ATRIAL FIBRILLATION ANALYSIS BY MEANS OF NONLINEAR METHODS

Vlad GHIZDOVAT[1], Maricel AGOP[2,3,*]

Abstract. *In this work we use nonlinear methods to study atrial fibrillations: we construct the 3D attractors, generate the 2D maps, and calculate the Hurst exponent and fractal dimension, respectively, for signals collected from a patient during an atrial fibrillation crisis. Our results show that the normal and pathological cardiac dynamics can be characterized through specific patterns (strange attractors).*

Keywords: atrial fibrillation; attractors; Hurst exponent; electrocardiogram

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Introduction

Heart rate dynamics is a subject of great interest in the field of medicine. Various mathematical procedures were proposed and used for analyzing heart-rate time series, in order to evaluate the heart-rate variability: autocorrelation function, bifurcation analysis, Lyapunov exponents, reconstructed phase-space analysis, recurrence plot, Hurst exponent, entropy etc. Accurate and complete descriptions of these methods can be found in specialized literature [1-4].
A crucial stage in the analysis of ECG signals involves extracting the clinically significant features that include all the pertinent information from the original ECG signal. These features serve as a representation of the signal for further analysis [5, 6]. The ECG signals may be analyzed to extract features using various approaches, such as time-domain analysis, frequency-domain analysis, combined time-frequency domain analysis, and nonlinear methods [7–9]. In recent years, researchers have shown particular interest in analyzing ECG data utilizing nonlinear signal processing approaches [7–9]. The ECG signal analysis techniques that are nonlinear in nature are inspired by the principles of nonlinear dynamics [10, 11]. This may be explained by the fact that biological signals, such as ECG, can be produced by nonlinear dynamical systems [12]. Nonlinear signal analysis approaches that have been extensively studied include reconstructed phase space analysis, Lyapunov exponents, correlation dimension, detrended fluctuation

[1] Biophysics and Medical Physics Department, "Grigore T. Popa" University of Medicine and Pharmacy, 700115 Iasi, Romania
[2] Academy of Romanian Scientists, 3 Ilfov, 050044, Bucharest, Romania
[3] Department of Physics, "Gheorghe Asachi" Technical University of Iasi, 700050 Iasi, Romania
[*] corresponding author, email: m.agop@yahoo.com

analysis (DFA), recurrence plot, Poincaré plot, approximation entropy, and sample entropy.
Taking the above into consideration, it can be said that an analysis of the ECG time series using nonlinear dynamics can yield insights into physiological processes beyond those related to cardiology. Experimental findings demonstrated that the pulse rate signal exhibits a low-dimensional chaotic condition during an epileptic seizure, characterized by a complicated and irregular pattern before and after the event [13]. Young and Benton demonstrated a connection between heart rate complexity and several cognitive functions such as reaction times, inhibition, and decision time. Their findings highlight the potential of nonlinear approaches for analyzing heart rate time series data in understanding the relationship between brain functioning and specific behaviors [14].
Although, as we mentioned, the nonlinear dynamics and complex system theory provided important information on many complex aspects in cardiology, the application of nonlinear systems analysis in therapy predictions is currently restricted in clinical practice [15]. Efforts should be made to enhance the existing procedures and devise novel approaches to enhance the efficacy of clinical applications.
This work presents a novel approach to assess atrial fibrillations, employing mathematical techniques tailored to nonlinear dynamics.

Materials and methods

We conducted an analysis on electrocardiograms obtained from the PhysioNet database. This database provides open access to a comprehensive collection of physiological signals [16] obtained from a diverse group of patients. It also offers specialized software for the visualization and analysis of these signals. It is freely accessible under the ODC Public Domain Dedication and License v1.0. Available resources are provided to encourage ongoing research in the field of analyzing intricate biomedical and physiological data.
The signal we examined possesses the subsequent characteristics: the recording has a duration of approximately 3 hours, with a sampling interval of 4 milliseconds and a sampling rate of 250 recordings per second. It consists of a total of 9,205,760 data points, with amplitudes ranging from -0.6 millivolts to 0.9 millivolts.
Figure 1 depicts the examination of the 1/R-R interval, which represents a single cardiac cycle. The analysis reveals three instances of crises, consisting of two atrial fibrillations and one flutter fibrillation. The analysis includes ECG fragments of 5 seconds each, representing the pre-crisis, first AFIB, atrial flutter (AFL), second AFIB, and post-crisis periods.

