Annals of the Academy of Romanian Scientists
Series on Science and Technology of Information
ISSN 2066 - 8562 Volume **4**, Number **2**/**2011**

PCG SIGNAL ANALYZING USING WAVELET TRANSFORM

S. GERGELY,[1] M. V. PUSCA,[2,3] M. N. ROMAN[3]

Abstract. *There are a very few PCG (Phonocardiography) signal characterization algorithms and also only a reduced number of devices capable for a complex pathology analysis. The analysis of the PCG signal is based on using of wavelet transform to achieve the full multi-resolution decomposition of the signal. The paper presents two such algorithms which are included in a portable device controlled by two microcontrollers.*

Keywords: phonocardiography, complex pathology analysis, digital signal processing

1. Introduction

Digital signal processing has a large number of analytical tools including the most important which is the Fourier analysis. Thus Fourier analysis is a technique that can transform a time domain signal into frequency domain. A number of applications in signal analysis require conservation or temporal information of a signal for which Fourier analysis encounters serious difficulties. To correct this deficiency Dennis Gabor introduced the concept of STFT (Short Time Fourier Transform) method which allows analysis of a shorter duration of the signal. This method solves in part the representation of the time-frequency signal, but there is a drawback by maintaining a constant time window throughout the frequency range for which the analysis is done. The appearance of wavelet transform allows the windows to be variable for the analyzed signal over the entire frequency spectrum. Therefore the first analyzing method displays a time-frequency scalogram of the PCG (Phonocardiography) signal. The second method enables the PCG signal analysis by converting two-dimensional convolution of a reference signal with the acquired signal [1]. By comparing the results of convolution it can be determined the correlation of each reference signal with the separately analyzed input signal.

The basic functions used in the Fourier analysis are periodic functions (sine) with infinite duration. Thus it is possible to obtain information only in the frequency domain and of course the provided analyzed signal type must be periodically. Fourier transform for aperiodic signals will generate a continuous spectrum. Figure 1.1. presents a non-stationary signal which can be seen [6] that the inverse

[1]National Institute for Research and Development of Isotopic and Molecular Technologies, Cluj-Napoca, Romania.
[2]Corresponding member of the Academy of Romanian Scientists.
[3]Technical University, Cluj-Napoca, Romania.

Copyright © Editura Academiei Oamenilor de Știință din România, 2011
Watermark Protected

Fourier transform permanently lose any information on the temporal position of the original signal.

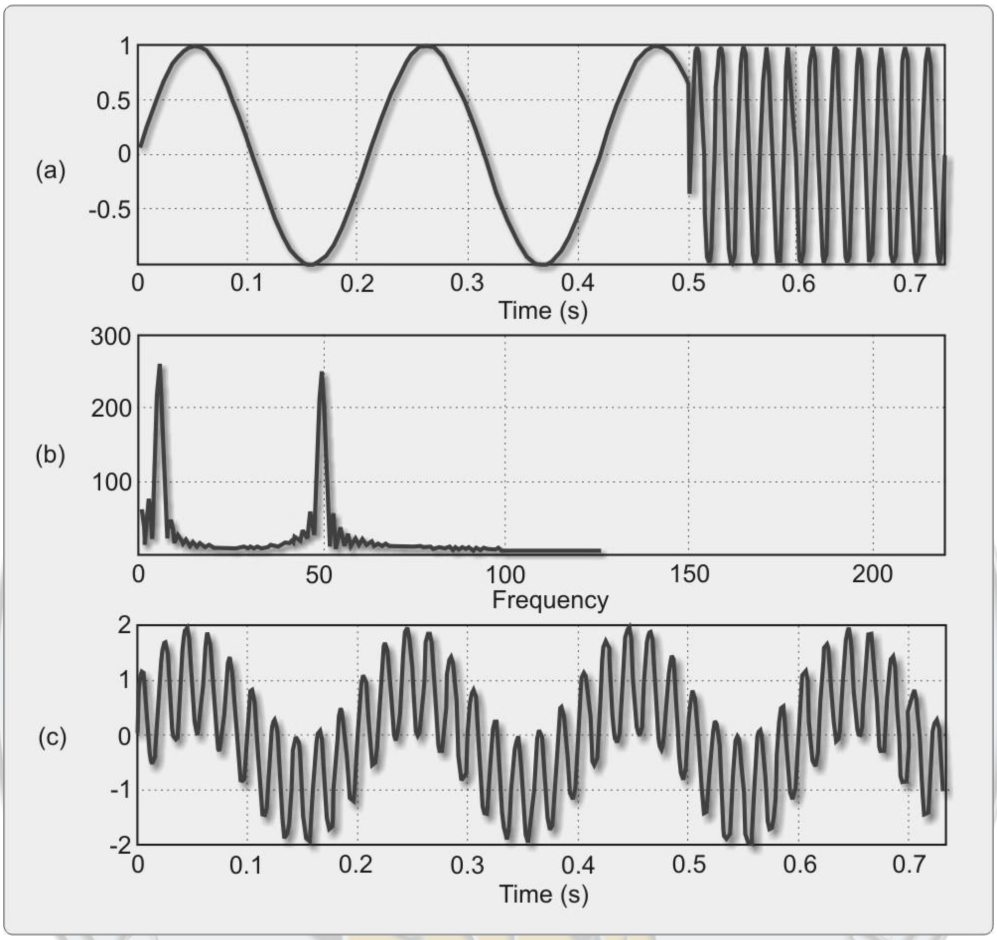
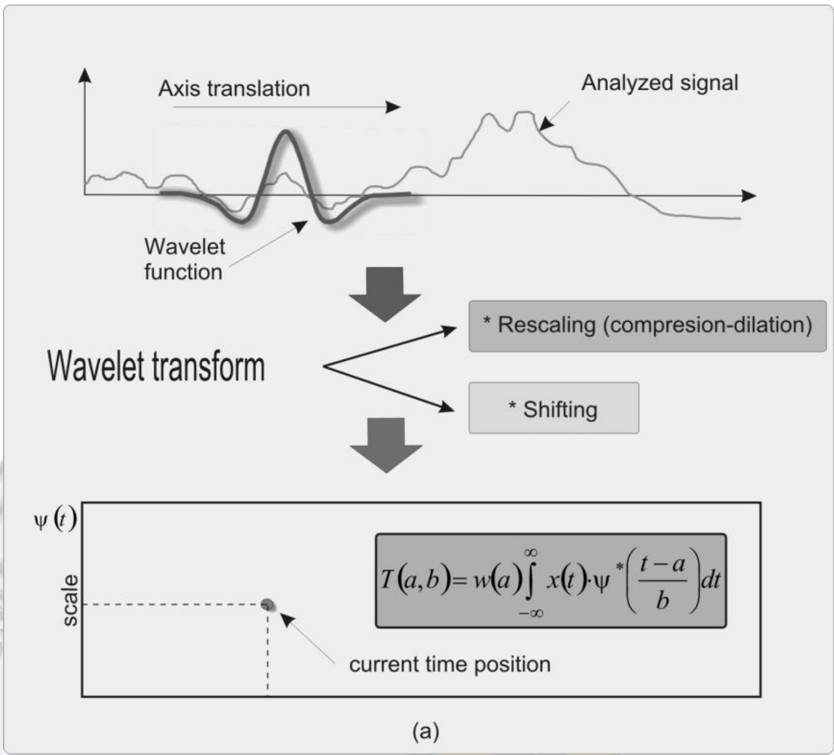


