THE CLASSIFICATION
OF THE HAMILTONIAN MECHANICAL SYSTEMS
USING
THE ALMOST SYMPLECTIC CONJUGATION CRITERION

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Rezumat. *În acest articol vom continua studiul din* [4], *cu aplicaţii la sistemele mecanice hamiltoniene. Vom obţine un criteriu de comparaţie al acestor modele.*

Abstract. *In this paper we will continue the study from [4] , with applications to the Hamiltonian mechanical systems. We will obtain a comparison criterion of these models.*

Key words: locating system**,** anisotropic source, energetic intensity distribution

1. Introduction.

The almost symplectic and the integrable almost symplectic structures play an important role in the theory of the geometrical models of the Hamiltonian mechanical systems.

Let us consider $M_n = (M, [A], R^n)$ a real, n-dimensional C^∞ differentiable manifold, which is paracompact, connected and $\xi = (E = TM, \pi, M)$, the tangent bundle of the M_n manifold and $\xi^* = \left(E^* = T^*M, \overset{*}{\pi}, M \right)$, the cotangent bundle.

In Hamiltonian mechanics, M is called the configuration space, TM is the speed space and T^*M is the phase space.

Generally speaking, a nondegenerate differential 2 form, on a differentiable manifold is an almost symplectic structure. The manifold dimension must be an even number so that the nondegenerate differential 2-form could exist. For ω to be integrable, from the point of view of topology, the manifold must be orientable. We have $\dim TM = 2n$ and $\dim T^*M = 2n$.

An almost symplectic structure defines an isomorphism between TM and T^*M so, for any vectorial field ξ, on M, there is a differential form, α, such that:

$$(1) \qquad \omega(\xi, Y) = \alpha(Y)$$

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Generally speaking, we note

$$\xi = X_\alpha$$

and we can write:

$$(2) \qquad i_{X_\alpha}\omega = \alpha$$

where i is the inner product.

We obtain:

$$(3) \qquad L_{X_\alpha}\omega = d\alpha + i_{X_\alpha}d\omega$$

where L_X is the Lie derivative and d is the exterior differential.

If ω is integrable (ω is a symplectic structure) then

$$d\omega = 0$$

so:

$$(3') \qquad L_{X_\alpha}\omega = d\alpha$$

Example (1). Let us consider H the Hamiltonian associated to a Hamiltonian mechanical system which is a function on the phase space:

$H = E + V$ (E = kinetic energy; V = potential function). We have:

$$(4) \qquad X_{dH}H = (dH)(X_{X_{dH}}) = \omega(X_{dH}X_{dH}) = 0 \qquad \text{(The law of energy)}$$

Let us consider X_α which is tangent to T^*M in $\alpha \in T^*M$ and

$$(5) \qquad \beta(X_\alpha) \overset{def}{=} \alpha(\pi_* X_\alpha). \text{ It results:}$$

$$(6) \qquad \omega = -d\beta; d\omega = 0$$

So ω is an integrable almost symplectic structure.

We obtain a well-known result:

Proposition (1). On the phase space, T^*M, there is an integrable, almost symplectic structure ω.

Example (2).

Because TM is paracompact, connected and C^∞- differentiable and $\dim TM = 2n$, there is a metric structure, g and an almost complex structure $F(F^2 = -I)$.

We define:

$$(7) \qquad G(XY) = g(XY) + g(FX, FY), X, Y \in \mathscr{X}(TM).$$

It results:

$$(8) \qquad G(FX, FY) = G(XY).$$

We obtain:

Proposition (2). There are almost hermitian G structures.

We define ω by:

$$(9) \qquad \omega(XY) = G(X, FY).$$

It results:

$$(10) \qquad \omega(XY) = -\omega(YX)$$

and ω is a nondegenerate, differential 2-form. It will be called natural, associated to (G, F).

2. Natural almost symplectic structures

Let us consider a natural, almost symplectic structure ω, defined by (9)§1.

Proposition (1). We have, for any linear connection D, on $E = TM$:

$$(1) \qquad (D_X \omega)(YZ) = (D_X G)(Y, FZ) + G(Y, (D_X F)(Z))$$

Proof. We have

$$(D_X \omega)(YZ) = X\omega(YZ) - \omega(D_X Y, Z) - \omega(Y, D_X Z) =$$
$$= XG(Y, FZ) - G(D_X Y, FZ) - \omega(Y, FD_X Z) =$$
$$= (D_X G)(Y, FZ) + G(Y, (D_X F)(Z))$$

because

$$(2) \qquad (D_X F)(Z) = D_X F(Z) - F(D_X Z).$$

It results:

Proposition (2). We have:

$$(3) \qquad (D_X \omega)(YZ) = (D_X G)(Y, FZ) \Leftrightarrow D_X F = 0$$

Definition (1). A linear connection**,** D, on TM with the property $DF = 0$ will be called F-connection.