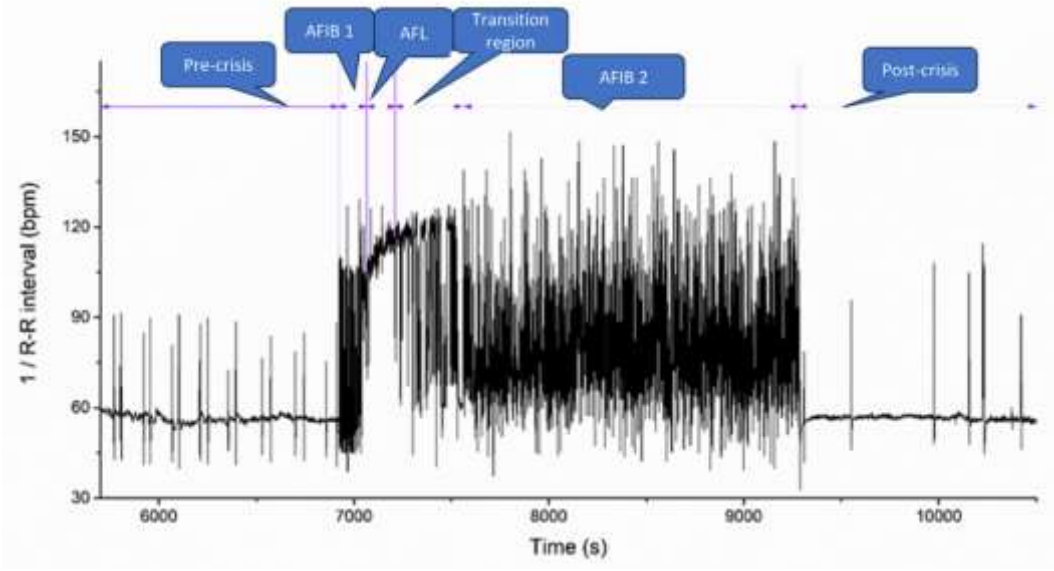


Fig. 1. Pulse time variation (1/R-R interval) during fibrillation crises

During both the pre-crisis and post-crisis periods, the signals remain within the expected range. During the initial episode of atrial fibrillation (AFIB), the heart rate gradually rises, followed by a sudden surge in atrial flutter (AFL), and then gradually decreases again in the subsequent episode of AFIB.

Results

The statistical analyses were performed using Origin Pro, version 9.6.5.169. Shapiro-Wilk test [17] was employed to assess the normality of the distribution of the measured data.

The histogram analysis of the signals from Figure 1, as depicted in Figure 2, reveals that during the initial phase, specifically the pre-crisis period, the pulse remains consistently constant at approximately 60 beats per minute. The heart rate typically rises during the initial episode of atrial fibrillation, reaching a peak of approximately 110 beats per minute. During atrial fibrillation, the pulse typically ranges between 100 and 130 beats per minute (bpm). The histogram representing the second instance of atrial fibrillation exhibits a distribution that closely resembles a Gaussian curve, with the highest frequency occurring between 60 and 80 beats per minute. Following the crisis, the pulse becomes stable and reaches a rate of 60 beats per minute.

