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EXISTENCE, STABILITY AND NUMERICAL ANALYSIS OF A FRACTIONAL NEUTRAL IMPLICIT DELAY DIFFERENTIAL SYSTEM WITH AN EXPONENTIAL KERNEL*

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Abstract

This article addresses the existence, uniqueness and stability analysis for various classes of implicit fractional neutral delay (finite and infinite) differential systems (IFNDDSs) employing the Caputo-Fabrizio operator (CFO). The findings rely on the application of specific fixed-point theorems. Additionally, illustrative numerical examples are presented in the concluding section to clarify and demonstrate the derived results.

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1 Introduction

Fractional calculus, an advanced mathematical framework, introduces the concept of derivatives and integrals with non-integer orders, providing a nuanced perspective on the behaviour of dynamic systems. This unconventional approach has garnered attention for its capacity to model complex phenomena in diverse scientific and engineering domains [32, 42, 44]. Unlike classical calculus, fractional calculus offers a more inclusive representation of dynamic behaviours by accounting for non-local and hereditary effects. Its applications extend to physics, biology, control theory, and signal processing, where it excels in capturing intricate features like memory and long-term dependencies [13, 39]. The smooth transition it facilitates between derivatives and integrals contributes to its versatility in solving differential equations, making fractional calculus a pivotal tool in understanding and addressing real-world intricacies. Recently, researchers have extensively explored the application of fixed point theory to establish results related to the existence, uniqueness, and controllability of solutions for various fractional initial value problems [7, 19, 21–23, 31, 37].

Implicit fractional differential equations (IFDEs) constitute a specialized class of mathematical expressions where the fractional derivatives are applied implicitly. In these equations, the unknown function and its derivatives are intertwined in a non-explicit manner, often involving fractional orders. This distinctive formulation adds complexity but captures intricate relationships in various systems [1, 5, 18]. The advantages of IFDEs include their enhanced capacity to model phenomena with non-local dependencies and complex dynamics. The implicit approach allows for a more accurate representation of systems with memory effects and long-term dependencies. These equations provide a versatile framework for describing processes exhibiting anomalous diffusion, viscoelastic behaviours, and other intricate dynamics that traditional differential equations may struggle to capture. IFDEs have proven effective in modelling real-world scenarios across disciplines such as physics, engineering, and biology, making them valuable in understanding and analyzing complex systems [25, 30, 34, 41].

A neutral differential system is a differential system that incorporates