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ON THE HEAT DISSIPATION FUNCTION FOR MAGNETIC RELAXATION PHENOMENA IN ANISOTROPIC MEDIA*

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Dedicated to Dr. Dan Tiba on the occasion of his 70^{th} anniversary

Abstract

Using the methods of classical irreversible thermodynamics with internal variables, the heat dissipation function for magnetizable anisotropic media, in which phenomena of magnetic relaxation occur, is derived. It is assumed that if different types of irreversible microscopic phenomena give rise to magnetic relaxation, it is possible to describe these microscopic phenomena splitting the total specific magnetization in two irreversible parts and introducing one of these partial specific magnetizations as internal variable in the thermodynamic state space. It is seen that, when the theory is linearized, the heat dissipation function is due to the electric conduction, magnetic relaxation, viscous, magnetic irreversible phenomena. This is the case of complex media, where different kinds of molecules have different magnetic susceptibilities and relaxation times, present magnetic relaxation phenomena and contribute to the total magnetization. These situations arise in nuclear magnetic resonance in medicine and biology and in other fields of the applied sciences. Also, the heat conduction equation for these media is worked out and the special cases of anisotropic Snoek media and

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anisotropic De-Groot-Mazur media are treated.

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1 Introduction

In some previous papers [1]-[8] a linear theory for magnetic relaxation phenomena in magnetizable continuous media was developed, that is based on thermodynamics of irreversible processes [9]-[15] with internal variables (see [16]). In [5]-[7] it was shown that, if an arbitrary number n of microscopic phenomena give rise to magnetic relaxation, it is possible to describe these microscopic phenomena introducing n macroscopic axial vectorial internal variables in the thermodynamic state vector and assuming that the specific magnetization axial vector $\mathbf{m}$ can be split in n+1 irreversible contributions, i.e.,

$$\mathbf{m} = \mathbf{m}^{(0)} + \sum_{k=1}^{n} \mathbf{m}^{(k)}, \tag{1}$$

where $\mathbf{m}^{(0)}$ and $\mathbf{m}^{(k)}$ $(k = 1, ..., n)$ are called partial specific magnetizations.

In the isotropic case (see [5], [6]) the following magnetic relaxation equation generalizing Snoek equation was obtained having the form of a linear relation among the magnetic field $\mathbf{B}$, the first n time derivatives of this field, the total magnetization $\mathbf{M}$ and the first n + 1 time derivatives of $\mathbf{M}$

$$\chi^{(0)}_{(BM)}\mathbf{B} + \chi^{(1)}_{(BM)}\frac{d\mathbf{B}}{dt} + ... + \chi^{(n-1)}_{(BM)}\frac{d^{n-1}\mathbf{B}}{dt^{n-1}} + \frac{d^{n}\mathbf{B}}{dt^{n}} =$$

$$\chi^{(0)}_{(MB)}\mathbf{M} + \chi^{(1)}_{(MB)}\frac{d\mathbf{M}}{dt} + ... + \chi^{(n)}_{(MB)}\frac{d^{n}\mathbf{M}}{dt^{n}} + \chi^{(n+1)}_{(MB)}\frac{d^{n+1}\mathbf{M}}{dt^{n+1}}, \tag{2}$$

where n is the number of phenomena that give rise to the total magnetization $\mathbf{M}$, with $\mathbf{M} = \rho\mathbf{m}$, being ρ the mass density of the medium, supposed a constant quantity, and $\chi^{(k)}_{(BM)}(k = 0, 1, ..., n-1)$ and $\chi^{(k)}_{(MB)}$ $(k = 0, 1, ..., n+1)$ are constant quantities. In particular, they are algebraic functions of the coefficients occurring in the phenomenological equations and in the equations of state.