Fig. 1.1. The Fourier transform of a aperiodic signal.

By performing a transformation which uses a periodic basic function, well put out frequency spectral components but inevitably the time location information of the signal is lost. Also in the presence of [5] Gaussian noise in the analyzed signal, the isolation of time components becomes impossible because the present noise in the whole band cannot be eliminated by Fourier transform. In order to characterize a signal in time domain, it must be decomposed with aperiodic signals as a basis for well positioned time components. Therefore the wavelet transform is used in particular for analyzing signals that can be characterized as aperiodic type, containing noise, intermittent or having abrupt transitions. The strong ability of this type of transform [12] is that it can simultaneously examine both the time domain and frequency domain. This is done in a different way than the classical Fourier analysis.

Copyright © Editura Academiei Oamenilor de Știință din România, 2011
Watermark Protected

a) 

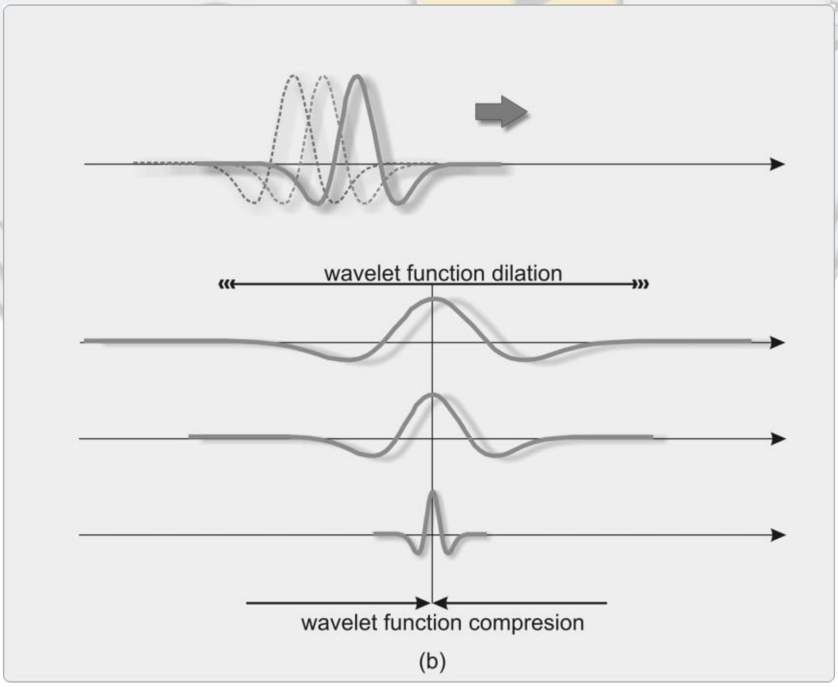
b) 

Fig. 1.2. (a) Correlation of wavelet function with signal;
(b) dilation-compression of wavelet function.

Copyright © Editura Academiei Oamenilor de Știință din România, 2011
Watermark Protected

Either mentioned case can create a match of graphic form, compared with the original signal. If the wavelet function has a matching shape, then we get a great value for the correlation. On contrary if the wavelet function does not correlate with the portion of the signal then the correlation value of the transform decreases. From the mathematical point of view were studied a large number of types of wavelet functions but in scientific and engineering applications are used only up to four such functions.

Of course it can be raised the question of why not to use the well-established Fourier methods for signal analysis. There are some major differences between Fourier analysis and wavelet transform performed by the method. Fourier analysis of the basic functions is localized in frequency but not in time. Local minor frequency changes are induced over the whole time domain of the Fourier transform. Wavelet functions are localized both in frequency and in time by shifting and scaling function without a projection over the entire transform. Another reason is that many functions classes (in our case signals) can be represented by this new method. Thus a signal having sharp discontinuities is using to approximate fewer wavelet functions than the sine-cosine functions of the typical Fourier transform.

2. The wavelet transform

The fundamental characteristic of wavelet functions is their ability to deal with noisy signals even covered by a lot of information, because in the end after the transform, the entire signal is compressed only to the wavelet coefficients only. Another important aspect is that in terms of number of necessary calculations for the Fourier transform there are required $O(n \cdot \log_2(n))$ operations, instead of $O(n)$ operations required by the wavelet transform. To complete the comparison between Fourier and wavelet transform should be noted that once the wavelet function $\psi(t)$ is defined it can take the translation and the dilation $\left\{\psi\left(\frac{x-b}{a}\right),(a,b)\in R^{+}\times R\right\}$. By defining the wavelet basis, a and b will take the values of $a = 2^{-j}$ and $b = k \cdot 2^{-j}$ where k and $j \in R$.

