

COMBINATORICS OF HANKEL RELATIONS*

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Abstract

We investigate the problem to determine the defining equations of the algebraic variety of Hankel two-planes in the projective space. We compute the first and the second partial lifting of the Machado's binomial relations, by applying techniques of Sagbi bases theory.

MSC: 13A02, 16W50, 14M15, 13P10

keywords: Sagbi basis, Grassmann variety, Hankel variety, Toric deformation of a variety.

1 Introduction

In the study of toric ideals and of canonical bases of subalgebras many authors are interested on the problem to degenerate an arbitrary parametrically presented variety X into a toric variety Y . The basic idea is to degenerate the algebra generators into monomials and therefore the algebra polynomial relations to binomials relations. We can see how this can be accomplished if X is a Grassmann variety, since there is a beautiful link between the toric ideal I of the toric degeneration of X and the Grassmann-Plücker ideal of X , whose initial ideal, with respect to a fixed weighted term order ([13]) on the monomials of the polynomial ring of the presentation of

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