ESTIMATION OF THE ELECTRODYNAMICAL FORCES NEAR FERROMAGNETIC COMPONENTS LOCATED INSIDE LOW VOLTAGE SWITCHING DEVICES

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Rezumat. *În interiorul aparatelor și echipamentelor electrice, conductoarele sunt adeseori plasate în vecinătatea unor pereți feromagnetici sau în interiorul unor nișe feromagnetice. Este necesară cunoașterea forțelor electrodinamice care apar în aceste situații, în vederea proiectării optimale a acestor mașini, aparate sau echipamente. Articolul de față prezintă o metodă generalizată, utilizată pentru calculul acestor forțe, bazată pe estimarea tensorilor lui Maxwell, care ia în considerare și dispunerea asimetrică a conductoarelor în interiorul unor nișe sau crestături rectangulare.*

Abstract. *Inside power apparatus, electric conductors are often placed nearby ferromagnetic walls or inside ferromagnetic slots. It is important to know exactly the forces which appear in those situations, in order to ensure an adequate design of all machines, apparatus and power equipment. This paper presents a more generalized method to compute these forces, based on Maxwell's tensors, and taking into consideration the asymmetry of the conductor placement inside a rectangular slot.*

Keywords: forces, electric conductors, ferromagnetic walls, ferromagnetic slots.

1. Introduction

As we know from [1], electric conductors crossed by different electric currents, are often located or placed nearby ferromagnetic walls. They are even placed inside ferromagnetic slots or compartments. In all these locations, they are subject to certain attraction forces.

The computational methods found in literature [1 - 6] deal with those situations in some particular cases like:

- the electric conductors are placed exactly in the symmetry axis or plan belonging to that geometrical structure;

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- the length of the ferromagnetic wall is considered as infinite;
- the ferromagnetic slots are rectangular or triangular ones only;

This paper presents a more general case, where the electric conductor is placed asymmetrically between two finite length ferromagnetic walls or in such a ferromagnetic slot.

2. Electric Conductor Placed in the Symmetry Axis of a Ferromagnetic Slot

We consider an electrical conductor crossed by a certain current, “i”. First, it is placed in a rectangular ferromagnetic slot. The geometrical dimensions are those represented in Figure 1 a, b.

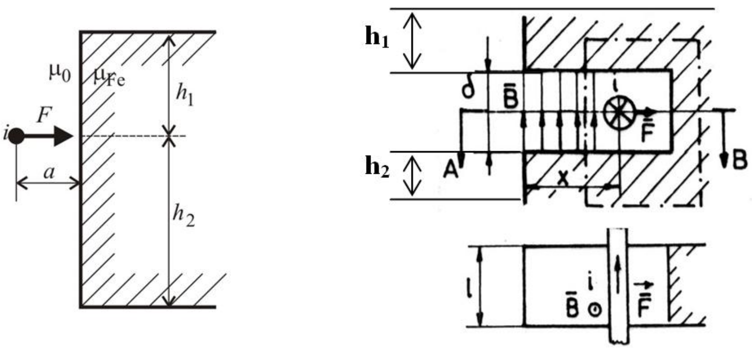

Fig. 1. Electric conductor placed close to a massive ferromagnetic piece, as described:
a. Ferromagnetic wall; **b.** Ferromagnetic slot.

By applying the generalized forces theorems for a specific case when the conductor is placed symmetrically inside the slot and the exterior walls are extended up to infinite (h_1=h_2= ∞), we will obtain the relation for the $\vec{\boldsymbol{F}}$ force which tries to push the conductor to the ferromagnetic wall [1], as shown in Fig.1b.

$$F=\left(\frac{\partial W_m}{\partial x}\right)_{i=ct.}=\frac{\mu_0}{2}\cdot\frac{\ell}{\delta}\cdot i^2 \tag{1}$$

where W_m is the magnetic energy.

Next, we consider that the length of the upper and lower ends of that slot are finite, considered as h_1 and h_2.