To fit a function in the class of wavelet type, they must meet the following conditions [15]:

a) The function must have finite energy:

$$E = \int_{-\infty}^{\infty} |\psi(t)|^2 \, dt < \infty \tag{2.1}$$

If the function $\psi(t)$ is complex then the amplitude is computed using both real and imaginary parts.

Copyright © Editura Academiei Oamenilor de Ştiinţă din România, 2011
Watermark Protected

b) if $\hat{\psi}(f)$ is the Fourier transform of function $\psi(t)$ then:

$$\hat{\psi} = \int_{-\infty}^{\infty} \psi(t) \cdot e^{-i(2\pi f)t} dt \tag{2.2}$$

And also the condition:

$$C_g = \int_{0}^{\infty} \frac{\left|\hat{\psi}(f)\right|^2}{f} df < \infty \tag{2.3}$$

Where the above condition is the admissibility function which implies that the function has no zero frequency components $\hat{\psi}(0) = 0$ or the average value of $\psi(t)$ must be equal zero. It turned to be obvious that each candidate to a wavelet based class will have an admissibility constant Cg.

For a function belonging to the space of finite energy $\psi \in L^2(R)$ we get

the transform:

$$T(a,b) = w(a) \int_{-\infty}^{\infty} x(t) \cdot \psi^* \left(\frac{t-a}{b} \right) dt \tag{2.4}$$

In order to ensure that at each scale factor "a" the wavelet function will have the same energy, energy conservation $w(a)$ introduces a coefficient with value $1/\sqrt{a}$.

Analog to the Fourier analysis, wavelet transform has the ability to reconstruct the original signal by computing the inverse wavelet transform of the analyzed signal. The mathematical form of the inverse wavelet transform is described by equation

the signal:

$$x(t) = \frac{1}{C_g} \int_{-\infty}^{\infty} \int_{0}^{\infty} T(a,b) \cdot \psi_{a,b}(t) \frac{dadb}{a^2} \tag{2.5}$$

where the C_g coefficient defines the admissibility condition. If we choose a narrower area of integration over the reverse wavelet transform it is obtained a filtering effect of the original signal. The above equation is called also a convolution therefore the final form of the wavelet transform is given by the following equation. This is also the well-known equation used in all practical applications:

$$\psi_{a,b}(t) = \frac{1}{\sqrt{a}} \psi \left(\frac{t-b}{a} \right) \tag{2.6}$$

Copyright © Editura Academiei Oamenilor de Știință din România, 2011
Watermark Protected

3. Implementation of the filtering algorithm

Stephane Mallat introduces the concept of filtering a signal through the discrete wavelet transformed in a multi-resolution decomposition. Each iteration of the process is called octave that can be assimilated to a set of two FIR filter type. By direct application of wavelet transform on a discrete signal, we obtain a double number of samples for the half of the transform vector. Because the number of points after each transform iteration is equal to the number of samples of the original signal, we had to proceed to decimation by two of the resulted vector.

The procedure of signal decomposition is the reverse process of synthesis by wavelet transform. This is done by inserting the inverse low pass filters respectively high pass filters. A low-pass filter generates a mean signal (of two adjacent samples), while a high pass filter generates the detail elements by making the difference between two adjacent samples [1]. If case of a low-pass filter with coefficients $\{1/2, 1/2\}$, the result is $(x[n]+x[n-1])/2$, this obviously is the average of two samples. A high-pass filter having the $\{1/2, -1/2\}$ yields the result of $(x[n]-x[n-1])/2$. The multi-resolution analysis introduces the average of a signal in a different set of filters as shown in figure 3.1.

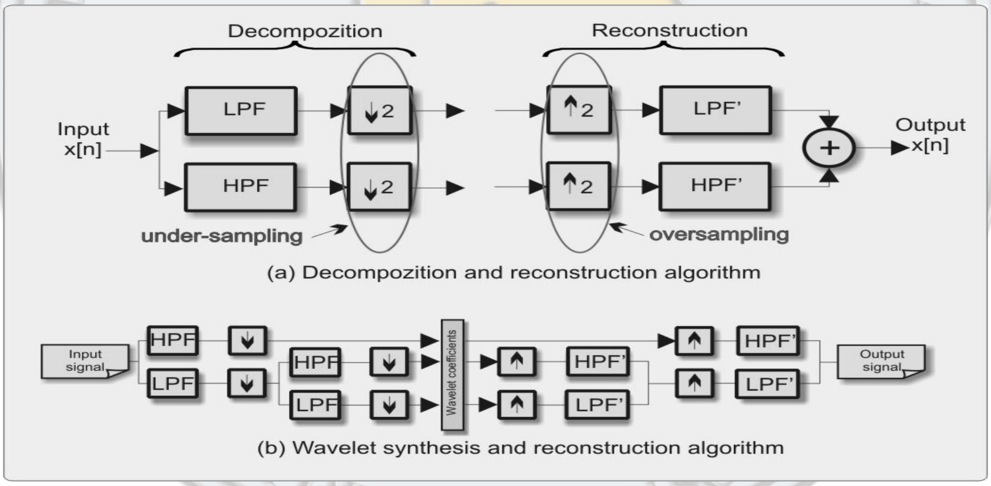


Fig. 3.1. (a) Decomposition algorithm (b) multi-resolution synthesis algorithm

The above described algorithm is part of a filtering process done by decimating the signal by two in which a sub-sampling is accomplished by removing sequentially one sample of the signal. The detail coefficients are obtained in a similar manner by using the pre-computed high pass filtering coefficients. The scaling sequence has a low pass filter behavior and the wavelet sequence is a low pass filter both where both are normalized for a coefficient sum equal with 2. In some applications the filtering sequences are normalized to have the sum equal with $\sqrt{2}$, to remain orthogonally to each other.

