Ann. Acad. Rom. Sci.
Ser. Math. Appl.
ISSN 2066-6594 Vol. 12, No. 1-2/2020

COMPARATIVE ANALYSIS BETWEEN TWO CASES OF RISK ASSESSMENT FOR E-BIKE CYCLING IN URBAN AREA*

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DOI https://doi.org/10.56082/annalsarscimath.2020.1-2.179

Dedicated to Dr. Vasile Drăgan on the occasion of his 70^{th} anniversary

Abstract

In this paper is presented comparative analysis between two cases of risk assessment for e-bike cycling in urban area. The input parameters taken into account in first case are the number of obstacles in each alternative branch of the route and in second case - the number of obstacles and road surface. Under obstacle in this study is considered any object that could either threaten the life of cyclist or present an obstacle in his way (potholes on the road or dangerous terrain, stray animals, hooligans, incompetent car drivers etc.). The researches is made as a stochastic problem with two stages. The first stage consists of a calculation of the average risk in each branch of the route. The input parameters are stochastic and discretized, preprocessed by statistic methodology. In the second stage the optimal route is found by application of the Bellman's optimally principle.
MSC: 91B05, 91G70.

*Accepted for publication in revised form on April 16, 2020
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keywords: risk assessment, e-bike cycling, route optimization, statistically approaches, Bellman's Principle of optimally

1 Introduction

In recent years, there has been considerable scientific interest in the assessment of risk and route optimization of various types of vehicles. The technological developments from the last years have given a new look to classical paradigms, such as the route optimization of personal vehicles. There is growing trend of electric and pedal-assisted bicycles usage as an environment-friendly and healthy alternative for urban mobility. These personal vehicles are becoming an interesting subject for scientific research in various fields of transportation, logistics and traffic safety. Therefore, by application of various mathematical methods and tools the risk assessment and route optimization for this newly emerged category of personal vehicles and their interaction with the urban environment and all the other actors in city traffic should be considered. Electric-powered cars are typically much more energy-efficient than fossil-fueled fuels. The increasing use of electric vehicles, and in particular those powered by renewable energy sources, can play an important role in achieving the EU's goal of reducing greenhouse gas emissions and moving towards a low carbon future.

Research into one direction for assessing the risk to the life and health of the cyclist in terms of pollution and exhaust emissions is done in [11]. There, the authors have made a study on the influence of exhaust gases such as CO, NOx and fine particulate matter of conventional vehicles (i.e. using internal combustion engines) resulting in serious health concerns for the general public. Another example is a research [4], showing that the mortality rate for people living in the most polluted cities can be 29% higher than those living in the least polluted cities based on data in the past several decades.

Research into another direction for assessing the risk to the life and health of a cyclist with regard to its technical characteristics of e-bike and environmental objects have been done in [8]. There are shown several types of stability are introduced and analyzed (average, mean-squared, almost-sure stability etc.) with the aim of widening the optimization scope. After numerous iterations of the optimization, the stability conditions are expressed with several levels of conservatism and feasibility. The authors in [6] have described the development of a useful tool for agencies and researchers for clustering of similar transportation patterns with respect to time-based events. The proposed supervision algorithm is conceived to take advantage of background knowledge of the dataset along with the similarity. Compared to analogous methods, this one stands out with scalarization and low computational complexity along with its other advantages. An intelligent control of the traffic lights is studied in [12]. A feed-forward neural network is adopted to accomplish the traffic signal controller. The proposed solution has several advantages compared to traditional methods and in particular the self-learning ability. The research in [10] is based on the Nash equilibrium for an infinite time horizon. Furthermore, various strategies are defined for the individual players in the competition. Their performance is then compared under equal conditions, with or without a feedback in the informational structure.

In this paper is presented comparative analysis between two cases of risk assessment for e-bike cycling in urban area. The input parameters taken into account in first case are the number of obstacles in each alternative branch of the route and in second case - the number of obstacles and road surface. Under obstacle in this study is considered any object that could either threaten the life of cyclist or present an obstacle in his way (potholes on the road or dangerous terrain, stray animals, hooligans, incompetent car drivers etc.). The researches is made as a stochastic problem with two stages. The first stage consists of a calculation of the average risk in each branch of the route. The input parameters are stochastic and discretized, preprocessed by statistic methodology. In the second stage the optimal route is found by application of the Bellman's optimally principle. The similar problems are analyzed and solved in [7] by using a multi-objective optimization approach. Various approaches to finding optimal routes by different criteria are described in [1], [2], [3], [5], [9], etc.

