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FROM PROPAGATION SYSTEMS TO TIME DELAYS AND BACK. CRITICAL CASES*

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Dedicated to Dr. Dan Tiba on the occasion of his 70^{th} anniversary

Abstract

The paper originates from the early ideas of A. D. Myshkis and his co-workers and of K. L. Cooke and his co-worker. These ideas send to a one-to-one correspondence between lossless and/or distortionless propagation described by nonstandard boundary value problems and a system of coupled differential and difference equations with deviated argument. In this way any property obtained for one mathematical object is automatically projected back on the other one. This approach is considered here for certain engineering applications. The common feature of these applications is the critical stability of the difference operator associated with the system with deviated argument obtained for each of the aforementioned applications. In fact the associated systems are of neutral type and, according to the assumption of Hale, only strong stability of the difference operator ensures robust asymptotic stability with respect to the delays. If the difference operator is in the critical case, the stability becomes fragile with respect to the delays. Based on some old results in the field, a conjecture concerning the (quasi)-critical modes of the system is stated; also a connection with the so called *dissipative boundary conditions* is suggested.

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1 Introduction. The methodology

Those who know the early history of the differential equations with deviated argument in the XXth century know also the very first book on such equations - the book of of A. D. Myshkis [1], more precisely its first edition of 1951, because a more recent revised edition was published in 1972. This book contains no reference to applications of such equations but another book published after 4 years does [2]. It is however not our intention to draw a history of these equations over the last 7 decades, but to point out a certain line of research and studies. It is firstly interesting to point out a series of three papers [3, 4, 5] where dynamics of controlled structures of thermal power engineering incorporated rather long steam or oil pipes inducing propagation effects. Linearization and use of the Laplace transform suggested characteristic quasi-polynomials which were specific to equations with deviated arguments with pointwise delays *of neutral type*.

Much later, in the 60ies of XXth century, a certain research made at IBM under the guidance of R. K. Brayton displayed similar aspects but for electrical circuits containing lossless transmission lines [6, 7, 8, 9]. Later it was pointed out [10] that pointwise delays are modeling what is called *lossless and/or distortionless propagation*.

The mathematical theory connected to those aspects grew in parallel. Firstly, the papers of Myshkis and his co-workers were published [11, 12]. In these papers there were considered hyperbolic partial differential equations in the plane - having as independent variables time and one space variables - in fact $1D$ systems. Their boundary conditions were non-standard since they contained Volterra operators. Somehow later the papers of K. L. Cooke were published: the first one, co-authored by D. W. Krumme, had clear reference to electrical circuits containing lossless transmission lines [13]. The second one [14] - less circulated (it looked more as a seminar exposition, with incomplete proofs) has little reference to applications. Worth mentioning that the papers of Cooke contain boundary problems of non-standard type but whose boundary conditions contain only differential equations, thus being less general than those considered by Myshkis and his co-workers.

Now, regardless the generality of the non-standard boundary conditions, *the methodology* of the two groups of papers is the same. Namely, making use of the fact that the Riemann invariants of the problems are constant