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AN OVERVIEW OF RESEARCH ON EVOLUTION EQUATIONS GOVERNED BY SUBDIFFERENTIAL OPERATORS*

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In memory of Haim Brezis (1944-2024), a brilliant mathematician with roots in Romania

Abstract

Let H be a real Hilbert space, and let $A : D(A) \subset H \to H$ be a (possibly multivalued) subdifferential operator. In this article we remind the most important results regarding the existence and asymptotic behavior for $t \to \infty$ of solutions to the evolution equation (inclusion) $u'(t) + Au(t) \ni f(t)$, $t > 0$, including contributions of J.-B. Baillon, H. Brezis, R.E. Bruck, Y. Kōmura, H. Okochi, and of the author. On this occasion, we show that a stability theorem of V. Barbu is in fact a particular case of a previous result of H. Brezis. Also, we extend that Brezis' result establishing the *weak convergence* as $t \to \infty$ of every (weak) solution to the above evolution inclusion.

Keywords: evolution equation, monotone operator, subdifferential operator, existence and asymptotic behavior of solutions as $t \to \infty$.

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1 Introduction

In this paper we are concerned with the following differential equation (inclusion)

$$u'(t) + Au(t) \ni f(t), \ t > 0,$$

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were $A : D(A) \subset H \to H$ is the (possibly multivalued) *subdifferential* of a function $\varphi : H \to (-\infty, +\infty]$ satisfying the following conditions

(H_φ) φ *is proper* (i.e., its effective domain $D(\varphi) = \{v \in H;\, \varphi(v) < +\infty\}$ is nonempty), *convex, and lower semicontinuous* (LSC).

The *subdifferential* of φ, denoted $A = \partial\varphi$, is defined by

$$\partial\varphi(x) = \{y \in H;\, \varphi(x) - \varphi(v) \le (y, x - v),\ \forall v \in D(\varphi)\}.$$

Obviously, $D(\partial\varphi) \subset D(\varphi)$. It is also easy to check that $A = \partial\varphi$ is a *monotone operator*, i.e., for every $x_1,\, x_2 \in D(A)$ and $y_1 \in Ax_1,\ y_2 \in Ax_2$, we have $(x_1 - x_2, y_1 - y_2) \ge 0$.

The class of monotone operators is wider than that of subdifferential operators. We continue with an important definition:

Definition 1. A monotone operator $A : D(A) \subset H \to H$ is said to be *maximal monotone* if its graph $G(A) = \{[x, y];\, x \in D(A),\, y \in Ax\}$ is not properly included into the graph of another monotone operator.

Theorem 1. (G. Minty, 1962) *A monotone operator $A : D(A) \subset H \to H$ is maximal monotone if and only if the range $R(I + \lambda A) = H$ for all $\lambda > 0$ (equivalently, for some $\lambda > 0$).*

Remark 1. We state the following:

(i) If $A : D(A) \subset H \to H$ is maximal monotone, then for every $x \in D(A)$

$$Ax = \{y \in H;\, (y - w, x - v) \ge 0,\ \forall [v, w] \in G(A)\},$$

so Ax is a closed convex set.

(ii) If $A : D(A) \subset H \to H$ is maximal monotone, so is the inverse operator A^{-1}, where A^{-1} is defined as follows

$$[y, x] \in G(A^{-1}) \iff [x, y] \in G(A).$$

(iii) For a maximal monotone operator A, the closure of $D(A)$, denoted $\overline{D(A)}$, is a convex set (see, e.g., G. Moroşanu [5, Theorem 1.3, p. 21]).

For other properties of maximal monotone operators, we refer the reader to H. Brezis [3] and G. Moroşanu [5]. We only recall the following important result:

Theorem 2. (H. Brezis [3, p. 25 and p. 29]) *Assume that (H_φ) above holds. Then, $A = \partial\varphi$ is a maximal monotone operaor, with $\overline{D(A)} = \overline{D(\varphi)}$.*

Now, let us consider the Cauchy problem associated with a maximal monotone operator $A : D(A) \subset H \to H$, denoted as (P),

$$\begin{cases} u'(t) & +Au(t) \ni f(t), \ t > 0, \\ u(0) & = u_0. \end{cases}$$

We denote by (P_T) the same Cauchy problem restricted to a bounded interval $[0, T]$, $T > 0$.