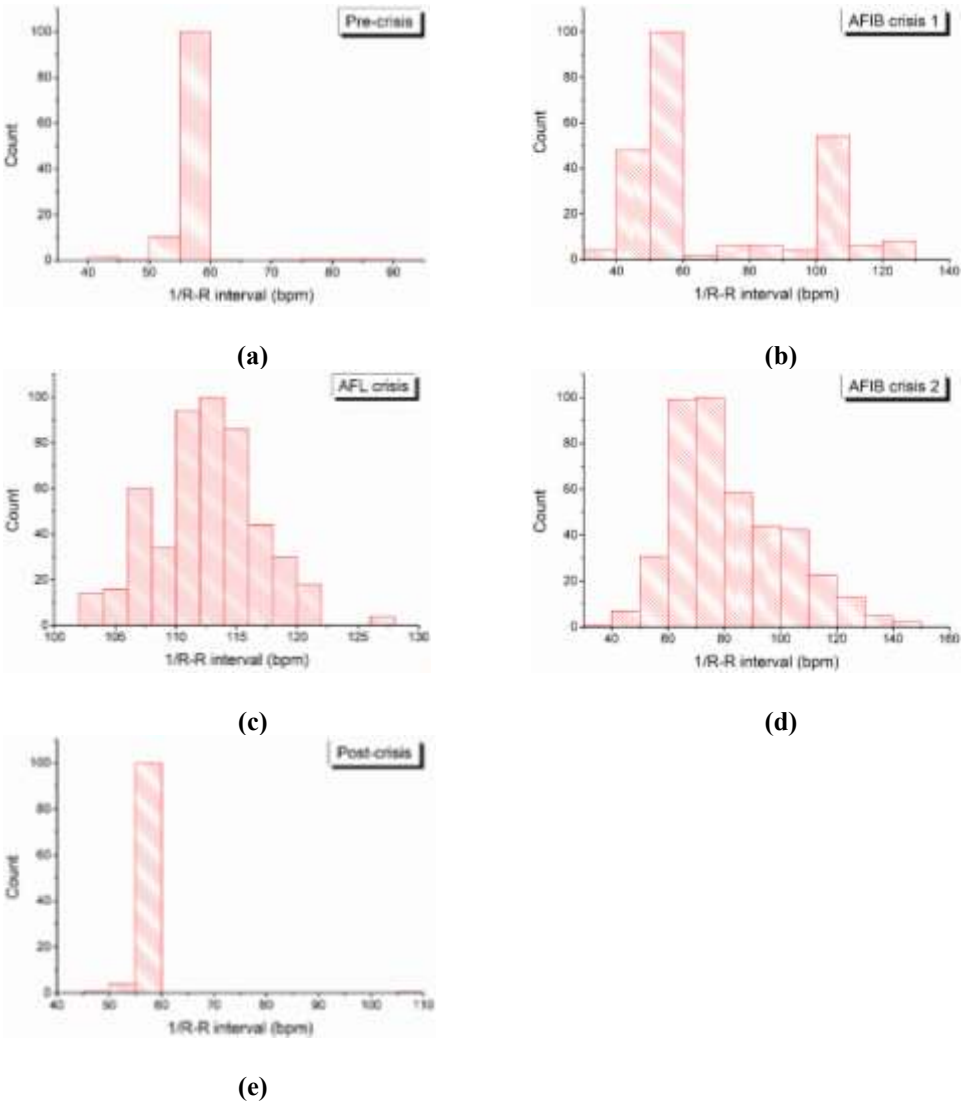


Fig. 2. Histograms associated to periods of pre-crisis (a), first AFIB (b), AFL (c), second AFIB (d), and post-crisis (e) (1/R-R intervals)

The Shapiro-Wilk test excluded the normal distribution for all five cases described in figure 2. Consequently, log-normal and gamma distributions were tested (see Figure 3). Anderson-Darling test rejected both distributions for all cases. According with the extensive analysis performed by Kula et al. [18], the median is more appropriate to be considered in data with unknown distribution to characterize the central tendency of data (real mean values).