In the article the research done is specific because it is another point of view of the the assigned problems. The problems arise from the increasing number of people in urban areas (urbanization) and the corresponding increase in human density. On the other hand, transport needs in the urban environment are specific (generally speaking). This means:

- A wide variety of vehicles (from e-bike to stand-alone trucks);
- Extensive amounts of data, such as vehicle technical information, travel profiles (route + driving style + vehicle);
- Share Common Resource - Road infrastructure is used simultaneously by objects that are different in size, weight and speed. Accordingly,

the interaction between road users is different from the point of view of the vehicle.

The significance of the research is that the benefit is maximum for cyclists who have to make a decision (related to the route) in an environment of uncertainty. Decision makers would like to assess the risks before they decide to understand the scope of the possible outcomes and the significance of the unwanted consequences.

2 Description and Aim of the two-stage problem

1st stage: Description of problem 1

cyclist is riding a pedal-assisted electric bicycle and is travelling from a certain point of departure to his destination. This can be performed via a number of routes including combination of their sections. The routes can be represented by a network model of an oriented graph $V(G, D)$, where $G = \{G_i\}_{i=1}^{k}$ are the nodes and $D = d_{ij},\ i = 1, \ldots, k-1; j = 2, \ldots, k; i < j$, are the graph arcs (figure 1).

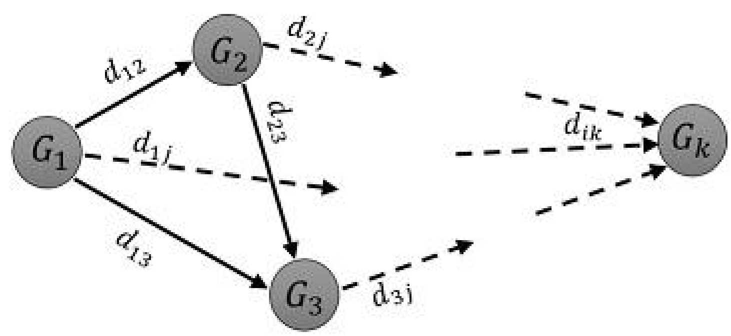

Figure 1: Network model of the oriented graph V(G,D)

The risk to the life and health of a cyclist during an electric bicycle management is directly related to:

1st case: the number of obstacles on the e-bike routes;

2nd case: the number of obstacles on the e-bike routes and the quality of the road surface.

Aim of problem 1

To determine the probability of occurrence of risk from the possible routes of the cyclist in depending on:

1st case: the number of obstacles;

2nd case: the number of obstacles and the quality of the road surface.

2nd stage: Description of problem 2

For each stretch of road known probability of lack of risk for:

1st case: the obstacles on the road;

2nd case: the obstacles on the road and poor quality of the road surface.

Aim of problem 2

The aim of the optimization problem is to determine a route from the departure to the final destination exposing the cyclist to a minimal the risk of

1st case: encountering obstacles;

2nd case: encountering obstacles and bad road surfaces.

3 Solution of the two-stage problem

Input data and processing:

1st case

X is a discrete random number characterizing the number of obstacles observed along each of the arcs $d_{ij},\ i = 1,\dots,k-1; j = 2,\dots,k; i < j$, for each hour of the day $l_t,\ t = 1,\dots,24$.

The data is processed statistically and for each of the sections $d_{ij},\ i = 1,\dots,k-1; j = 2,\dots,k; i < j$, and the frequency of occurrence is expressed for a number of equal intervals $[x_r;;x_{r+1}],\ r = 1,\dots,R$ (table 1).

Table 1: Frequency of occurrence of the random event X

	Interval $[x_r;x_{r+1}]$	**Average Value** $\overline{x_r}^*$	**Absolute Frequency of occurrence** L_r	**Relative frequency of occurrence** P_r^*	$\overline{x_r}^*.P_r^*$
1	$[x_1;x_2)$	$\overline{x_1}^*$	$L_\mathbf{1}$	$P_1^*=\frac{L_1}{L}$	$\overline{x_1}^*.P_1^*$
2	$[x_2;x_3)$	$\overline{x_2}^*$	L_2	$P_2^*=\frac{L_2}{L}$	$\overline{x_2}^*.P_\mathbf{2}^*$
...	...	...	...	...	...
R	$[x_R;x_{R+1}]$	$\overline{x_R}^*$	L_R	$P_R^*=\frac{L_R}{L}$	$\overline{x_R}^*.P_R^*$
			$\sum_{r=1}^{R} L_r=L$	$\sum_{r=1}^{R} P_r^*=1$	$\sum_{r=1}^{R} \overline{x_r}^*.P_r^*$

For each arcs $d_{ij},\ i = 1,\dots,k-1; j = 2,\dots,k; i < j$, the average number of obstacles is expressed:

$$Ex_{ij} = \sum_{r=1}^{R} \overline{x_r}^*.P_r{}^* = EX^*,\ i = 1,\ldots,k-1; j = 2,\ldots,k; i < j. \quad (1)$$