Definition 2. Let T be a positive number and $f \in L^1(0, T; H)$. A function $u \in C([0, T]; H)$ is said to be a *strong solution* of problem (P_T) if

- u is absolutely continuous on every compact subinterval of $(0, T)$;
- $u(t) \in D(A)$ for a.a. $t \in (0, T)$;
- $u(0) = u_0$ and u satisfies equation (E) for a.a. $t \in (0, T)$.

Definition 3. Let T be a positive number and $f \in L^1(0, T; H)$. A function $u \in C([0, T]; H)$ is called a *weak solution* of problem (P_T) if $(\exists)$ some sequences $\{u_n\} \subset W^{1,\infty}(0, T; H)$ and $\{f_n\} \subset L^1(0, T; H)$ such that

- $u_n'(t) + Au_n(t) \ni f_n(t)$, for a.a. $t \in (0, T)$, $n = 1, 2, \ldots$;
- $u_n \to u$ in $C([0, T]; H)$;
- $u(0) = u_0$ and $f_n \to f$ in $L^1(0, T; H)$.

In both cases the solution is unique (as follows by a Gronwall type lemma in H. Brezis [3, Appendix, Lemma A.5, p. 157]; see also G. Moroşanu [5, Lemma 2.1, p. 47]). For $f \in L^1_{loc}(0, \infty; H)$ one can naturally extend the notions of *strong solution* and *weak solution* to problem (P), i.e., $u = u(t)$, $t \geq 0$, is a strong (weak) solution of problerm (P) if the restriction $u_{[0,T]}$ is a strong (respectively, weak) solution of problem (P_T) for all $T \in (0, +\infty)$.

Now let us recall an existence result which is essentially attributed to Y. Kōmura, being later enriched by H. Brezis [2, Proposition 3.3, p. 68].

Theorem 3. (see, e.g., G. Moroşanu [5, Theorem 2.1, p. 48]) *Let $A : D(A) \subset H \to H$ be a maximal monotone operator. If $u_0 \in D(A)$ and $f \in W^{1,1}(0, T; H)$ for some $T > 0$, then problem (P_T) has a unique strong solution $u \in W^{1,\infty}0, T; H)$. Moreover, u is differentiable from the right at any point in $[0, T)$ and*

$$\frac{d^+u}{dt}(t) = \big(f(t) - Au(t)\big)^0, \ \textit{for all } 0 \leq t < T,$$

$$|(d^+u/dt)(t)| \le |\big(f(0) - Au_0\big)^0| + \int_0^t |f'(s)|\,ds, \text{ for all } 0 \le t < T,$$

where $\big(f(t) - Au(t)\big)^0$ *is the element of minimum norm of the closed convex set* $f(t) - Au(t)$ *(the case* $t = 0$ *being included).*

A consequence of Theorem 3 is the following.

Theorem 4. (H. Brezis [3, Theorem 3.4, p. 65], G. Moroşanu [5, Theorem 2.2, p. 55]) *Let* $A : D(A) \subset H \to H$ *be a maximal monotone operator. For every* $u_0 \in \overline{D(A)}$ *and* $f \in L^1(0,T;H)$ *for some* $T > 0$*, problem* (P_T) *has a unique weak solution* u *and, for all* $0 \le s \le t \le T$, $[x,y] \in G(A)$,

$$\frac{1}{2}|u(t) - x|^2 \le \frac{1}{2}|u(s) - x|^2 + \int_s^t (f(\tau) - y, u(\tau) - x)d\tau.$$

Let us mention an interesting (nontrivial) result for the case when H is finite dimensional.

