In figure 2, the temperature of the tissue (°C) at the point of tumour x = 0.006 m rises to desired temperature 43 °C at the end of duration of the process T = 800 s for the application of

$$Q(t) = 1525\,\mathrm{Wm}^{-2} = 0 \le t \le 600\ \mathrm{s};$$

$$Q(t) = 300\ \mathrm{Wm}^{-2},\ 600\mathrm{s} \le t \le 800\mathrm{s};$$

and

$$u(t) = 10.59\,^{\circ}\mathrm{C},\ 0 \le t \le 500\mathrm{s};$$

$$u(t) = 7.25\,^{\circ}\mathrm{C},\ 500\mathrm{s} \le t \le 800\mathrm{s}.$$

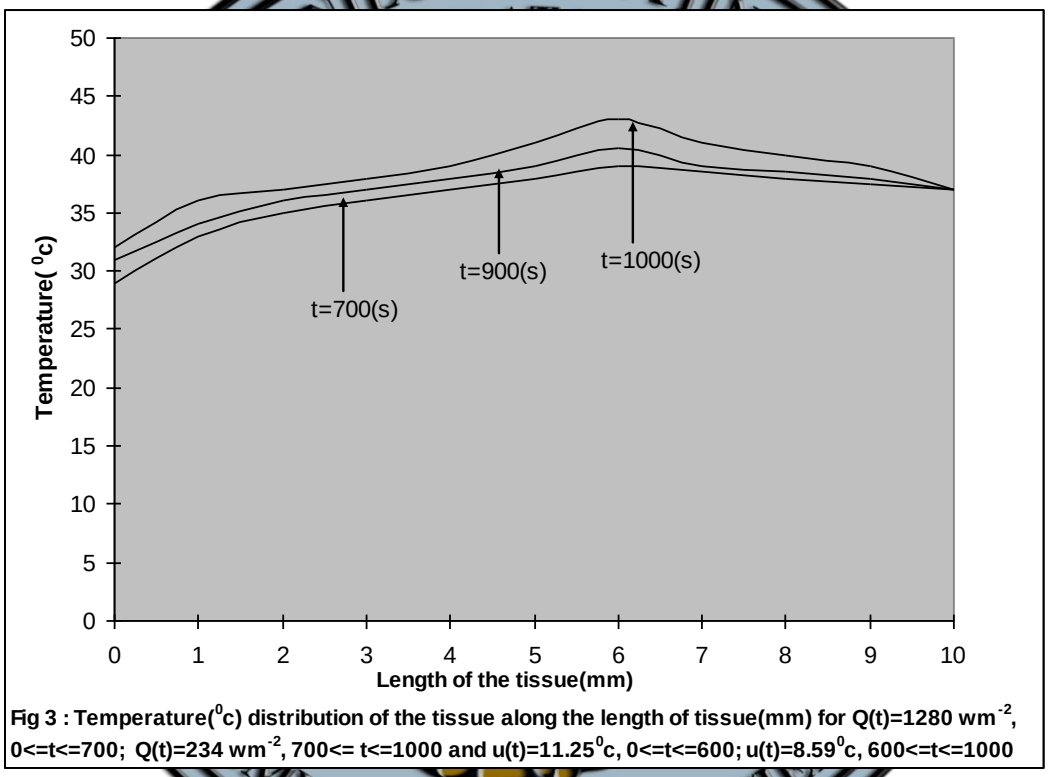

Fig 3 : Temperature(^{0}c) distribution of the tissue along the length of tissue(mm) for Q(t)=1280 wm^{-2}, 0<=t<=700; Q(t)=234 wm^{-2}, 700<= t<=1000 and u(t)=11.25^{0}c, 0<=t<=600; u(t)=8.59^{0}c, 600<=t<=1000

Figure 3 displays the temperature of the tissue [°C] along its length [mm] due to application of

$$Q(t) = 1280\ \mathrm{Wm}^{-2},\ 0 \le t \le 700\mathrm{s};\ Q(t) = 234\ \mathrm{Wm}^{-2},\ 700\mathrm{s} \le t \le 1000\mathrm{s}$$

and

$$u(t) = 11.25\,^{\circ}\mathrm{C},\ 0 \le t \le 600\mathrm{s};\ u(t) = 8.59\,^{\circ}\mathrm{C},\ 600\mathrm{s} \le t \le 1000\mathrm{s}.$$

Here, the desired temperature 43 °C is attained on the tumour location x = 0.006 m at the end of operation of the forces T = 1000 s.

4. Conclusions

It is observed that the temperature of tissue increases on the left side of the tumour at $x = 6$ mm till it attains the beneficial desired temperature $43\,°\text{C}$ at the end of operation of the process and then the temperature of the tissue on the right side of the tumour decreases steadily to $37\,°\text{C}$ (arterial temperature). Further, it is to note that as the total time of operation of the process increases from $T = 600$ s to 1000 s, the first time segment of operation of the process $(0, t_1)$ increases with the corresponding decrease of $Q(t)$ [Wm^{-2}] in this segment for the switching time t_1. The surface cooling temperature $u(t)$ [°C] increases in the first time segment of operation of the process $(0, t_2)$ as the total time of operation increases from $T = 600$ s to 1000 s for the switching time t_2.

Again, it is seen that the temperature of the healthy tissue on the both sides of the tumour are less than desired rise of temperature $43\,°\text{C}$ and thus the damage of the healthy tissue is avoided due to overheating.

This analytical study of the optimal control problem may be used in case of computer- aided therapy planning in hyperthermia treatment. It can further be developed at different points of location of the tumour along with different length of tissue which may focus as a useful guideline to illustrate the versatility of the computer program.

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