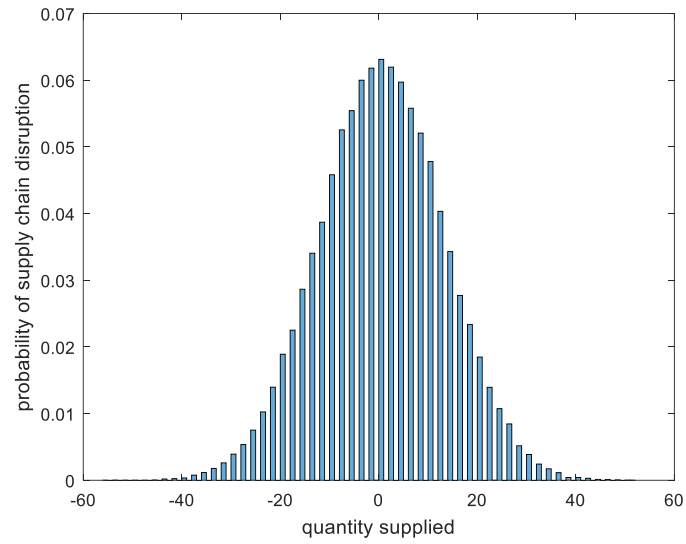


Fig. 7. Probability of supply chain risk management; manufacturing processes=6

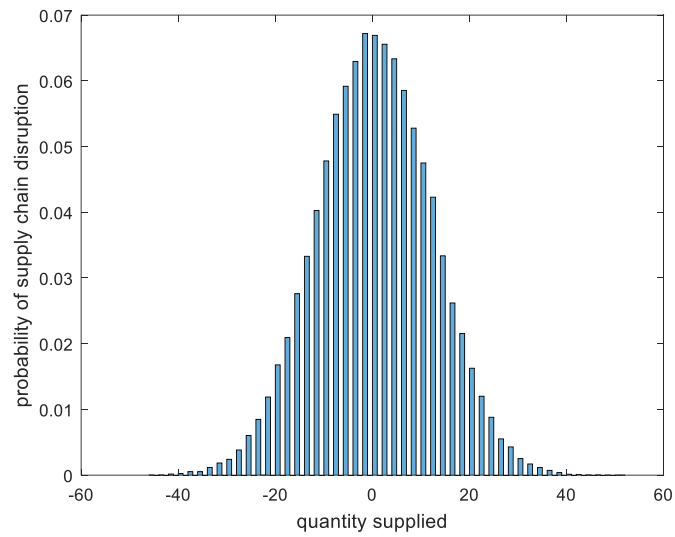


Fig. 8. Probability of supply chain risk management; manufacturing processes=8

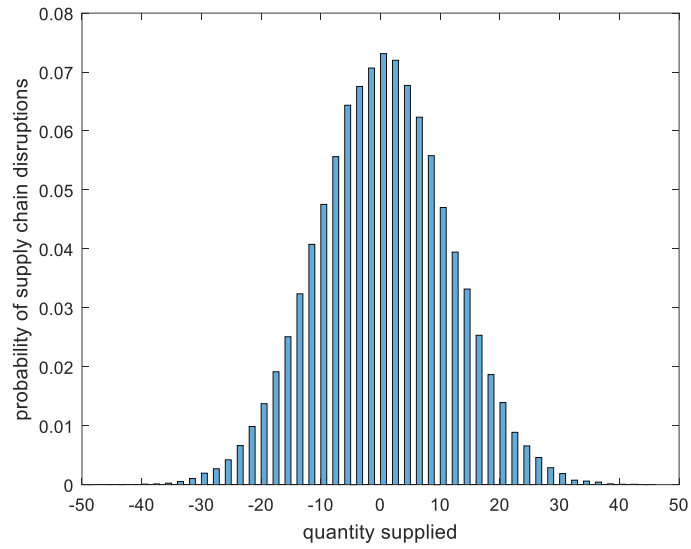


Fig. 9. Probability of supply chain risk management; manufacturing processes=10

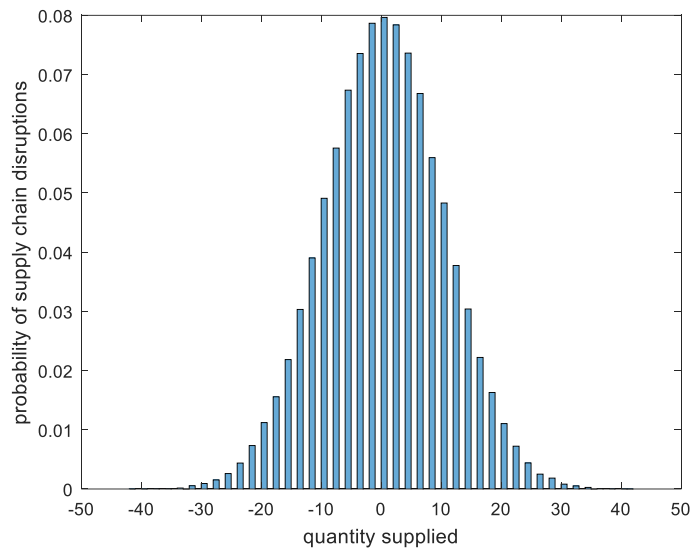


Fig. 10. Probability of supply chain risk management; manufacturing processes=12

Figure 9 presents the probability of the supply chain disruptions for a manufacturing process consisting of 10 different processes/stages. The analysis also shows that the increase of the number of manufacturing processes increases the probability of supply chain disruptions, $P=0.75$ and thus, it would put more strain on the supply chain. Further increase of the number of manufacturing processes increases further the probability of supply chain risk management. Therefore, the analysis of a process consisting of 12 manufacturing processes, Figure 10, reveals the increase in the probability of the supply chain disruptions, $P=0.8$.

Overall the analysis of the stochastic modeling of supply chain reveals that the increase of the number of the manufacturing processes increases the probability of supply disruptions and thus, it makes the supply chain more prone to blockage. Therefore, it is recommended that a lower number of manufacturing processes be employed, whenever it is possible without affecting the safety and quality of the product. It is also recommended that in the cases when the tolerances and surface finishes are not of critical importance these manufacturing processes be eliminated so that the risk of the supply chain management is minimized.

4. Conclusions

A stochastic model using the theory of probability is developed and proposed for the analysis of the supply chain risk management. The study shows that stochastic modeling is a powerful tool that can be used in the analysis of the supply chain risk management. The stochastic modeling of supply chain management can help in enhancing the supply chain management by predicting the disruptions of the supply chain.

The current research shows that the increase in the number of manufacturing processes increases the probability of the supply chain disruptions. Therefore, the larger the number of manufacturing processes the higher the probability of the supply chain disruptions.

Therefore, it is recommended that the number of manufacturing processes be kept at a minimum admissible such that the disruptions of the supply chain would be lowest. However, it is also recommended that the decisions regarding the number of manufacturing processes are made such that the quality and safety of the final product are not compromised.

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