Theorem 5. (H. Brezis [3, Proposition 3.8, p. 82]) *Assume that* H *is a Hilbert space of finite dimension, and let* $A : D(A) \subset H \to H$ *be a maximal monotone operator. Then, for every* $(u_0, f) \in \overline{D(A)} \times L^1(0,T;H)$ *for some* $T > 0$*, problem* (P_T) *has a unique strong solution.*

Theorem 6. (H. Brezis [3, Theorem 3.2, p. 57 and Theorem 3.6, p. 72]) *Let* $A = \partial\varphi$*, with* φ *satisfying* (H_φ)*. Then, for every* $u_0 \in \overline{D(A)} = \overline{D(\varphi)}$ *and* $f \in L^2(0,T;H)$ *for some* $T > 0$*, problem* (P_T) *has a unique strong solution* u*, with* $t^{1/2}u' \in L^2(0,T;H)$*. If* $u_0 \in \overline{D(\varphi)}$ *and* $f \equiv 0$*, then problem* (P) *has a unique strong solution* u *which is differentiable from the right on* $(0, +\infty)$*,* $u(t) \in D(A)$ *for all* $t > 0$*, and*

$$|(d^+u/dt)(t)| = |A^0u(t)| \le |A^0v| + \frac{1}{t}|u_0 - v|, \forall t > 0,\ v \in D(A), \tag{1}$$

where A^0 *is the minimal section of* A *(i.e.,* A^0v *is the element of minimum norm of the closed convex set* Av*, for all* $v \in D(A)$*).*

Remark 2. It follows from Theorem 6 above that in the case $A = \partial\varphi$, with φ satisfying (H_φ), for every $u_0 \in \overline{D(A)} = \overline{D(\varphi)}$ and $f \in L^2_{loc}(0,\infty;H)$, problem (P) has a unique strong solution. If, in addition, $f \in W^{1,1}_{loc}(0,\infty;H)$ (including the case $f \equiv 0$!), then $u(t) \in D(A)$ for all $t > 0$ and u is differentiable from the right on $(0, +\infty)$.

2 Asymptotic behavior of solutions to problem (P)

Let us start with a simple example. Let $H = \mathbb{R}^2$ equipped with the usual scalar product and Euclidean norm. Define $A : D(A) = H \to H$ by

$$A(x_1, x_2) = (-x_2, x_1),\ \forall (x_1, x_2) \in D(A) = \mathbb{R}^2.$$

It is easy to check that A is monotone, and even maximal monotone, since $R(I + A) = \mathbb{R}^2$, i.e., Minty's surjectivity condition is fulfilled. Consider the homogeneous differential equation

$$\frac{d}{dt}(u_1, u_2) + A(u_1, u_2) = (0, 0),\ t \geq 0. \tag{2}$$

Notice that the components $u_1(t)$, $u_2(t)$ of the solutions are linear combinations of $\sin t$ and $\cos t$, so the solutions do not converge as $t \to \infty$. However, for every solution $u(t) = (u_1(t), u_2(t))$ of equation (2), we have convergence of the average as $t \to \infty$:

$$\lim_{t\to\infty} \frac{1}{t} \int_0^t u(s)\, ds = (0, 0).$$

In fact, we have the following general result:

Theorem 7. (G. Moroşanu [5, Theorem 1.2, p.73]) *Let $A : D(A) \subset H \to H$ be a maximal monotone operator, with $F := A^{-1}0 \neq \emptyset$. Assume in addition that $f \in L^1(0, \infty; H)$. Then, for every weak solution $u = u(t)$, $t \geq 0$, of problem (P), $(\exists)$ $p \in H$ such that the weak limit*

$$w - \lim_{t\to\infty} \frac{1}{t} \int_0^t u(s)\, ds = p, \tag{3}$$

and

$$\lim_{t\to\infty} |Proj_F u(t) - p| = 0 \tag{4}$$

(hence p belongs to the closed convex set F).

Remark 3. The proof of this theorem is based on the technique used by J.-B. Baillon and H. Brezis [1] for the case $f \equiv 0$.