We will denote by $D(F)$ the set of F –connections on $E = TM$.

We have:

$$(3') \quad (D_X \omega)(YZ) = (D_X G)(Y, FZ), \forall D \in D(F)$$

Definition (2). A linear connection, D, with the property

$$(4) \quad D_X F = \rho(X) F \qquad \rho \in \Lambda_1(TM)$$

will be called $\rho - F$ linear connection.

We will denote by $D(\rho, F)$ the set of $\rho - F$-linear connections. It results:

Proposition (3). We have:

$$(5) \quad (D_X \omega)(Y, Z) - \rho(X)\omega(Y, Z) = (D_X G)(YFZ) \quad \forall D \in D(\rho; F)$$

Let us associate the Nijenhuis tensor to the almost complex structure F:

$$(6) \quad N_F(X,Y) = [FX, FY] - F[FX, Y] - F[X, FY] + F^2[X,Y]$$

$$(F^2 = -I)$$

Proposition (4). We have:

$$(7) \quad N_F(X,Y) = (D_{FX} F)(Y) - (D_{FY} F)(X) - F(D_X F)(Y) + F(D_Y F)(X) -$$
$$- \{T(FX, FY) - FT(FX, Y) - FT(X, FY) - T(X, Y)\}$$

Proof. From (6) and from

$$[XY] = D_X Y - D_Y X - T(XY)$$

$$(D_X F)(Y) = D_X F(Y) - F(D_X Y) \text{ it results (7).}$$

Proposition (5). If there exists a linear torsion-free (symmetric) connection, $D \in D(F)$, then the almost complex structure F is integrable.

So F is a complex structure.

Proof. From (7) with $T = 0$ and $DF = 0$, it results $N_F = 0$. So F is a complex structure and, therefore, (G, F) is a hermitian structure.

Proposition (6). If $D \in D(F)$ and $T = 0$ then:

     a)   The structure (G, F) is a hermitian structure.

     b)   We have:

$$(8) \quad (d\omega)(XYZ) = \sum_{(XYZ)} (D_X G)(Y, FZ)$$

Proof. From the general relation

$$(9) \qquad (d\omega)(XYZ) = \sum_{(XYZ)} \{(D_X\omega)(YZ) + \omega(T(XY),Z)\}$$

and from (1), because $T = 0$, it results (8).

Because $D \in D(F)$ and $T = 0$, from (7) we obtain

$$N_F = 0.$$

So F is integrable and therefore, F is a complex structure.

The structure (G, F) is a hermitian structure.

Proposition (7). Let us consider the Levi-Civitta connection defined by $G\left(\nabla G = 0; \overset{*}{T} = 0\right)$.

We have:

$$(10) \qquad (\nabla_X\omega)(Y,Z) = G_X\left(Y,(\nabla_X F)(Z)\right)$$

$$(11) \qquad N_F(X,Y) = (D_{FX}F)(Y) - (D_{FY}F)(X) - F(D_X F)(Y) + F(D_Y F)(X)$$

Proof. From (1) it results (10). From (10) it results (11).

Definition (3). A linear connection D will be called ω-compatible if we have:

$$(12) \qquad D_X\omega = 0 \qquad \forall X. \text{ It results:}$$

Proposition (8). If D is ω-compatible and torsion free then ω is a symplectic structure.

Proof. From (7) it results $d\omega = 0$, so ω is a symplectic structure.

The structure $(G; F, \omega)$ is an almost kählerian structure.

Proposition (9). The Levi-Civitta connection, ∇, is a F-connection if and only if the structure $(G; F, \omega)$ is a kählerian structure.

Proof. Let us consider the Levi-Civitta connection. From (10) if $\nabla F = 0$ it results:

$$(13) \qquad (\nabla_X\omega)(YZ) = 0$$

From (9) (13) it results $d\omega = 0$. From (7) and $\nabla F = 0$ we obtain $N_F = 0$. So $(G; F, \omega)$ is a kählerian structure.

Conversely: If $(G; F, \omega)$ is a kählerian structure, then we have

$$d\omega = 0; \; N_F = 0 \,.$$

We have (10) (11). It results:

$$(14) \quad (\nabla_X \omega)(YZ) = (d\omega)(XYZ) - \frac{1}{2} G(FX, N_F(Y,Z))$$

and therefore

$$\nabla_X \omega = 0 \,.$$

From (10) it results $\nabla F = 0$.