Copyright © Editura Academiei Oamenilor de Știință din România, 2011
Watermark Protected

4. Choosing the optimal wavelet function for the analysis of PGC signals

Every application which requires the using of a wavelet transform requires the adaptation or finding of a certain type of wavelet function. The first criterion is defined by the correlation factor between the wavelet function and the analyzed signal. Unfortunately the shape of the correlated wavelet function is capable to indicate only the family of wavelet functions but gives no indication regarding the number of the wavelet coefficients. To compute the necessary coefficient numbers, we used the wavelet toolbox of the Matlab program. A key criterion in determining the type of wavelet function consists in evaluation the overall reconstruction error, for the given type of signals that will appear in the application. So the evaluation must be made for signal reconstruction at the extreme in terms of spectral content. We imposed this condition for the decomposition algorithm to use a minimum number of coefficients, to optimize the convolution calculation time. Figure 4.1 is presenting the minimal set necessary for the calculation of the reconstruction error.

```
load Normal;                   %Load file
s=Normal(1:32768);
[C,L]=wavedec(s,3,'db2');
A3=wrcoef('a',C,L,'db2',3)     %Computing the decomposition of level 3
D1=wrcoef('d',C,L,'db2',1);
D2=wrcoef('d',C,L,'db2',2);
D3=wrcoef('d',C,L,'db2',3);
sig=A3+D3+D2+D1;               %Reconstruction from signal parts
err=max(abs(s-sig));           %Error computation
```

Fig. 4.1. Matlab program for wavelet reconstruction error computing.

After computing the wavelet function we tested the validity of the selection by taking on all detail components of reconstructed signal and computing the reconstruction error. The goal was to find a maximum error therefore the maximum sensitivity of the wavelet function to the given input signal. The tables are presenting the overall computed errors which must be the lowest possible and the component extraction error which must indicate the highest sensitivity of the wavelet functions.

These two conditions are met by the Daubeschies db2 function. It seemed that this function did answer very well to the PCG signal which is a strongly polynomial type due the massive presence of arbitrary heart murmur signals. Due to the Matlab convention the db2 wavelet function is mathematically defined by four coefficients which are computed by using the orthogonally conditions along with a specially imposed condition which gives the smoothness of the Daubeschies functions. The signal is reconstructed by using the equation:

$$S_{reconstrut} = A\langle level\rangle + D\langle level\rangle + D\langle level-1\rangle + D\langle level-2\rangle \tag{4.1}$$

Copyright © Editura Academiei Oamenilor de Știință din România, 2011
Watermark Protected

Table 4.1. Reconstruction error computation for a normal PCG signal (without pathology)

DAUBESCHIES WAVELETS	S=A3+D3+D2	S=A3+D3+D1	S=A3+D1+D2	AVERAGE ERROR	OVERALL ERROR S=A3+D3+D2+D1
DB2 (*MATLAB*)	0.095	0.0168	0.0688	0.0317	1.46E-13
DB3 (*MATLAB*)	0.0099	0.0138	0.0328	0.0188	3.73E-12
DB4 (*MATLAB*)	0.022	0.0137	0.022	0.0192	7.025E-13
DB5 (*MATLAB*)	0.076	0.0116	0.0196	0.035	1.43E-12
DB6 (*MATLAB*)	0.0075	0.0123	0.0206	0.0133	7.75E-13

Table 4.2. Reconstruction error computation for a PCG signal with Aortic regurgitation

DAUBESCHIES WAVELETS	S=A3+D3+D2	S=A3+D3+D1	S=A3+D1+D2	AVERAGE ERROR	OVERALL ERROR S=A3+D3+D2+D1
DB2 (*MATLAB*)	0.0678	0.1796	0.5198	0.255	1.168E-12
DB3 (*MATLAB*)	0.035	0.151	0.5226	0.2362	1.116E-11
DB4 (*MATLAB*)	0.0408	0.1015	0.3732	0.171	2.068E-12
DB5 (MATLAB)	0.0596	0.0761	0.3775	0.171	3.12E-12
DB6 (*MATLAB*)	0.0646	0.0869	0.3311	0.16	1.793E-12

Since the reconstruction is made more difficult for signals with a higher spectral content so the finally chosen wavelet function is the db2. The reconstruction of the initially decomposed signal by using the wavelet transform is done through the reconstruction filters. The number of coefficients which are defining the wavelet function has a major influence over the construction of the reconstruction filter. According to Strang and Nguyen the reconstruction filter presented in figure 2.10, is a quadrature filter used very frequently in signal processing. Decimating the signal with a factor of two induces a series of aliased samples in the original signal which had to be eliminated through a anti-alias filter before the sampling process. The reconstruction signal [7] in terms of levels of decomposition is shown in figure 2.15.

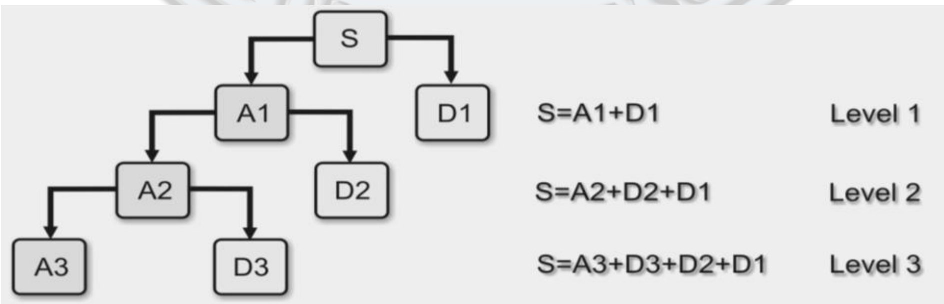


Fig. 2.15. Reconstruction of signal in terms of decomposition levels

Copyright © Editura Academiei Oamenilor de Știință din România, 2011
Watermark Protected

Through the PCG signal analysis, ultimately the reconstruction algorithm is used only in the process of choosing the right wavelet function for the multi-resolution decomposition.

5. Algorithms for the characterization of the PCG signals

Due to the extremely complex structure of the PCG signal, in analogy with an information carrier signal, it is characterized by a low frequency amplitude modulation, followed by a frequency modulation of a "carrier" generated by the cardiac activity at different times in relation to the movement of valves, and finally the overall signal is phase modulated between all components. The properties of the PCG signal in the frequency domain are the best starting point in the characterization of these signals. The most important parameter is the frequency domain of the signal which is situated in the range of 62-800 Hz. Figures 5.1 and 5.2 are showing the frequency spectrum domain for two cases.