We continue with a comparison criterion:

Theorem 8. (G. Moroşanu [5, Theorem 1.3, p. 77]) *Assume that $A : D(A) \subset H \to H$ is a maximal monotone operator, with $F := A^{-1}0 \neq \emptyset$, and $f \in L^1(0, \infty; H)$. Denote by (P_0) the problem (P) with $f \equiv 0$.*

If for every $u_0 \in \overline{D(A)}$ the weak solution $u = u(t; u_0)$ of problem (P_0) converges strongly (weakly) as $t \to \infty$, then every weak solution of problem (P) converges strongly (respectively, weakly) as $t \to \infty$.

Remark 4. According to Theorem 7 above, the limits we are talking about in Theorem 8 belong to the set $F := A^{-1}0$.

In the case $A = \partial\varphi$, H. Brezis stated the following important result:

Theorem 9. (H. Brezis [3, Theorem 3.11, p. 90]) *Let $\varphi : H \to (-\infty, +\infty]$ be a function satisfying (H_φ). Let $f_\infty \in R(\partial\varphi)$, and let $f = f(t)$ be a function such that $f - f_\infty \in L^1(0, +\infty; H)$. Assume in addition that*

$$\forall C \in \mathbb{R},\ \textit{the set } \{v \in H;\ \varphi(v) + |v|^2 \leq C\} \textit{ is (strongly) compact.} \tag{5}$$

Then, for every weak solution u of the equation $u' + \partial\varphi(u) \ni f(t)$, $t > 0$, there exists $\lim_{t\to+\infty} u(t) = u_\infty$ (strongly) with $u_\infty \in (\partial\varphi)^{-1}(f_\infty)$.

Remark 5. Theorem 9 was later published by V. Barbu [2, Theorem 2.4, p. 195], who did not mention the source used and imposed additional conditions on the function f as follows:

- $f : [0, \infty) \to H$ is absolutely continuous on every compact of $[0, \infty)$ such that $f' \in L^1(0, \infty; H)$;
- $\lim_{t\to\infty} f(t) = f_\infty \in R(\partial\varphi)$;
- $f - f_\infty \in L^1(0, \infty; H)$.

Notice that $\lim_{t\to\infty} f(t)$ does exist, since $f' \in L^1(0, \infty; H)$, and the limit is precisely f_∞, since $f - f_\infty$ is assumed to belong to $L^1(0, \infty; H)$. So, Barbu's conditions above can simply be expressed as follows:

$$f - f_\infty \in W^{1,1}(0, \infty; H), \text{ with } f_\infty \in R(\partial\varphi).$$

Anyway, Brezis' conditions are enough to guaranty the conclusion of Theorem 9, which is valid for all weak solutions, not just for some strong solutions as in Barbu's version.

Remark 6. One can replace f_∞ in Theorem 9 with 0. Indeed, it is enough to replace function φ with ψ, defined by $\psi(v) := \varphi(v) - (f_\infty, v),\ v \in H$. Notice that condition (5) is satisfied by the new function ψ, since any proper, convex and LSC function is bounded from below by an affine function (see, e.g., G. Moroşanu [5, Theorem 1.8, p. 33]).

Remark 7. Notice that in Theorem 9 the compactness condition (5) is needed to derive the strong convergence of solutions as $t \to \infty$ (see Brézis' proof in [3, Theorem 3.11, p. 90]). In the absence of such a condition, we need to consider the convergence of solutions in the weak topology of H. This idea was considered by R.E. Bruck [4] for the class of the so-called *demipositive operators*, which includes the set of subdifferential operators. More precisely, we have the following important result:

Theorem 10. (R.E. Bruck [4]) *Let $A = \partial\varphi$, with $F := A^{-1}0 \neq \emptyset$, where $\varphi : H \to (-\infty, +\infty]$ is a proper, convex and lower semicontinuous function. Then, for every $u_0 \in \overline{D(A)} = \overline{D(\varphi)}$, the solution of problem (P_0) (i.e., problem (P) with $f \equiv 0$) converges weakly as $t \to \infty$ to a point $p \in F$ (i.e., p is a minimum point of φ).*

Now, let us state a new result, which is an extension of Brezis' theorem (Theorem 9 above, reformulated with $f_\infty = 0$).