Each section $d_{ij},\ i = 1,\ldots,k-1; j = 2,\ldots,k; i < j$, with an average number of obstacles $Ex_{ij},\ i = 1,\ldots,k-1; j = 2,\ldots,k; i < j$, is associated to the probability of being risky $q_{ij} \in [0;1]\,,\ i = 1,\ldots,k-1; j = 2,\ldots,k; i < j$, where:

$$q_{ij} = \frac{Ex_{ij}}{\sum_{i=1}^{k-1}\sum_{j=2}^{k} Ex_{ij}},\ i = 1,\ldots,k-1; j = 2,\ldots,k; i < j. \quad (2)$$

The solution of the first stage consists in expressing the probability $d_{ij},\ i = 1,\ldots,k-1; j = 2,\ldots,k; i < j$, of encountering obstacles such as: pedestrians, animals, vehicles in each section $q_{ij} \in [0;;1]\,,\ i = 1,\ldots,k-1; j = 2,\ldots,k; i < j$, of the route. These obstacles represent a risk for the cyclist, because the probability of an accident is proportional to the number of obstacles in the e-bike cyclist path.

2nd case

Let X be a discrete random value characterizing the hourly number of obstacles, counted by discrete observations for S days, in each arc of the route: $d_{ij},\ i = 1,\ldots,k-1; j = 2,\ldots,k; i < j$ (table 2).

Table 2: Data obtained by observation

Day	***Hour***			
	00:00	***01:00***	...	***23:00***
	Number of obstacles			
1	$x_{1,1}$	$x_{1,2}$	...	$x_{\mathbf{1},24}$
2	$x_{2,1}$	$x_{2,2}$	...	$x_{2,24}$
...	...	...	...	...
S	$x_{S,1}$	$x_{S,2}$	...	$x_{S,24}$
$EX_t \rightarrow$ t=1,...,24	$EX_1 =$ $= \frac{1}{S}\sum_{s=1}^{S} x_{s,1}$	$EX_2 =$ $= \frac{1}{S}\sum_{s=1}^{S} x_{s,2}$	...	$EX_{24} =$ $= \frac{1}{S}\sum_{s=1}^{S} x_{s,24}$

Then for each arc $d_{ij},\ i = 1,\ldots,k-1; j = 2,\ldots,k; i < j$, the average number of obstacles is:

$$EX_{ij} = \frac{1}{24}\sum_{t=1}^{24} EX_t = \frac{1}{24.S}\sum_{\mathbf{t=1}}^{\mathbf{24}}\sum_{\mathbf{s=1}}^{\mathbf{S}} \mathbf{x_{s,t}}. \quad (3)$$

And for each arc of the route the average number of obstacles $Ex_{ij},\ i = 1,\ldots,k-1; j = 2,\ldots,k; i < j$, is associated to the probability of being risky due to the number of obstacles that can appear by cycling through it. Therefore, the probability is expressed by $q_{ij} \in [0;1]\,,\ i = 1,\ldots,k-1; j = 2,\ldots,k; i < j$, where

$$q_{ij} = \frac{Ex_{ij}}{\sum_{i=1}^{k-1}\sum_{j=2}^{k} Ex_{ij}},\ i = 1,\ldots,k-1; j = 2,\ldots,k; i < j. \tag{4}$$

For each arc of the route d_{ij} the road quality (for cycling) $b_{ij},\ i = 1,\ldots,k-1; j = 2,\ldots,k; i < j$, is rated by $b_{ij} = 1,\ldots,U$. A higher value of b_{ij} means a worse quality of the pavement. The value of d_{ij}, is associated with a corresponding weight $w_{ij},\ i = 1,\ldots,k-1; j = 2,\ldots,k; i < j$, characterizing the road pavement:

$$w_{ij} = \frac{b_{ij}}{U},\ i = 1,\ldots,k-1; j = 2,\ldots,k; i < j. \tag{5}$$

Generally speaking, risk occurs when certain decision has to be taken and the results are uncertain, at the contrary – there is no risk if no uncertainty in the results of an action exist. The risk is more or less subjective, but the uncertainty is impartial. The lack of information (which is also objective and can be assessed) results in a risk. As the uncertainty is a source of risk it can be minimized by obtaining more information (and in an ideal case uncertainty could be eliminated at all). In practice it is rarely possible to reduce all uncertainty. As a result, every decision that has to be taken in an uncertain environment can be treated as a risk assessment problem.

In decision theory risk can be used to quantify uncertainty and is often defined as a deviation from the expected result. Based on this, mathematical methods for estimation of the risk can be implemented. Therefore, the risk for an e-bike cyclist is proportional to the number of obstacles and depend from road surface on his route, in other words a route with higher average number of obstacles and bad road surface is riskier for the cyclist.