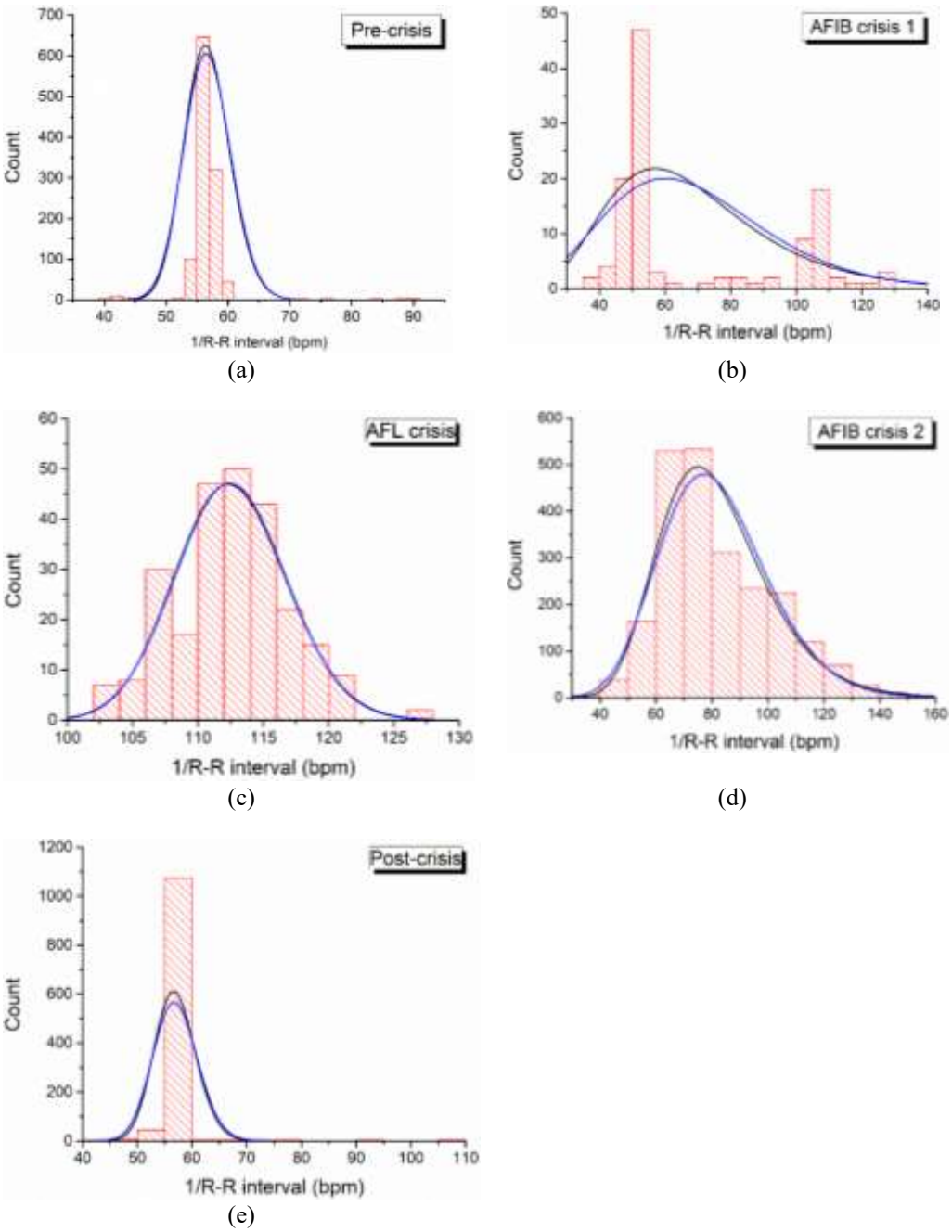
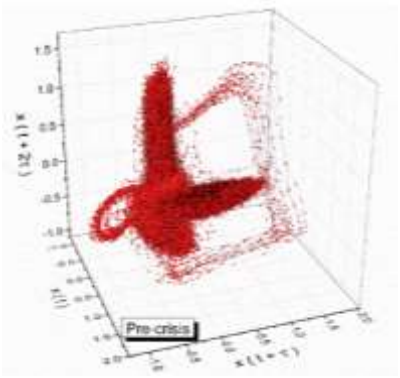
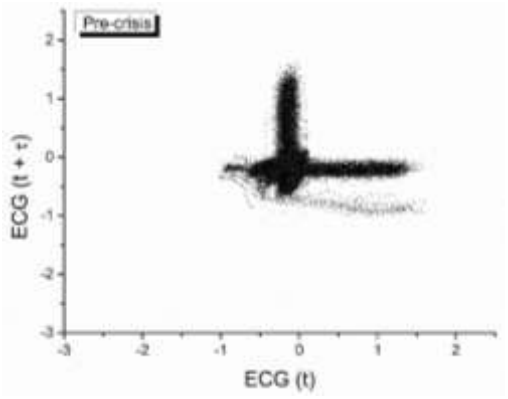