When we use the Maxwell Tensors Method for computing the same force, taking into consideration the magnetic induction B, we will obtain this relation:

$$F=F_1+F_2=\frac{\mu_0\cdot\ell}{2\cdot\delta}\cdot i^2\cdot\left(\frac{h_1}{2\cdot h_1+\delta}+\frac{h_2}{2\cdot h_2+\delta}\right) \tag{2}$$

In Fig. 1a. the conductor is placed close to a ferromagnetic wall. By using the electrical image method, we obtained a relation for the specific attraction force:

$$f_{\infty} = \frac{F}{l} = \frac{\mu_0}{4 \cdot \pi \cdot a} \cdot i^2 \tag{3}$$

where l is the length of the conductor and the heights h_1 and h_2 are extended to infinite.

When h_1 and h_2 are finite, (3) becomes:

$$f = f_{\infty} \cdot \varphi_{h1} \cdot \varphi_{h2} \tag{4}$$

where φ_{h1} and φ_{h2} are shape coefficients which took into consideration the heights h_1 and h_2. They are given by:

$$\varphi_{h1} = \frac{\pi \cdot h_1 - 2 \cdot a}{\pi \cdot h_1 + 2 \cdot a} \tag{5}$$

And:

$$\varphi_{h2} = \left(1 + \frac{2 \cdot \pi \cdot a}{\pi \cdot h_1 - 2 \cdot a} \cdot \frac{h_2 - h_1}{\frac{3 \cdot \pi}{2} \cdot h_2 + 2 \cdot a - \frac{\pi}{2} \cdot h_1} \right) \tag{6}$$

All relations starting from (1) up to (6) are particular cases, where the conductor is placed close to a ferromagnetic wall.

3. Electric Conductor Asymmetrically Placed between Two Ferromagnetic Walls

We also consider an electrical conductor crossed by a certain current, “i”. It is asymmetrically placed between two ferromagnetic walls A and B, as shown in Fig. 2.

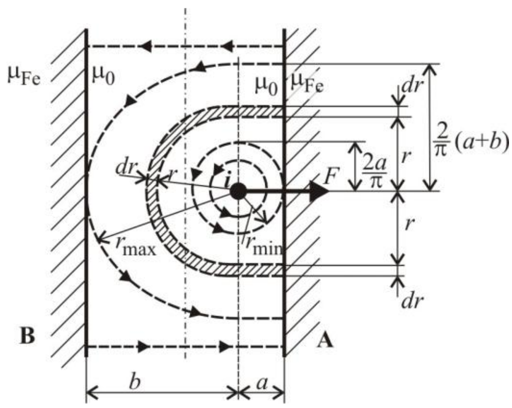

Fig. 2. Electric conductor placed between two ferromagnetic walls.

We appreciate that with the right approximation the magnetic lines are closing according to Fig. 2. The two walls are extended up to the infinite. The length of an ordinary field line, starting from A, continuing in the air, which does not reach the wall B (shape Γ on Fig. 2) is given by:

$$l_\Gamma = 2 \cdot a + \pi \cdot r \tag{7}$$

By applying the magnetic circuit law for the closed loop Γ, we will obtain the magnetic field's intensity H_m as well as the magnetic field's induction B_M, normal in point M, on the A surface:

$$B_M = \mu_0 \cdot H_M = \frac{\mu_0}{\pi \cdot r + 2 \cdot a} \cdot i \tag{8}$$

Consequently, the Maxwell tensor around point M, normal on A is [1]:

$$f_M = \frac{\mu_0}{2} \cdot \frac{i^2}{(\pi \cdot r + 2 \cdot a)} \tag{9}$$

The force applied by the ferromagnetic wall on the electric conductor is the sum of all Maxwell tensors generated by that field lines starting on wall A and not reaching wall B. The minimum radius r_{min} of a field line reaching wall A is:

$$\pi \cdot r_{\min} = 2 \cdot a \tag{10}$$

And the maximum radius of a field line which does not reach wall B, r_{max}, is satisfying the next relation:

$$\pi \cdot r_{\max} = 2 \cdot (a + b) \tag{11}$$

We consider an elementary surface dS_A, on wall A, given by:

$$dS_A = l \cdot dr \tag{12}$$

The integration limits are from r_{min} up to r_{max}. Taking into consideration the conditions established by (10) and (11), the *F* force could be written as:

$$F = 2 \cdot \int_{r\min}^{r\max} f_m dS_a = \int_{\frac{2a}{\pi}}^{\frac{2(a+b)}{\pi}} \frac{\mu_0 \cdot l}{2} \cdot i^2 \cdot \frac{dr}{(\pi \cdot r + 2 \cdot a)^2} \tag{13}$$

