

FREDHOLM TOEPLITZ OPERATORS ON THE WEIGHTED BERGMAN SPACES*

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Abstract

In this paper we have shown that if $\phi \in (L_h^2(dA_\alpha))^\perp \cap L^\infty(\mathbb{D})$ and $\text{Range}T_\phi^{(\alpha)}$ is closed, then the Toeplitz operator $T_\phi^{(\alpha)} \in \mathcal{L}(L_a^2(dA_\alpha))$ is a Fredholm operator of index zero and $T_\phi^{(\alpha)}$ is not of finite rank. Several applications of the result were also obtained. We further show that if $\phi \in L_{M_n}^\infty(\mathbb{D})$ is such that T_ϕ is Fredholm and of index zero in $\mathcal{L}(L_a^{2, \mathbb{C}^n}(dA_\alpha))$ then there exists $\psi \in E_{n \times n} = E \otimes M_n$ such that $T_{\phi + \delta\psi}$ is invertible for all sufficiently small nonzero δ . Here E is a total subspace of $L^\infty(\mathbb{D})$ and M_n is the set of all $n \times n$ matrices with complex entries.

MSC: 47B38, 47B32

keywords: Weighted Bergman spaces, Finite rank operator, Toeplitz operator, Little Hankel operator, Bounded harmonic functions.

1 Introduction

Let $dA(z) = \frac{1}{\pi} dx dy = \frac{1}{\pi} r dr d\theta$ be the normalized area measure on the open unit disk $\mathbb{D} = \{z \in \mathbb{C} : |z| < 1\}$ in the complex plane \mathbb{C} . For $\alpha > -1$, let

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