Theorem 11. *Let $A = \partial\varphi$, with $F := A^{-1}0 \neq \emptyset$, where $\varphi : H \to (-\infty, +\infty]$ is a proper, convex and lower semicontinuous function, and let $f \in L^1(0, \infty; H)$. Then, for every $u_0 \in \overline{D(A)} = \overline{D(\varphi)}$, the (weak) solution of problem (P) converges weakly as $t \to \infty$ to a point $p \in F$ (i.e., p is a minimum point of φ).*
If in addition condition (5) is fulfilled, then $u(t)$ converges strongly to p.

Proof. For the first part (concerning the weak convergence of solutions) we simply use Theorem 7, Remark 4, and Theorem 10 above. For the second part of the theorem, it is enough to consider the case $f \equiv 0$ (by virtue of Theorem 8). So, let $u = u(t)$ be the solution of problem (P_0) for an arbitrary $u_0 \in \overline{D(A)} = \overline{D(\varphi)}$. We know from the first part that $u(t)$ converges weakly to a minimum point p of φ as $t \to \infty$, so the set $\{u(t);\, t \geq 0\}$ is weakly bounded. According to the uniform boundedness principle, we obtain

$$\sup_{t \geq 0} |u(t| < \infty. \tag{6}$$

On the other hand, since $-(d^+u/dt)(t) \in \partial\varphi(u(t))$, $t > 0$, we have

$$0 \leq \varphi(u(t)) - \varphi(p) \leq \big(-\frac{d^+u}{dt}(t), u(t) - p\big),\ \forall t > 0,$$

which combined with inequality (1) with $v = p$, and (6), shows that $\varphi(u(t)) + |u(t)|^2$ is bounded. Thus, condition (5) and the weak convergence of $u(t)$ to

p as $t \to \infty$ lead us to the conclusion that $u(t)$ converges strongly to p as $t \to \infty$. $\square$

Remark 8. If φ is as in the statement of Theorem 11, then condition (5) is equivalent to the following one

$$\forall c > 0, \text{ the set } \{v \in H;\, |v| \le c,\, \varphi(v) \le c\} \text{ is (strongly) compact.}$$

To prove this equivalence one can use the fact that φ is bounded from below by an affine function.

In what follows we shall talk about another condition on function φ ensuring strong convergence of solutions to problem (P), as $t \to \infty$. More precisely, we have :

Theorem 12. *Let $A = \partial\varphi$, where $\varphi : H \to (-\infty, +\infty]$ is a proper, convex, lower semicontinuous, and even function (i.e., $\varphi(v) = \varphi(-v)$, for all $v \in H$), and let $f \in L^1(0, \infty; H)$. Then, the set F of minimum points of φ is nonempty (containing at least 0), and every weak solution of problem (P) converges strongly, as $t \to \infty$, to a minimum point of φ.*

The proof is provided in G. Moroşanu [5, Remark 2.1, p. 105]. It is based on contributions due to J.-B. Baillon, R.E. Bruck, and the author (see G. Moroşanu [5, Theorem 1.3, p. 77; Proposition 1.2, p. 86; Theorem 2.1, p. 99].

Remark 9. Theorem 12 remains valid if the condition that φ is an even function is replaced by the following more general assumption proposed by H. Okochi [6]:

$$D(\varphi) = -D(\varphi) \text{ and } \varphi(v) - \varphi(0) \ge \alpha\big(\varphi(-v) - \varphi(0)\big),\ \forall v \in D(\varphi)\,, \tag{7}$$

where α is a positive number. The proof in the case $f \equiv 0$ can be found in G. Moroşanu [5, Theorem 2.4, p. 113], and the conclusion for the case $f \in L^1(0, \infty; H)$ follows by Theorem 8.

Notice that if (7) holds with $\alpha = 1$, then obviously φ is an even function.