After computing the integral, we will obtain:

$$F = \frac{\mu_0 \cdot i^2 \cdot l}{4 \cdot \pi \cdot a} \cdot \frac{b - a}{b + a} \tag{14}$$

And the specific attraction force:

$$f = \frac{F}{l} = \frac{\mu_0 \cdot i^2}{4 \cdot \pi \cdot a} \cdot \frac{b - a}{b + a} = f_\infty \cdot \varphi_{ab} \tag{15}$$

where:

$$\varphi_{ab} = \frac{b - a}{b + a} \tag{16}$$

and f_∞ is given by the same relation (3).

Relations (14) and (15) allow us to make some significant observations:

- We observe that the presence of a second infinite ferromagnetic wall, called B in this case, diminishes the attraction force applied on the conductor by the ferromagnetic wall A, with a diminution coefficient φ_{ab} given by relation (16).
- If the conductor is placed symmetrically ($a = b$), then the attraction force becomes zero, which is logical, because the attraction forces applied by both walls are cancelling each other.
- If dimension “*a*” becomes greater than “*b*”, the attraction force changes its sign, and the conductor will be attracted by the B wall, instead of A, which is also a logic thing.
- If wall B is taken away ($b>>a$), then $\varphi_{a\,b} \rightarrow 1$ and relation (15) becomes relation (3), which is the one deduced for a single wall.
- If we look at the integration limits, relations (14) and (15) are not necessarily applied for infinite walls. They are used for walls having a minimum height equal to $2(a+b)/\pi$.

4. Electric Conductor Asymmetrically Placed Between Two Ferromagnetic Walls

We consider an electrical conductor asymmetrically placed in a ferromagnetic rectangular slot, as shown in Fig. 3 a and b.

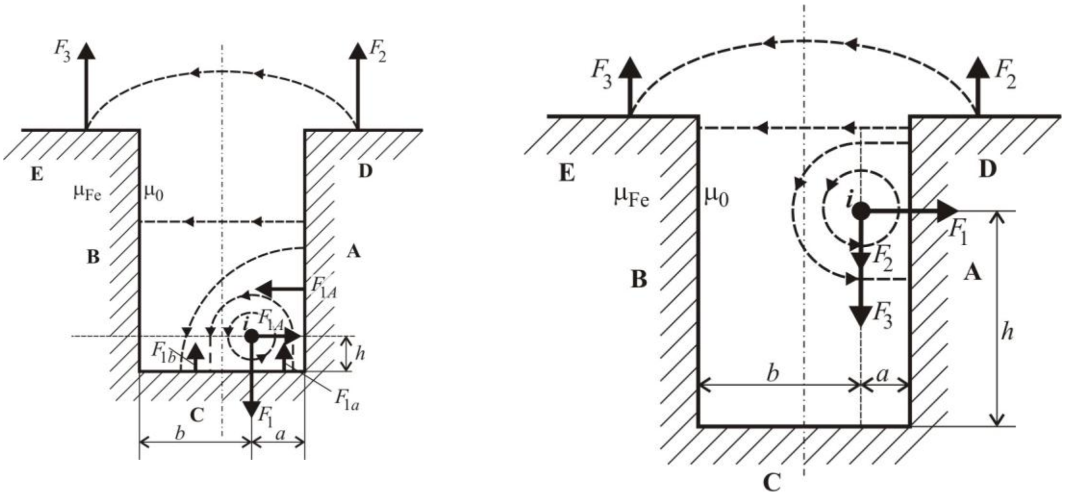

Fig. 3. Electric conductor placed inside a ferromagnetic slot.
a. Placed far from the bottom; **b.** Placed close to the bottom.

The situation described in Fig. 3a could be treated starting from the previously presented case, when the conductor was placed between two ferromagnetic walls. The main condition is that the “h” distance between the conductor and the bottom of the slot, named as C, must fulfil the condition given by (11), which means to be at least equal to 2(a+b)/π.

In this case, the attraction force F1, exerted by the A wall on the conductor, is given by a relation similar to (14), which is:

$$F = \frac{\mu_0 \cdot i^2 \cdot l}{4 \cdot \pi \cdot a} \cdot \frac{b-a}{b+a} \tag{27}$$

Forces F_2 and F_3, generated by Maxwell’s tensors are also applied on the conductor. They are produced on the exterior D and E surfaces of the slot, and they are pushing the conductor to the bottom C of the slot.