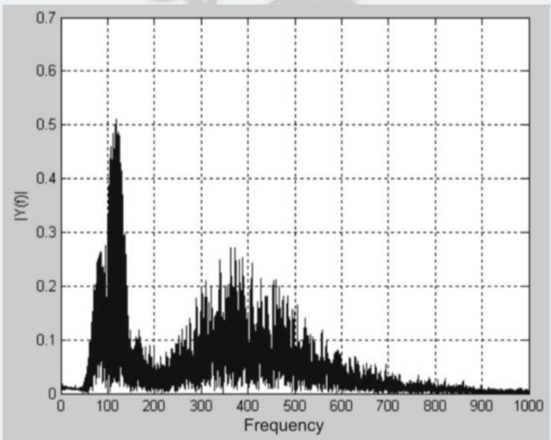


Fig. 5.1. Normal heart PCG signal frequency spectrum.

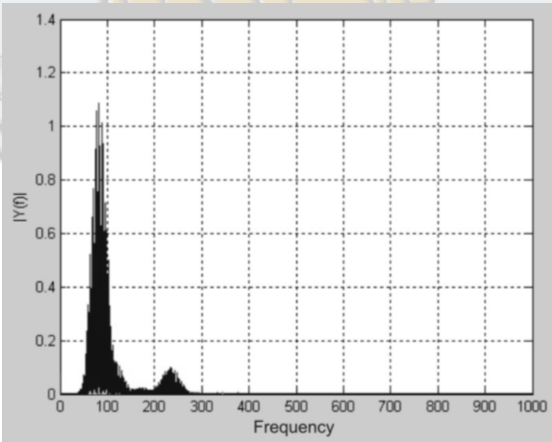


Fig. 5.2. Aortic Regurgitation PCG signal frequency spectrum.

Copyright © Editura Academiei Oamenilor de Ştiinţă din România, 2011
Watermark Protected

Figure 5.1 and 5.2 are showing that the frequency spectrum for the PCG signals are situated in the low and in the middle of the 62-800 Hz domain, regardless the type of the pathology. So the presentations of analysis algorithms of the PCG signals are presented through the following steps:

- determination of the spectral content of the wavelet packets
- precise detection of the heart beat rate (HBR) for the signals containing certain pathologies
- PCG signal rescaling through multirate sampling
- PCG signal envelope detection
- Correlation of the two dimensional converted PCG signals

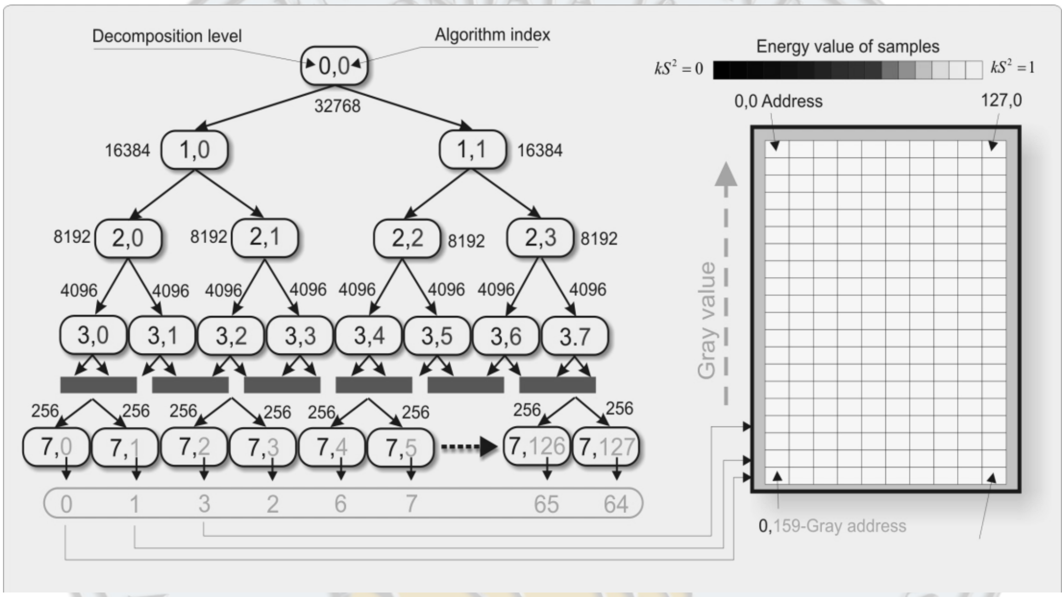


Fig. 5.3. Computing algorithm of the time-frequency scalogram display addreses.

Displaying of the time-frequency scalogram requires the construction of a special algorithm for the computing of addresses for each pixels of the color LCD display. The origin of the first displayed value is situated in the upper left corner of the display panel. The mentioned starting point is not appropriate for the display of the scalogram due the mathematical configuration of the time-frequency representation. The scalogram changes the standard displaying mode to a custom designed algorithm which modifies both horizontal (time) and vertical (frequency) coordinates. The main issue is the correct representation of the frequency coordinates, due to the interlaced addressing of the pixels. This addressing mode is imposed by the changing of the usual order of the computed frequency domains. The changing of the domain order follows a Gray code pattern; therefore each Gray code of the address value is first converted to its decimal value. The new value represents the address of the real pixel rows of the

Copyright © Editura Academiei Oamenilor de Știință din România, 2011
Watermark Protected

time-frequency scalogram. The LCD display has 128 horizontal pixels and 160 vertical pixels. Therefore finally the y coordinate yields by computing $INDEX_{adress\breve{a}} = 159 - val_{Gray}$.

The value written at a specific address is given by the energy of the aquired samples, where the energy is given by the squared value of each filtered sample. The energy values are scaled to different colors which are represented in a color matrix coded in a RGB565 configuration.

The scalogram algorithm is presented in figure 5.3.