Now, if (7) is satisfied with $\alpha > 1$, then φ is constant on $D(\varphi)$. To show this, we first derive from (7)

$$\varphi(v) - \varphi(0) \ge \alpha^2\big(\varphi(v) - \varphi(0)\big),\ \forall v \in D(\varphi)\,,$$

which implies

$$\varphi(v) \le \varphi(0),\ \forall v \in D(\varphi).$$

Assuming by contradiction that $(\exists)\, w \in D(\varphi)$, such that $\varphi(w) < \varphi(0)$, we would obtain

$$\varphi(0) = \varphi\Big(\frac{1}{2}(w + (-w))\Big) \le \frac{1}{2}\big(\varphi(w) + \varphi(-w)\big) < \varphi(0),$$

which is impossible. Thus, $\varphi(v) = \varphi(0),\ \forall v \in D(\varphi)$, as claimed.

Thus, the only possibility to have a convex function φ which satisfies (7) but is not even is to choose $\alpha \in (0,1)$.

For example, the function $\varphi : \mathbb{R} \to \mathbb{R}$, defined by

$$\varphi(x) = \begin{cases} x^2, & \text{if } x \ge 0\,, \\ \frac{2}{3}x^2, & \text{if } x < 0\,, \end{cases}$$

is convex and satisfies (7) for every $\alpha \in (0, 2/3]$, but is not even.

An interesting example that comes as a follow-up is the *canonical extension* of the function φ above to the space $H = L^2(\Omega) := L^2(\Omega; \mathbb{R})$, where Ω is a Lebesgue measurable subset of $\mathbb{R}^N$ for some $N \ge 1$, with $0 <$ meas $(\Omega) < \infty$, this space being equipped with the usual scalar product

$$(v_1, v_2) = \int_\Omega v_1(s) v_2(s)\, ds, \quad v_1, v_2 \in H = L^2(\Omega),$$

and the induced Hilbertian norm.

The related canonical extension of φ, denoted Φ, is defined by

$$\Phi(v) = \int_\Omega \varphi(v(s))\, ds, \quad \text{for all } v \in H = L^2(\Omega).$$

Obviously, this function Φ is everywhere defined on $H = L^2(\Omega)$, convex and continuous. Moreover, Φ satisfies condition (7) for every $\alpha \in (0, 2/3]$ and $\Phi(0) = 0$, since φ satisfies these conditions. On the other hand, using constant functions v we easily find that Φ is not even.

Summarizing, Φ is everywhere defined on $H = L^2(\Omega)$, convex, continuous, and satisfies condition (7), but is not even. So Φ is a good candidate for applying Okochi's result.

In fact, we have the following general result:

Theorem 13. *Let $(H, (\cdot,\cdot), |\cdot|)$ be a real Hilbert space as above, let Ω be a Lebesgue measurable subset of $\mathbb{R}^N$ for some $N \ge 1$, with $0 <$ meas $(\Omega) < \infty$,* *and consider the real Hilbert space* $\mathbb{H} = L^2(\Omega; H)$ *equipped with the scalar product*

$$< v_1, v_2 >= \int_\Omega (v_1(s), v_2(s))\, ds, \quad v_1, v_2 \in \mathbb{H},$$

and the induced Hilbertian norm. Let $\varphi : H \to (-\infty, +\infty]$ *be a proper, convex, lower semicontinuous function, which satisfies condition* (7) *for some* $\alpha \in (0, 1)$*, and is not even. Define* $\Phi : \mathbb{H} \to (-\infty, +\infty]$ *by*

$$\Phi(v) = \begin{cases} \int_\Omega \varphi(v(s))\, ds, \ \ if \quad v \in \mathbb{H} \ \ and \ \varphi \circ v \in L^1(\Omega), \\ \\ +\infty, \ \ otherwise \end{cases}$$

(note that Φ *cannot assume the value* $-\infty$ *since* φ *is bounded from below by an affine function). Then* Φ *is a proper, convex, lower semicontinuous function satisfying condition* (7) *with the same* α*, and is not even.*

Proof. Φ is proper (since $D(\Phi)$ contains constant functions $v \equiv h$ with $h \in D(\varphi)$), convex, lower semicontinuous (see, e.g., G. Moroşanu [5, Proposition 1.5, p. 44]), satisfies condition (7) with the same α, and is not even (that follows by using constant functions). □

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