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THE BEST SPECTRAL CORRECTION OF DMDY CONJUGATE GRADIENT METHOD*

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Abstract

In this paper, we present an enhanced spectral correction for the DMDY conjugate gradient method. Our approach involves integrating a third term and determining its parameter through three different approaches. The primary objective is to ensure the sufficient descent condition. By applying the Wolfe line search conditions, we establish the global convergence property for all three proposed algorithms. Numerical tests conclusively demonstrate the superior efficiency of our algorithms, surpassing that of existing methods.

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1 Introduction

The purpose of utilizing Nonlinear Conjugate Gradient (NCG) methods, extensively studied in [1, 2], is to minimize unconstrained optimization problems formulated in the following manner:

$$\min_{x \in \mathbb{R}^n} f(x), \tag{1}$$

where, $n \in \mathbb{N}^*$ is supposed to be very large and $f : \mathbb{R}^n \to \mathbb{R}$ is continuously differentiable function.
To solve the problem (1) starting form an initial point $x_0 \in \mathbb{R}^n$, the NCG method generates a sequence of points $\{x_k\}_{k \in \mathbb{N}}$ defined by

$$x_{k+1} = x_k + \alpha_k d_k, \tag{2}$$

where, the stepsizes $\alpha_k \in \mathbb{R}_+^*$ are determined by some line search and are very important for global convergence of conjugate gradient methods. In our work, we use line search to satisfying the Wolfe conditions [3, 4]

$$f(x_k + \alpha_k d_k) - f(x_k) \leq \rho \alpha_k g_k^t d_k, \tag{3}$$

$$g_k^t d_{k-1} \geq \sigma g_{k-1}^t d_{k-1}, \tag{4}$$

where, $0 < \rho < \sigma < 1$, and $\delta < \sigma < 1$. $d_k \in \mathbb{R}^n$ are search directions given by

$$\begin{cases} d_0 = -g_0, \\ d_k = -g_k + \beta_k d_{k-1}, \quad k \geq 1, \end{cases} \tag{5}$$

where, $g_k = g(x_k) = \nabla f(x_k)$ is the gradient of the function f in the point x_k and $\beta_k \in \mathbb{R}^*$ is a scalar called the conjugate gradient parameter. In the following table, we recall some famous formulas of this parameter