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COUNTING PATHS OF GRAPHS VIA INCIDENCE MATRICES*

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Abstract

Operating only by means of the incidence matrix of a connected graph G, a new algebraic combinatorial method for determining the paths of length $(q-1)$ of G together with the generators of the corresponding generalized graph ideal $I_q(G)$ is discussed and developed. The stated formulae are obtained and shown even by changing techniques appropriately when the difficulties of calculation increased.

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Introduction

Algebraically speaking, determining some paths of length $(q-1)$, q positive integer, of a connected undirected graph G, means to find generators of a monomial ideal to which G can be associated, the generalized graph ideal $I_q(G)$ (see [3, 4, 5]).
The problem of computing, using only the incidence matrix of G, the number and structure of paths of fixed length in G, and the generators of the relative generalized graph ideal, presents aspects useful in various scientific and statistical research areas.

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