If we consider surfaces D and E extended to the infinite, according to (1), the main forces are:

$$F_2 = F_3 = \frac{\mu_0 \cdot i^2 \cdot l}{4 \cdot (a+b)} \tag{38}$$

Consequently, the total force F, which is pushing the conductor up to the bottom C of that slot, is given by:

$$F = F_2 + F_3 = \frac{\mu_0 \cdot i^2 \cdot l}{2 \cdot (a+b)} \tag{49}$$

We noticed that if $a = b$, then F_1 becomes zero, so only the F force is applied to the conductor. We are finding relation (1).

The case b in Fig. 3 does not satisfy the relation from which the distance between the conductor and the bottom must be higher than $2(a+b)/\pi$.

In this situation, the conductor will be attracted to the bottom of the slot with a F_1 force and it will be additionally pushed by a force $F = F_2 + F_3$. It will be also attracted by the A wall with an additional F_{1A} force.

Because forces F_2 and F_3 are the same like in case a), they are given by the previous relation (18).

The F_1 force is given by the sum of F_{1a} and F_{1b}, forces that appear on the surfaces having an “*a*” and a “*b*” length, belonging to the C wall.

By using the Maxwell’s tensors on the field lines having the specified shape like in Fig. 3b, we will obtain:

a. For the F_1a force, the average field line is considered as:

$$l_{1a} = 2 \cdot h + \pi \cdot r \tag{20}$$

By repeating all computational procedures like previously described in chapter 3, it results:

$$F_{1a} = 2 \cdot \int_{r\min}^{r\max} f_m dS_C = \int_{\frac{2h}{\pi}}^{a-h} \frac{\mu_0 \cdot l}{2} \cdot i^2 \cdot \frac{dr}{(\pi \cdot r + 2 \cdot h)^2} = \\ = \frac{\mu_0 \cdot l \cdot i^2}{4 \cdot \pi \cdot h} \cdot \frac{\pi(a-h)+2h}{\pi(a-h)+2h} \tag{21}$$

b. For the F_{1b} force, the average field line is considered as:

$$l_{1b} = h + \frac{\pi}{2} r + a \tag{22}$$

By repeating all computational procedures like previously described in chapter 3, it results:

$$r_{min} = a\text{-}h \\ r_{max} = b\text{-}h \tag{23}$$

It results:

$$F_{1b} = \int_{a-h}^{b-h} \frac{\mu_0 \cdot l \cdot i^2}{2} \cdot \frac{dr}{(\frac{\pi}{2} r + h + a)^2} = \\ = \frac{\mu_0 \cdot l \cdot i^2}{2} \cdot \frac{b-a}{\frac{\pi}{2}(a-h)+h+a} \cdot \frac{b-a}{\frac{\pi}{2}(b-h)+h+a} \tag{24}$$

c. The F_{1A} force is equal to F_{1b}, but oriented like in Fig. 3b.

On Fig. 4, we can distinctly represent the composition of all forces applied on the electric conductor placed inside the ferromagnetic slot, like in the next Fig. 3b.

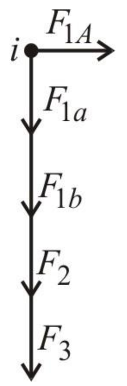

Fig. 4. The forces applied on the electric conductor asymmetrically placed near the bottom of the ferromagnetic slot.

According to (24), we still notice that if $a = b$, then $F_{1b} = 0$ and $F_{1A} = 0$, which is perfectly logical.

5. Conclusions

The situation of the electrical conductors placed near ferromagnetic walls (or inside ferromagnetic slots) is a case frequently seen inside all the electrical machines and apparatus, as well as transformers and power equipment.

To know exactly the forces acting in these cases is a priority for all designers of such equipment.

The computational method based on Maxwell's tensors offers a more precise point of view by comparing the classical methods based on the magnetic energy and using many simple models.

All the computational processes are carried out by considering a more general case, when the electric conductor is placed asymmetrically between two ferromagnetic walls.

The new formulas deduced by using those considerations are perfectly compatible with the older cases, all simplified older formulas are particular cases of the newest ones.

R E F E R E N C E S

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