The risk to the life and health of the cyclist during the driving of an electric bicycle is directly dependent on the number of obstacles on the bike path and the quality of the road surface, is characterized by probability

$$Q_{ij} = q_{ij}.\ w_{ij} = \frac{Ex_{ij}}{\sum_{i=1}^{k-1}\sum_{j=2}^{k} Ex_{ij}}.\frac{b_{ij}}{\sum_{u=1}^{U} u}, \tag{6}$$

$$i = 1,\ldots,k-1; j = 2,\ldots,k; i < j.$$

Solution of problem 2

The probability of the absence of risk (the inverse of risk occurrence) for each arc of the route $d_{ij},\ i=1,\dots,k-1; j=2,\dots,k; i<j$, is:

1st case:

$$p_{ij} = 1 - q_{ij} = 1 - \frac{Ex_{ij}}{\sum_{i=1}^{k-1}\sum_{j=2}^{k} Ex_{ij}}, \tag{7}$$

$$i=1,\dots,k-1; j=2,\dots,k; i<j.$$

2nd case:

$$p_{ij} = 1 - Q_{ij} = 1 - q_{ij}.\ w_{ij} = \frac{Ex_{ij}}{\sum_{i=1}^{k-1}\sum_{j=2}^{k} Ex_{ij}}.\frac{b_{ij}}{U}, \tag{8}$$

$$i=1,\dots,k-1; j=2,\dots,k; i<j.$$

A network model of the route is developed (figure 2) where each arc $d_{ij},\ i=1,\dots,k-1; j=2,\dots,k; i<j$, is characterized by the probability $p_{ij},\ i=1,\dots,k-1; j=2,\dots,k; i<j$.

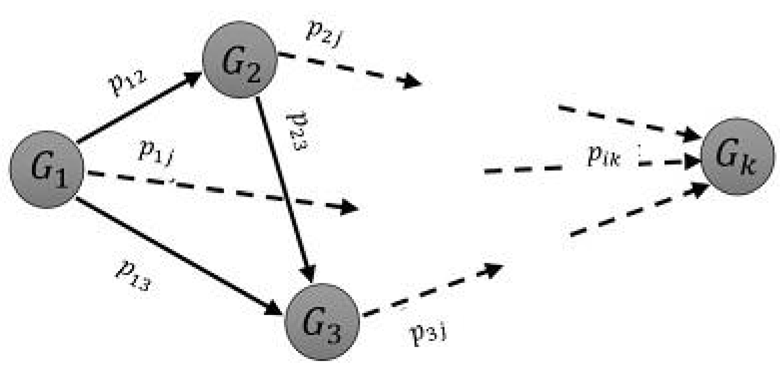

Figure 2: Network model of the oriented graph $V(G,P)$

The problem of finding the less risky route from the starting point to the final destination is modeled as a network problem, but in fact it is also a reliability problem. This complex problem can be solved by a dynamic programming method by decomposing it into sub-problems which are easier to solve. The decomposition consists in dividing the solution into stages and formulation of optimization problems for each stage that are less complex than the global problem. For each stage there is a scalar (control) variable whose value can be optimized and then the results are linked by a recursive algorithm. Therefore, the solution of the global problem is obtained finally after consecutive solution of a number of sub-problems. This method, based on recursive iterations relies on the Bellman optimally principle that states: 'The optimal strategy is composed of optimal sub-strategies'.

The objective function if a generalized characteristic of the decisions taken and the results obtained by solving the problem. It reflects the way in which the global problem is decomposed in less complex sub-problems. In the problem of optimal path, the objective function is multiplicative and the global result is a product of the results obtained each stage. It is similar to the reliability of a system built by consecutive addition of a number of elements (building blocks).

The number of stages after decomposition of the main problem is k. At each stage $E_n,\ n = 1, \ldots, k$, the problem of finding the less risky path between nodes G_1 and $G_n,\ n = 1, \ldots, k$. A Bellman's function $f_N,\ N = 0, 1, \ldots, k$, is introduced. It gives a quantitative measure of the less risky way from the initial point to the nth $(n = 1, \ldots, k)$ and is defined by a recursive dependence:

$$f_j = \max_{i<j} \{p_{ij}.f_i\}\ ,\ i = 1, \ldots, k-1; j = 2, \ldots, k; \tag{9}$$

$$f_0 \equiv 1,\ f_1 = 1, f_k = f_{max}. \tag{10}$$

By inversion of this principle, the less risky for the cyclist path from his initial point of departure to his destination is obtained (figure 3).

