Ann. Acad. Rom. Sci.
Ser. Math. Appl.
ISSN 2066-6594 Vol. 16, No. 1/2024

DECAY RATES AND INITIAL VALUES FOR TIME-FRACTIONAL DIFFUSION-WAVE EQUATIONS*

Masahiro Yamamoto [1,2,3] †

Dedicated to Dr. Dan Tiba on the occasion of his 70th anniversary

Abstract

We consider a solution $u(\cdot,t)$ to an initial boundary value problem for time-fractional diffusion-wave equation with the order $\alpha \in (0,2) \setminus \{1\}$ where t is a time variable. We first prove that a suitable norm of $u(\cdot,t)$ is bounded by the rate $t^{-\alpha}$ for $0 < \alpha < 1$ and $t^{1-\alpha}$ for $1 < \alpha < 2$ for all large $t > 0$. Second, we characterize initial values in the cases where the decay rates are faster than the above critical exponents. Differently from the classical diffusion equation $\alpha = 1$, the decay rate can keep some local characterization of initial values. The proof is based on the eigenfunction expansions of solutions and the asymptotic expansions of the Mittag-Leffler functions for large time.

MSC: 35R11, 35B40, 35C20
DOI https://doi.org/10.56082/annalsarscimath.2024.1.77
keywords: fractional diffusion-wave equation, decay rate, initial value

*Accepted for publication on November 24-th, 2023

†myama@ms.u-tokyo.ac.jp [1] Graduate School of Mathematical Sciences, The University of Tokyo, Komaba, Meguro, Tokyo 153-8914, Japan [2] Zonguldak Bülent Ecevit University, Turkey [3] Honorary Member of Academy of Romanian Scientists, Ilfov, nr. 3, Bucuresti, Romania [4] Correspondence Member of Accademia Peloritana dei Pericolanti, Palazzo Università, Piazza S. Pugliatti 1 98122 Messina, Italy. Paper written with financial supports of Grant-in-Aid for Scientific Research (A) 20H00117 and Grant-in-Aid for Challenging Research (Pioneering) 21K18142 of Japan Society for the Promotion of Science.