Fig. 3. Fitting of the data with log-normal (black line) and gamma (blue line) distributions, corresponding to the five cases described in Figure 3

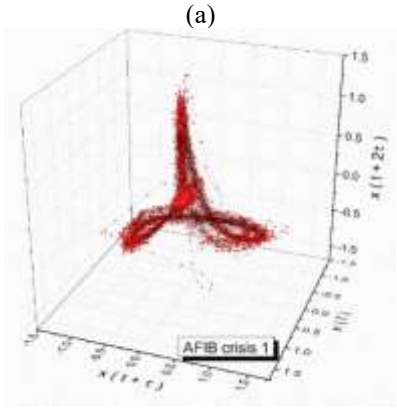
By employing the auto-correlation function, we developed specific attractors within the phase space (reconstructed using the delay time approach) for each stage of the heart dynamics: pre-crisis, AFIB crises 1 and 2, AFL crisis and post-crisis. These attractors and the corresponding 2D maps are shown in Figure 4.



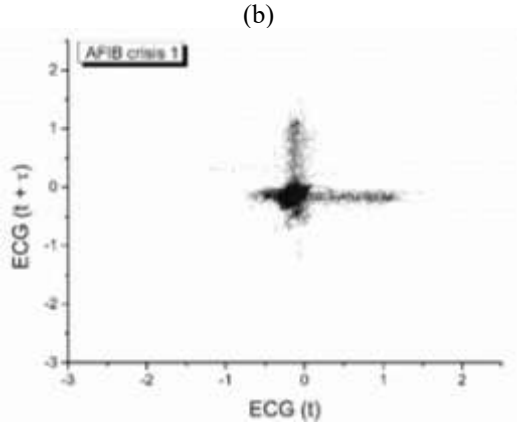
(a)



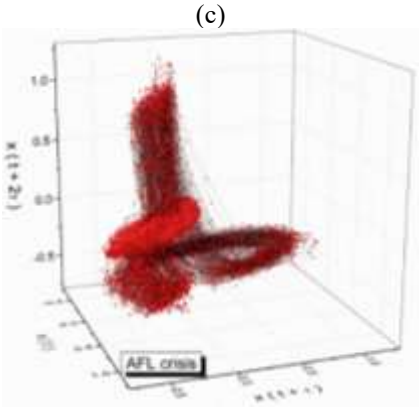
(b)



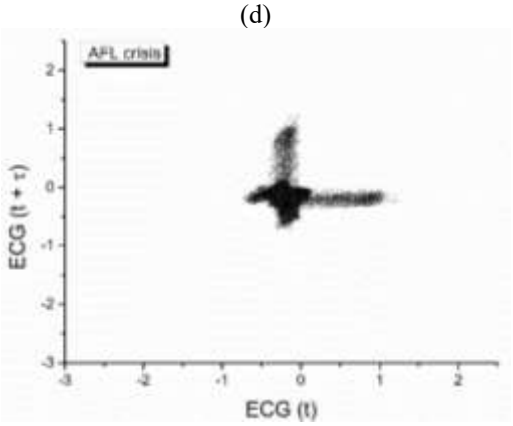
(c)



(d)



(e)



(f)

