

THE BEREZIN TRANSFORMATION ON $L^2(\mathbb{U}_+)^*$

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Dedicated to Dr. Dan Tiba on the occasion of his 70th anniversary

Abstract

Let $L_a^2(\mathbb{U}_+)$ be the Bergman space of the upper half plane \mathbb{U}_+ . In this paper, we consider the integral operator H from $L^2(\mathbb{U}_+)$ into $L^2(\mathbb{U}_+)$ defined by $(Hf)(w) = \tilde{f}(w) = \int_{\mathbb{U}_+} f(s) |d_{\bar{w}}(s)|^2 d\tilde{A}(s)$, $w \in \mathbb{U}_+$,

where $d_{\bar{w}}(s) = \frac{1}{\sqrt{\pi}} \frac{w+i}{\bar{w}-i} \frac{(-2i)\text{Im } w}{(s+w)^2}$ and $d\tilde{A}$ is the area measure on \mathbb{U}_+ . We refer the map H as the Berezin transformation defined on $L^2(\mathbb{U}_+)$. We have derived various algebraic properties of the operator and showed that $\|H\| \leq \frac{3\pi}{4}$ considered as an operator on $L_a^2(\mathbb{U}_+)$.

Keywords: Bergman space, upper half plane, integral operators, Berezin transformation, reproducing kernel.

MSC: 47B38, 30H20, 45P05.

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1 Introduction

The Berezin transform was first introduced by F.A. Berezin [1] as a tool in quantization [2]. It has since found applications in many areas of mathematics and mathematical physics [3]. The Berezin transform was studied

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