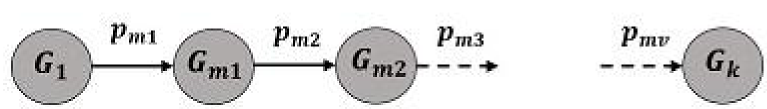

Figure 3: Optimal cycling path

Then

$$f_{max} = \prod_{V=1}^{v} p_{mV}. \tag{11}$$

In the second stage of the problem, the optimal path is found. The objective function is minimization of the risk for the cyclist during his way from the starting point to the destination.

Problem 2 is solved using an iterative method. Through the appropriate elements of the model, the complex problem is decomposed into simpler ones, which are solved almost independently. The solutions found at each stage are optimal and acceptable. This is due to the fact that the problem of the size of the problem is solved by being reduced in stages through the recurrent dependence. And this increases the capabilities of using this method to solve complex problems.

4 Numerical realizations

The proposed method is applied to this particular problem. In reality the e-bike cyclist is choosing his route just before or during the cycling, but when he arrives he can recapitulate his path and analyze if it has been the less risky from all possible routes. In order to assist the cyclist in choosing his route is developed the presented methods for optimization of the risk for electric bike cyclists. In figure 4 is depicted an example of departure and destination of an e-bike cyclist and the possible routes that he can choose. The number of routes is finite and includes combinations of their arcs.

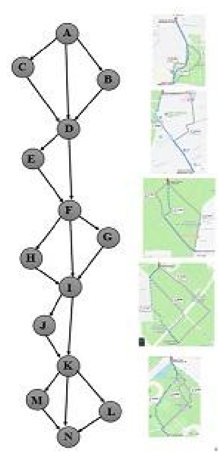

Figure 4: Routes considered for the numerical example

Input numerical data and processing:

1st case:

After determination of the possible routes, input data of the hourly number of obstacles (entire number - integer) is collected for each arc of the routes. In this numerical example random data is generated and each arc of the routes includes an hourly number of obstacles in the range from 0 to 120 (table 3).

Table 3: Random generated hourly number of obstacles for each arc

	0:00	***1:00***	***2:00***	***3:00***	***4:00***	***5:00***	***6:00***	***7:00***
ABD	11	63	19	114	75	52	45	54
AD	81	110	25	35	7	17	58	60
ACD	86	27	6	27	121	4	7	99
DF	95	72	83	73	95	122	23	106
DEF	116	82	37	83	96	39	86	13
FGI	128	4	70	54	8	38	43	23
FI	100	80	90	27	112	43	117	47
FHI	76	47	65	123	121	61	15	7
IK	121	6	70	11	128	84	128	68
IJK	75	64	58	14	112	3	70	44
KLN	2	25	16	18	102	109	92	23
KN	16	16	64	22	67	73	130	27
KMN	112	27	111	81	23	111	37	118
	8:00	***9:00***	***10:00***	***11:00***	***12:00***	***13:00***	***14:00***	***15:00***
ABD	88	9	39	107	90	104	43	78
AD	61	32	8	44	72	95	85	103
ACD	119	7	39	38	52	7	97	48
DF	14	57	6	97	8	9	76	27
DEF	97	2	66	1	101	12	96	11
FGI	96	117	99	6	44	104	31	100
FI	73	26	82	87	79	123	96	27
FHI	24	12	12	78	**96**	89	126	50
IK	**78**	**40**	**11**	68	14	17	113	72
IJK	39	59	101	95	17	94	11	30
KLN	17	13	118	92	71	14	48	83
KN	28	129	69	102	63	15	48	63
KMN	116	43	14	37	116	83	89	20

	16:00	***17:00***	***18:00***	***19:00***	***20:00***	***21:00***	***22:00***	***23:00***
ABD	102	56	35	1	41	41	20	59
AD	13	90	5	55	41	22	50	55
ACD	38	99	88	85	28	81	21	47
DF	31	56	56	94	33	128	99	73
DEF	69	85	59	69	116	22	113	97
FGI	12	14	79	14	91	34	46	55
FI	53	121	8	82	72	52	89	56
FHI	14	24	41	16	24	10	38	16
IK	15	35	100	17	28	89	69	3
IJK	102	104	91	13	10	52	108	38
KLN	38	63	16	18	119	128	78	41
KN	78	100	17	22	92	52	44	85
KMN	125	51	12	26	73	81	39	124

The input data given in table 3 is processed and characterized by frequency of occurrence of obstacles for each arc of the route (table 4).