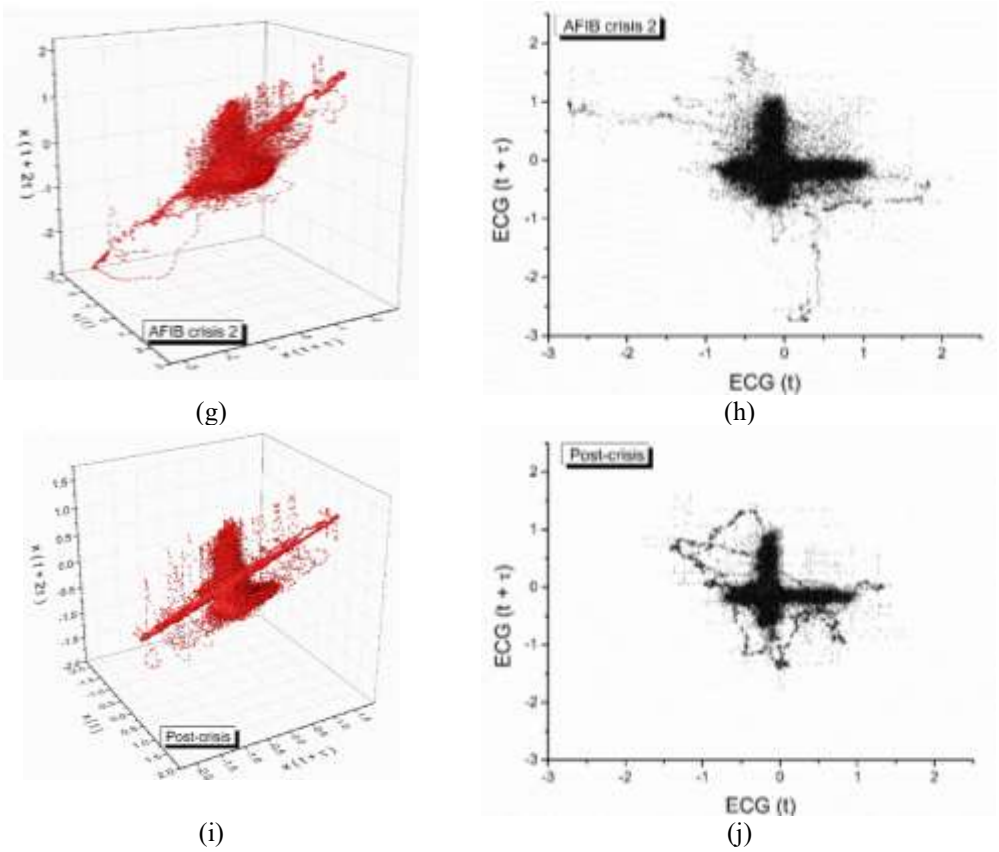


Fig. 4. Attractors for Pre-crisis (a), AFIB 1 (c), AFL (e), AFIB 2 (g) and Post-crisis (i) and the corresponding 2D maps for Pre-crisis (b), AFIB 1 (d), AFL (f), AFIB 2 (h) and Post-crisis (j)

AFTER we built the attractors and generated the corresponding 2D maps, we were able to calculate the Hurst exponents and fractal dimension for each stage. They are presented in Table 1.

Table 1. The calculated Hurst exponents for each stage of the heart dynamics

Stage	Hurst exponent	Fractal dimension
Pre-crisis	0.7739	0.2261
AFIB crisis 1	0.6772	0.3228
AFL crisis	0.6680	0.332
AFIB crisis 2	0.7871	0.2129
Post-crisis	0.8390	0.161

Discussions

As can be seen from Table 1, the Hurst exponent for all the stages is subunitary. Furthermore, we can observe, by means of the fractal dimension variation, that the physical processes (in our case, atrial fibrillations) that disrupt the normal functioning of the heart produce a fractal pattern in time. The largest fractal dimension is present in the AFL crisis, and the lowest obtained values are present in pre- and post-crisis, respectively. These facts show that the cardiac dynamics is strongly chaotic during the AFL crisis, while before and after the crisis, these dynamics tend to have a more regular behavior.
We must mention that several recent works have simulated ECG signals, in order to analyze cardiac rhythms by employing reduced-order mathematical models composed of oscillators with time-delayed couplings. The models were able to accurately represent the key elements of the dynamic response of the heart, generating electrocardiograms for several scenarios of both normal and abnormal heart rhythms [19, 20].
Our approach was different, because we analyzed an actual ECG recording. However, our study has several limitations, including the fact that we only examined a solitary instance and relied on data sourced from an open-access database. To enhance the credibility of our results, it is necessary to acquire data gathered in a controlled clinical environment.

Conclusions

We proposed a new method for analyzing pathological cardiovascular dynamics, in particular atrial fibrillation, by using non-linear procedures: variance, geometric standard deviation, histograms, attractors, 2d maps etc. In such context, by using the self-similar correlations of the Hurst exponent and fractal dimension, we propose a first step toward developing a non-linear theory of the physiological mechanisms that generate these life-threatening cardiac events.

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