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# ON THE HEAT CONDUCTION EQUATION AND THE HEAT DISSIPATION FUNCTION IN ANISOTROPIC REACTING FLUID MIXTURES WITH MAGNETIC RELAXATION*

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**Abstract**

In some previous papers, within the framework of the thermodynamics of irreversible processes with internal variables, a linear theory for magnetic relaxation phenomena in anisotropic mixtures, consisting of $n$ reacting fluid components, was developed. In particular, assuming that the macroscopic magnetization $\mathbf{m}$ can be split in two irreversible parts $\mathbf{m} = \mathbf{m}^{(0)} + \mathbf{m}^{(1)}$ a generalized Snoek equation was derived. In this paper we derive for these reacting anisotropic mixtures the heat conduction equation. We show that the heat dissipation function is due to the chemical reactions, the magnetic relaxation, the electric conduction, the viscous, magnetic, temperature fields and the diffusion and the concentrations of the $n$ fluid components. Also, the Snoek and De Groot special cases are studied. The obtained results find applications in nuclear resonance, in biology, in medicine and other fields, where different species of molecules have different magnetic susceptibilities and relaxation times and contribute to the total magnetization.

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## 1 Introduction

In [1]-[7] a linear theory for magnetic relaxation phenomena in magnetizable continuous media was developed, based on the thermodynamics of irreversible processes with internal variables [8]-[13]. In particular, in [1], in linear approximation Kluitenberg, assuming that the total specific magnetization is given by the sum of one reversible part and one irreversible part, derived for magnetizable isotropic media the *classical Snoek equation* describing magnetic relaxation phenomena [14]. Subsequently, in [2], assuming that the total specific magnetization $\mathbf{m}$ is composed of two irreversible parts, i.e.,

$$\mathbf{m} = \mathbf{m}^{(0)} + \mathbf{m}^{(1)}, \tag{1}$$

Kluitenberg obtained a more general magnetic relaxation equation. In [4], in the assumption that an arbitrary number $n$ of microscopic phenomena give rise to the total specific magnetization $\mathbf{m}$, that can be split in $n+1$ irreversible parts, i.e.

$$\mathbf{m} = \mathbf{m}^{(0)} + \sum_{k=1}^{n} \mathbf{m}^{(k)}, \tag{2}$$

in the isotropic case a generalized Snoek equation was obtained by Kluitenberg and one of the authors, having the form of a linear relation among the magnetic field $\mathbf{B}$, the first $n$ time derivatives of this field, the total magnetization $\mathbf{M} = \varrho\mathbf{m}$, with $\varrho$ the mass density, and the first $n+1$ time derivatives of $\mathbf{M}$, being $n$ the number of phenomena giving rise to the magnetization. In [5] and [6]) reviews about the results obtained in [4] were done.

In [7] the behaviour of anisotropic reacting fluid mixtures with magnetic relaxation was investigated. The irreversible microscopic phenomena giving rise to magnetic relaxation are described, assuming that the total specific magnetization $\mathbf{m}$ given by two irreversible parts $\mathbf{m}^{(0)}$ and $\mathbf{m}^{(1)}$ as in (1), and in the linear case the magnetic relaxation equation was derived.

In [15], [16] analogous studies for dielectric relaxation phenomena in polarizable media with internal variables were performed by using the same methods of the classical thermodynamics of irreversible processes with internal variables.