Table 4: Frequency analysis for the arc FHI

	Interval $[\mathbf{x_r};;\mathbf{x_{r+1}}]$	**Average value** $\overline{\mathbf{x_r}}^*$	**Absolute frequency** $\mathbf{L_r}$	**Relative frequency** $\mathbf{P_r}^*$	$\overline{\mathbf{x_r}}^*.\mathbf{P_r}^*$
1	[0;10)	5	0	0	0
2	[10;20)	15	2	0.0244	0.3659
3	[20;30)	25	6	0.0732	1.8293
4	[30;40)	35	3	0.0366	1.2805
5	[40;50)	45	1	0.0122	0.5488
6	[50;60)	55	3	0.0336	2.0122
7	[60;70)	65	0	0.0000	0.0000
8	[70;80)	75	2	0.0244	1.8293
9	[80;90)	85	2	0.0244	2.0732
10	[90;100)	95	1	0.0122	1.1585
11	[100;110)	105	1	0.0122	1.2805
12	[110;120]	115	0	0.0000	0.0000
				$\sum P_r{}^*$=1	EX_{FHI}=12.3781

For each arc of the route the average number of obstacles is associated with a probability of risk. In table 5 are presented the average number of obstacles and the probabilities of risky and risk-free ride in each arc.

Table 5: Risk associated to each section of the route

	ABD	***AD***	***ACD***	***DF***	***DEF***	***FGI***	
E	53.0488	17.8049	14.8780	18.2927	17.5000	15.6098	
q	0.2201	0.0739	0.0617	0.0759	0.0726	0.0648	
p	0.7799	0.9261	0.9383	0.9241	0.9274	0.9352	
	FI	***FHI***	***IK***	***IJK***	***KLN***	***KN***	***KMN***
E	17.6829	12.3781	13.0488	18.4756	14.5732	16.8293	10.9756
q	0.0734	0.0514	0.0541	0.0767	0.0605	0.0698	0.0455
p	0.9266	0.9486	0.9459	0.9233	0.9395	0.9302	0.9545

2nd case:

The random data is generated and each section of the routes includes an hourly number of obstacles in the range from 0 to 120 (table 6).

Table 6: Randomly generated distribution of the number of obstacles in some arcs

ABD	***00:00***	***01:00***	***02:00***	***03:00***	***04:00***	***05:00***	***06:00***	***07:00***
1	12	85	109	64	99	15	1	113
2	72	83	6	6	112	31	35	92
3	120	27	1	89	52	23	66	70
4	82	114	35	54	51	15	38	9
5	12	98	76	84	44	56	46	40
6	59	72	115	74	34	67	115	119
7	35	119	53	100	50	39	80	64
8	98	75	69	3	108	111	116	23
9	9	6	112	22	33	49	48	58
10	97	36	4	5	106	58	54	36
ABD	***08:00***	***09:00***	***10:00***	***11:00***	***12:00***	***13:00***	***14:00***	***15:00***

1	23	97	108	100	110	111	95	109
2	60	56	1	106	20	104	39	50
3	33	87	22	12	31	66	29	49
4	41	55	27	105	40	55	41	65
5	95	29	105	27	12	24	59	119
6	16	1	36	65	26	94	52	100
7	103	40	57	33	9	78	56	78
8	22	31	100	47	2	90	33	95
9	43	59	23	3	91	53	114	30
10	64	48	19	43	118	65	98	92
ABD	***16:00***	***17:00***	***18:00***	***19:00***	***20:00***	***21:00***	***22:00***	***23:00***
1	3	89	97	62	4	39	80	87
2	26	52	86	29	17	21	8	116
3	36	29	101	6	49	45	61	105
4	8	92	13	18	86	30	55	54
5	12	103	37	53	62	27	74	24
6	3	67	5	62	64	77	58	116
7	119	52	113	90	57	14	6	93
8	98	74	47	48	99	75	45	117
9	34	54	91	56	10	57	25	74
10	92	80	92	73	44	59	82	79
...	...	...	...	...	...	...	...	...
FI	***16:00***	***17:00***	***18:00***	***19:00***	***20:00***	***21:00***	***22:00***	***23:00***
1	57	96	61	104	11	89	48	10
2	76	87	109	26	15	58	95	24
3	100	109	61	45	101	110	68	56
4	116	70	35	1	90	49	111	99
5	76	110	95	86	95	14	102	67
6	21	73	112	107	62	39	81	112
7	44	65	93	77	23	24	39	61
8	45	52	52	38	92	63	65	60
9	98	26	62	83	118	20	51	21
10	101	110	7	36	36	3	91	5

For each arc of road, the average number of obstacles is associated with the probability of a risk area. Table 6 contains the average number of obstacles, the probability of risk and non-risk area.

For each section of the road input data are processed according to the methodology described above, and the results are presented in table 7.