As a result of the full decomposition, the displayable resolution is given by equation 5.1:

$$\Delta f = \frac{\Delta f_{Band}}{nr\left[INDEX_{dec}\right]} = \frac{800-62}{128} = 5.76Hz \tag{5.1}$$

By adding new levels to the decomposition obviously the frequency resolution does increase by adding more frequency bands but also the time resolution decreases due the decreasing of the samples number. This is a practical prove of the Heisenberg principle of uncertainty which states that a time-frequency representation has a fundamental limitation because the product of time resolution Δt and frequency resolution Δf is given by the equation 5.2:

$$\Delta t \cdot \Delta \omega > \frac{1}{2} \tag{5.2}$$

Where:

$$\Delta t = \sqrt{\frac{\int t^2 \left|\psi(t)\right|^2 dt}{\int \left|\psi(t)\right|^2 dt}} \tag{5.3}$$

$$\Delta \omega = \sqrt{\frac{\int \omega^2 \left|\Psi(\omega)\right|^2 d\omega}{\int \left|\psi(\omega)\right|^2 d\omega}} \tag{5.4}$$

and where $\Psi(\omega)$ is the Fourier transform of basis function $\psi(t)$.

Therefore Heisenberg's uncertainty principle states that the frequency resolution can be achieved at the expense of resolution in time and vice versa. This is because the resolutions cannot be arbitrarily small because their product has a finite value. In figure 5.4 are presented some scalograms achieved by simulating the decomposition algorithm presented above. Simulations have shown the need for a suitable scaling corresponding to the color values assigned to the displayed energy.

Copyright © Editura Academiei Oamenilor de Ştiinţă din România, 2011
Watermark Protected

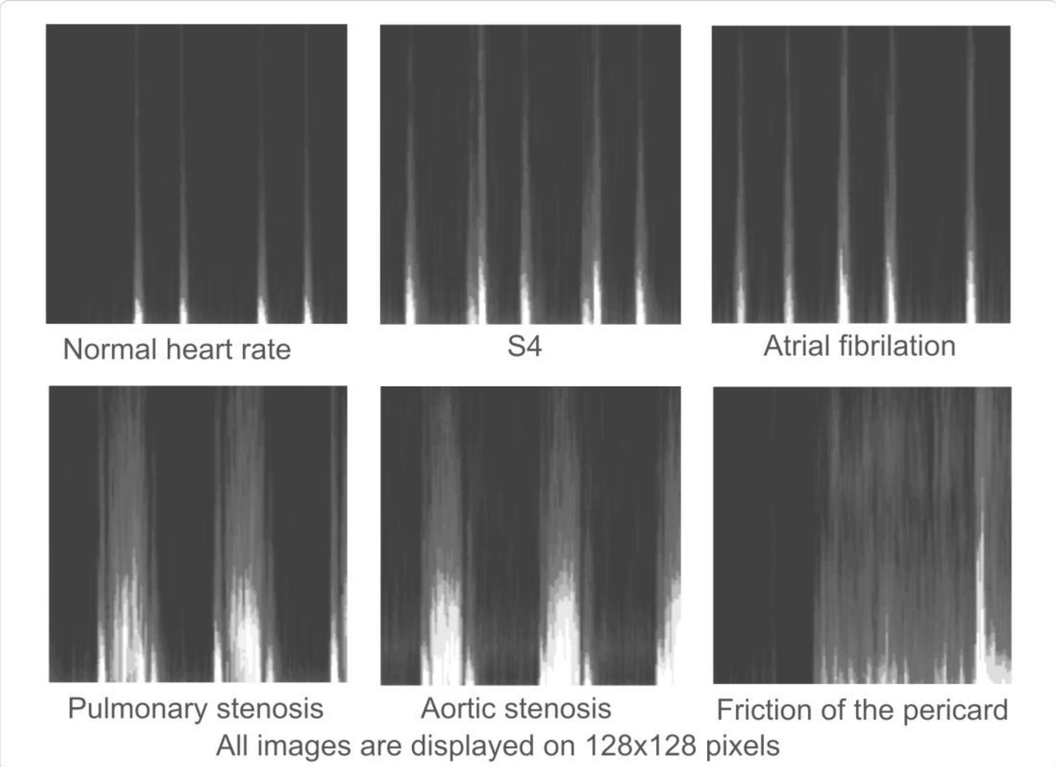


Fig. 5.4. Time-frequency scalograms obtained using algorithm presented in figure 5.3.

The final form of the PCG signal correlation does include some intermediary algorithms presented in figure 5.5.

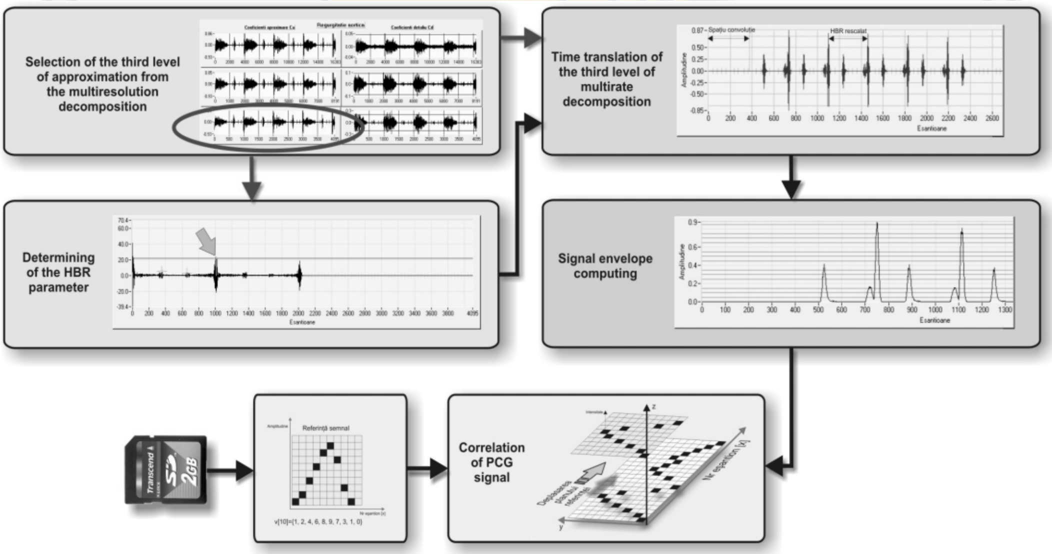


Fig. 5.5. PCG signal correlation algorithm.

Copyright © Editura Academiei Oamenilor de Ştiinţă din România, 2011
Watermark Protected

Due to the large overall mathematical complexity, the algorithms are only enumerated in this paper. The above presented scalogram and the next algorithms are by now implemented in a portable medical device capable to characterize a number of pathologies which are present in the PCG signals.

Simulations showed that the classical correlation method between a one-dimensional signal vectors with another reference signal, did not give the best results. The problems were caused by the presence of relative symmetry of the shape in the reference signal. For example, even if the reference signal contains two asymmetric peaks with values which are very different, the same set of values can be found in any other signal due to the summing operation of the correlation process.

Due to the complex nature of the PCG signal characterized by the presence of several peaks situated at relatively the same distance but in different pathology, it was imposed the introduction of an additional information which to indicate the relative position of signal components. Dividing the signal into successive epochs by applying threshold values and classical comparing applied to ECG signals can not be used. Tests have shown that signal synchronization using threshold method can be successful only when there's no pathology presence, and only to determine heart rate. Accurate and nondestructive detection by filtering the envelope of the PCG signal, pave the way for much closer analysis to a visual perception so that formal representation can be similar to an ECG signal. This requires the obtaining of separate images for each type of pathology stored in the SD card.

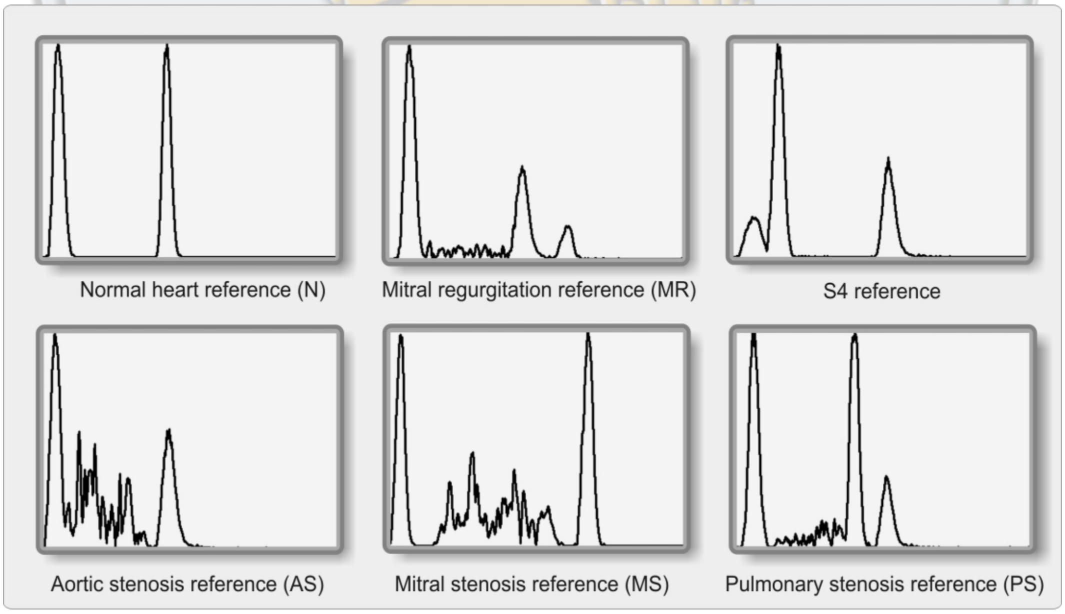


Fig. 5.6. PCG signal pathologies references.

Copyright © Editura Academiei Oamenilor de Ştiinţă din România, 2011
Watermark Protected

Defining certain pathologies is done by manually extracting the desired signal sequence. The image extraction has to be done by a medically trained person who knows the components that make up a given pathology. The images were extracted from the demodulated signals, for which it was wrote an own program. Each reference image is extracted for a specific cardiac pathology. This can build a database with various pathologies to be compressed and stored.

The necessary memory to store a pathology image is approximately 2 Ko. For a correct reference image generation, the computer requires as input parameter only the value of heart rate taken at time of the signal acquisition. The program extracts from the PCG signal a number of samples by moving two cursors on the signal and after that it allows to save as a file in a reference data or graphical bmp format. Some of the images corresponding to the pathologies references are shown in figure 5.6.

During the researches we applied a new method to analyze the PCG signal by introducing an additional coordinate to the signal vector. The idea is that we switched to two-dimensional analysis similar to pattern recognition located in an image. We converted from one-dimensional vector v [x] to the vector w[y][x] by marking the position (indexes) sample with value 1, if the signal from that amplitude value is in the rescaled amplitude domain. Samples that fall outside the scope of amplitude are marked with 0. Thus the points defined in terms of measured values X0Y are introducing a second dimension of the signal and can thus be interpreted as an image. Reducing the number of calculations required to scale the amplitude vector x [t] for a number of dots (pixels). By extending the method, if the marked pixel value changes from 1 to higher value, the vector w[y][x], receives the z coordinate, i.e. the intensity that we refer to a grayscale or color image.

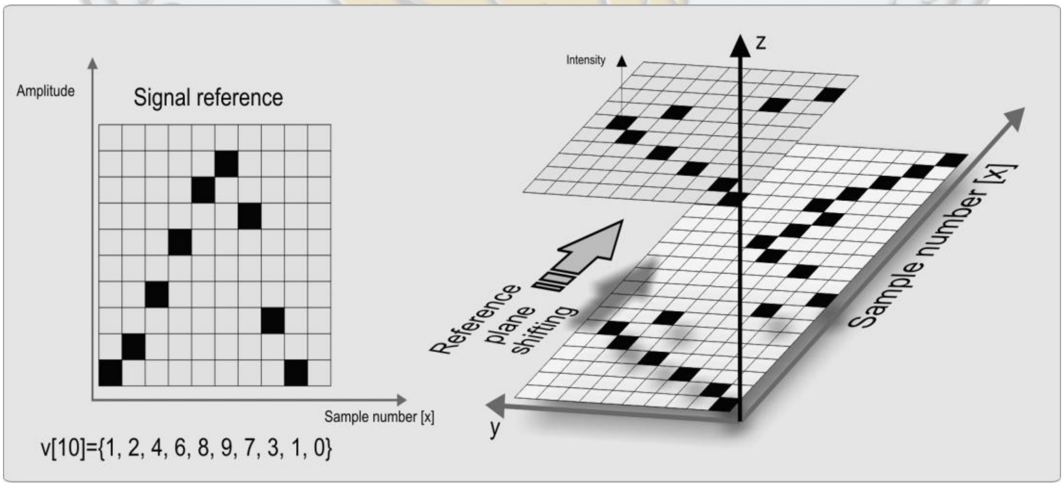


Fig. 5.7. Two-dimensional conversion of the PCG signal.