Table 7: Results after input data processing: q_{ij} – probability for risky section because of an obstacle; w_{ij} – weight characterizing the quality of the road pavement; Q_{ij} – probability for introducing an obstacle because of the pavement quality

	$\mathbf{q_{ij}}$	$\mathbf{w_{ij}}$	$\mathbf{Q_{ij}=q_{ij}.w_{ij}}$	$\mathbf{p_{ij}=1-Q_{ij}}$
ABD	0.08096	0.17560	0.01422	0.98578
AD	0.08173	0.75345	0.06158	0.93842
ACD	0.08170	0.56482	0.04615	0.95385
DF	0.08187	0.67520	0.05528	0.94472
DEF	0.08255	0.56892	0.04696	0.95304
FGI	0.08453	0.27531	0.02327	0.97673
FI	0.08680	0.87253	0.07574	0.92426
FHI	0.08523	0.24567	0.02094	0.97906
IK	0.08190	0.67289	0.05511	0.94489
IJK	0.08550	0.34958	0.02989	0.97011
KLN	0.08286	0.75987	0.06296	0.93704
KN	0.08235	0.35746	0.02944	0.97056
KMN	0.08453	0.68715	0.05809	0.94191

5 Results

The solution of the risk optimization problem is obtained according to a multiplicative objective function and the result is a product of the sub-problem solutions. It reflects the reliability of a system constructed by a number of separate elements. By using this method, the numerical solutions are depicted in figure 5.

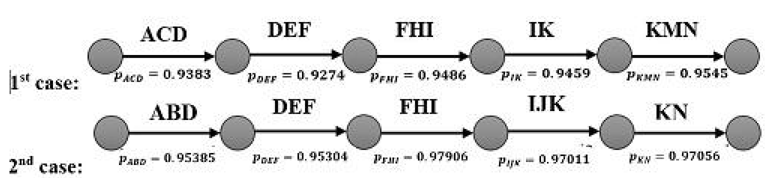

Figure 5: Optimal cycling route

The numerical realizations are implemented and solved by using the software environments MatLab and Maple.

6 Discussion of the results

Regarding the results obtained by application of the Bellman's optimization principle, the following observations can be made:

Figure 6 gives a histogram of the results obtained in the numerical realizations in both cases, which shows that in 1st case the best chance of no risk is 75%, but only one factor directly affecting the risk is taken - number of obstacles; in 2nd case - is 87%, but to this factor of 1st case is added another - quality of the road surface, i.e. the factors are already two. This is due to the fact that, when considering each factor, the aim is to eliminate or reduce the risk elements. Then adding factors eliminates or reduces the risk components, which in turn leads to an improvement in the probability of lack of risk. Ideally, this probability should be 100%, but it is actually impossible. The probability characterizing the least risky route in the both cases is not far from the theoretical maximum: 100% lack of risk. The result is adequate and reflects the real situation, given the inherent uncertainty due to the urban environment combined with segments located in a large park, including a dense forest.

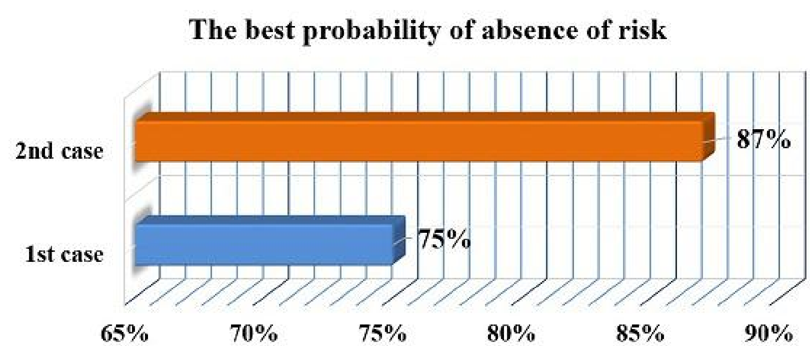

Figure 6: Histogram of the results obtained - Comparison for the best probability of absence of risk between 1st and 2nd case

The shortest path from the numerical realization has a length of 5.15 km and the length of the least risky one is 5.60 km in 1st case and 5.75 km in 2nd case. It turns out that the shortest route is not the most risk free (figure 7).

The duration of the fastest route is 39 min and the time needed for travelling along the least risky route is 46 min in 1st case and 43 min in 2nd case. It turns out that the fastest route is not the most risk free (figure 8).

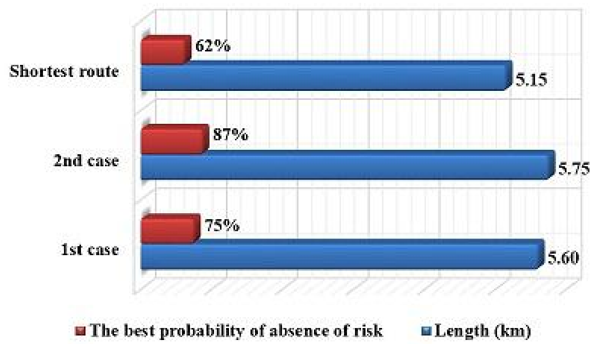

Figure 7: Histogram of the results obtained - Comparison for the best probability of absence of risk and length between 1st and 2nd case and shortest route