Copyright © Editura Academiei Oamenilor de Știință din România, 2011
Watermark Protected

This type of resulted image can be processed by using advanced image processing methods. On a closer analysis, there is an important feature which allows to limit one of the coordinates of the vector w[*y*][*x*]. It can be observed that both signals involved in the algorithm are scaled to the same number of values on the *Y* coordinate, which provides a significantly higher computing speed. The signal conversion process is shown in figure 5.7.

The references of pathology images were extracted from real signals which are shown in figure 5.6 proves how easy it can be identified with the original pathology. Of course, the graphics are enhanced by the computer through the interpolation procedure where pairs of neighboring points are plotted using anti-aliased filter to reduce the pixelization. In fact, without being processed the signals are the same as in the matrix shown in figure 5.7. The construction of the above shown algorithm is achieved through a very simple calculation routine that consists only of a few lines of program. Reference signals were rescaled to 1 Volt for a total of 50 points. The x axis number of points remains unchanged against the original signal and dynamically allocated between 100 and maximum 350 points. Under these conditions the reference image is represented in a matrix of 50×350 points. To test the recognition algorithm, the routine uses computer generated pixels of the image without intensity parameter *Z* (pixel has value 1) and no interpolation therefore a minimum number of points. The lowest number of points (42) is contained by the reference of normal heart rhythm. Analogous to the procedure for the reference signal, it applies the same above procedure for the analyzed signal. As a pattern recognition method, we have used the computation of the Euclidian distance between the analyzed signal and the stored references. The Euclidian distance is computed using the equation 5.5:

$$d\left(p_{ij},q_{ij}\right)=\sqrt{\sum_{i=1}^{m}\ \sum_{j=1}^{n}\left(q_{ij}-p_{ij}\right)^2} \tag{5.5}$$

6. Conclusions

It was presented a set of algorithms for processing of the PCG, which are part of two main methods of pathologic classification, along with the obtained results. The first method involved the analysis of spectral content of the PCG signal and then based on a complex algorithm; the frequency spectrum is displayed on a color LCD display. This method of evaluation of the signal is addressed to medical personnel who can understand the time-frequency scalogram created. The PCG signal correlation method includes an algorithm for accurate heart rate detection using autocorrelation function. Determining the precise heart rate is dictated by the need of temporal rescaling of the wavelet decomposition at level three, to bring the acquired signal to the same HBR value of the reference signal.

Copyright © Editura Academiei Oamenilor de Știință din România, 2011
Watermark Protected

The last step is to precisely detect the envelope of the PCG signal by using the discrete signal Hilbert transform. The PCG signal correlation is prepared by converting the signal into a two-dimensional vector, making the actual signal in a two-dimensional image. This method opened the way to the methods of imaging processing especially for shape recognition. The sum of these presented numerical processing methods is an important step in the processing of a signal virtually unapproachable by traditional methods of numerical processing.

REFERENCES

[1] A. Jensen, A. la Cour-Harbo, *Ripples in mathematics, The discrete wavelet transform*, Springer, ISBN: 3-540-41662-5, **2001**.

[2] Abbas K.Abbas, Rasha Bassam, *Phonocardiography signal processing*, MCP, ISBN: 9781598299762, **2009**.

[3] Abdallah et al., *Arterial stenosis murmurs: an analysis of flow and pressure fields*, Journal Acoust. Soc Am. 83(1):318-34, **1988**.

[4] Addison S. Paul, *The illustrated wavelet transform handbook*, IOP, ISBN: 0-7503-0692-0, **2002**.

[5] Albert Boggess, Francis J. Narcowich, *A first course in wavelets with Fourier analysis*, ISBN: 978-0-470-43117-7, **2009**.

[6] Andreas Antoniu, *Digital signal processing: signals systems and filters*, McGraw-Hill, ISBN: 0-07-145424-1, **2006**.

[7] Andrew K.Chan, Steve J.Liu, *Wavelet toolware*, ISBN 0121745953, **1998**.

[8] Anke Meyer-Base, *Pattern recognition for medical imaging*, Elsevier, ISBN: 0-12-493290-8, **2004**.

[9] Bachman George, Lawrenc Narici, Edward Beckenstein, *Fourier and wavelet analysis*, Springer, ISBN:0-387-98899-8, **2000**.

[10] Broersen Piet M.T., *Automatic Autocorrelation and Spectral Analysis*, Springer, ISBN-10: 1-84628-328-0, **2006**.

[11] Bu-Chin, *Radar image processing*, Wiley &Sons, ISBN: 978-0-470-18092-1, **2008**.

[12] Burrus C. Sidney, Ramesh A. Gopinath, *Introduction to Wavelets and wavelet transforms*, Prentice-Hall, ISBN: 0-13-489600-9, **1998**.

[13] Cipra A. Barry, *A healthy heart is a fractal heart*, SIAM, Vol. 36, No. 7, **2003**.

[14] Cory L. Clark, *LabView digital signal processing and digital communications*, McGraw-Hill ISBN: 0-07-146966-4, **2005**.

[15] D Sundararajan, *The discrete Fourier Transform, Theory Algorithms and applications*, ISBN: 981-02-4521-1, **2001**.

[16] Dag Stranneby, *Digital signal processing & applications*, ISBN: 0-7506-48112, **2001**.

Copyright © Editura Academiei Oamenilor de Știință din România, 2011
Watermark Protected