7 Conclusion and future works

The cyclist would like to make a decision and assess the risks of possible routes before making a decision to understand the ranges of possible outcomes and the importance of unwanted consequences. Decision making is a study to identify and select alternatives, which choice is based on the preference of the decision maker. Also, decision making is a process of sufficiently reducing uncertainty and suspicion of alternatives in order to make it possible to make a reasonable choice from among them. The information gathering function is important when making a decision. It should be noted that the uncertainty is reduced and not eliminated. Thus, any solution is associated with a certain risk. If there is no uncertainty, no decision is needed, and an algorithm already exists - a set of steps or a recipe that is followed in order to achieve a certain result. The work proposes a methodology that specifies the parameters of the specific studies proposed, their boundaries, what is the necessary information to be collected and processed so that it is in favor of the cyclist to decide on which route to pass before he left.

Route optimizations in the proposed cases are based on a dynamic programming approach, adding additional factors that characterize the cycling of electric bikes and pedal bicycles in urban environments. Due to the significant urban traffic dynamics, the input data of the optimization problem are highly variable in the daily and hourly scales and are also influenced by weather conditions and other factors or events.

The next stages of the research will be aimed at optimizing the energy

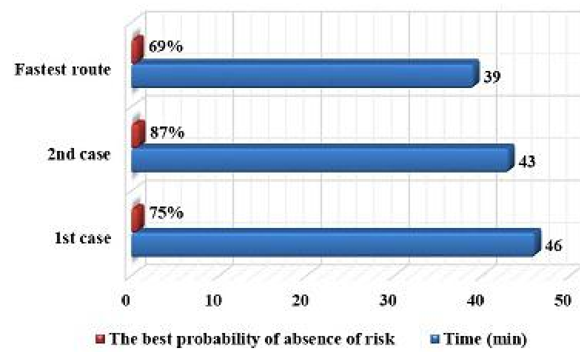

Figure 8: Histogram of the results obtained - Comparison for the best probability of absence of risk and time between 1st and 2nd case and fastest route

needs of the bicycle and other vehicles.

8 Acknowledgement

This research is funded and carried out in the framework of project 'Optimal Control of the Energy Flows in Electric Vehicles by means of Artificial Intelligence', contract 1910018-07, Scientific and Research Sector of the Technical University of Sofia.

References

[1] Dr Pradeep J Jha. *Operations Research.* L J Institute of engineering and Technology, Ahmedabad, India, 2014.

[2] G. Srinivasan. *Operations Research – Principles and Applications.* Third Edition, Delhi - 110092, 2017.

[3] H. A. Eiselt, Carl-Louis Sandblom. *Operations Research – A model – Based Approach.* Second Edition, Springer, e-book, 2012.

[4] J. O. Anderson, J. G. Thundiyil, and A. Stolbach. *Clearing the air: a review of the effects of particulate matter air pollution on human health.* Journal of Medical Toxicology, vol. 8, N. 2, pp. 166–175, 2012.

[5] Jan A. Snyman, Daniel N. Wilke. *Practical Mathematical Optimization – Basic Optimization Theory and Gradient – Based Algorithms.* Second Edition, Springer International Publishing, part of Springer Nature, e-book, 2018.

[6] Mohamed Haitam Laarabi.*Multi-criteria route optimization for dangerous goods transport using fuzzy risk assessment and agent-based traffic simulation.* Business administration, Ecole Nationale Suprieure des Mines de Paris, 2014.

[7] Mohammed H. Almannaa, Mohammed Elhenawy, and Hesham A. Rakha, Senior Member.*A Novel Supervised Clustering Algorithm for Transportation System Applications.* Transactions on intelligent transportation systems, IEEE, 2019.

[8] P. Bolzern, P. Colaneri, G. De Nicolao.*Stochastic stability of positive Markov jump linear systems.* Automatica, 50:1181-1187, 2014.

[9] S. I. Gass & M. Fu, eds.*Encyclopedia of Operations Research and Management Science.* Springer, New York, 2013.

[10] T. Azevedo-Perdicoulis, G. Jank.*Linear Quadratic Nash Games on Positive Linear Systems.* European Journal of Control, 11:1-13, 2005.

[11] Yingqi Gu, Mingming Liu, Matheus Souza and Robert N. Shorten. *On the Design of an Intelligent Speed Advisory System for Cyclists.* 21st International Conference on Intelligent Transportation Systems (ITSC), Maui, Hawai, USA, November 4-7, 2018.

[12] Yujie Dai, Jinzong Hu, Dongbin Zhao, Member and Fenghua Zhu. *Neural Network Based Online Traffic Signal Controller Design with Reinforcement Training.* 14th International IEEE Conference on Intelligent Transportation Systems, Washington, DC, USA, October 5-7, IEEE, 2011.