3. Natural almost symplectic conjugations ω-compatible models

Any geometric model of an Hamiltonian mechanical system contains a differentiable manifold E, an almost symplectic structure, ω and a linear connection, D, on E. So it is a space with a linear connection, $\overset{(1)}{D}$, denoted by $\left(E, \overset{(1)}{D} \right)$ equipped with an almost symplectic structure ω. This model will be denoted by $\overset{(1)}{L} = \left(E, \omega; \overset{(1)}{D} \right)$.

If we change $\overset{(1)}{D}$ with $\overset{(2)}{D}$ we will obtain another model, $\overset{(2)}{L} = \left(E, \omega; \overset{(2)}{D} \right)$.

The linear and symmetric connections D on E play an important role in the mathematical modelling. There are such connections. But a linear connection D of the model can be chosen such that D be ω-compatible, i.e. $D\omega = 0$.

In Lagrange models, based on a Riemannian metric G and on a relativistic (pseudoriemannian) metric G, respectively, there is always a linear connection, ∇, which is symmetric and G-compatible ($\nabla G = 0$). This is the already mentioned Levi-Civitta connection.

Proposition (1). Let us consider (E, ω), where ω is an almost symplectic structure.

    a) There are symmetric linear connections D.

    b) There are ω-compatible, linear connections D.

    c) There are no linear connections D, on E, which have both properties a) and b).

Proof. For any linear connection, D, on E, the connection $\overline{D}$, defined by:

$$(1) \qquad \overline{D}_X Z = D_X Z - \frac{1}{2} T(XZ), \forall XZ \in \mathscr{X}(E)$$

is symmetric $\left(\overline{T} = 0\right)$.

We can associate to any connection D on E a new connection, $\overline{D}$, defined by:

$$(2) \qquad \omega\!\left(Y, \overline{D}_X Z\right) = \omega(Y, D_X Z) + \frac{1}{2}(D_X \omega)(YZ)$$

and we have:

$$(3) \qquad \overline{D}_X \omega = 0$$

so $\overline{D}$ is ω-compatible.

But $\overline{D}$ defined by (1), does not have the property $\overline{D}_X \omega = 0$ or $\overline{D}$, defined by (2) does not have the property $\overline{T} = 0$, because from

$$(4) \qquad (d\omega)(XYZ) = \sum_{(XYZ)} \left\{ \left(\overline{D}_X \omega\right)(YZ) + \omega\!\left(\overline{T}(XY), Z\right) \right\} \text{ it will result}$$

$$(5) \qquad d\omega = 0 \text{ if}$$

$$\overline{D}\omega = 0, \overline{T} = 0.$$

So ω will be necessarily integrable.

But generally speaking, we have $d\omega \neq 0$ for an almost symplectic structure.

Definition (1). We will say that we obtain the connection $\overline{D}$ which is ω-compatible, by (2), by the ω-compatibilisation process of the connection D.

Example (1). If ω is natural, i.e. is defined by (9)§1, then the Levi-Civitta connection, ∇, is torsion free: $\left(\overset{\nabla}{T} = 0\right)$ but is not ω-compatible, if ω is not integrable.

Having two models $\overset{(1)}{L} = \left(E, \omega; \overset{(1)}{D}\right)$, $\overset{(2)}{L} = \left(E, \omega; \overset{(2)}{D}\right)$ **of an Hamiltonian mechanical system, there must exist a natural criterion of comparison (compatibility) for them.**

Definition (2). Two 1-dimensional distributions $\overset{(1)}{\underset{(1)}{D}}, \overset{(2)}{\underset{(1)}{D}}$ on E are called ω-conjugated if, in every $u \in E$ is preserved:

$$(6) \qquad \omega\left(\overset{(1)}{V}, \overset{(2)}{V}\right) = 0, \forall \overset{(1)}{V} \in \underset{(1)}{\mathscr{D}}, \overset{(2)}{V} \in \underset{(1)}{\mathscr{D}}$$

at the parallel transport of distribution $\overset{(1)}{\underset{(1)}{D}}$ with respect to $\overset{(1)}{D}$ and of distribution $\overset{(2)}{\underset{(1)}{D}}$, with respect to $\overset{(2)}{D}$.

In this case we will say that the two models $\overset{(1)}{L} = \left(E, \omega; \overset{(1)}{D}\right)$, $\overset{(2)}{L} = \left(E, \omega; \overset{(2)}{D}\right)$ are ω-compatible or $\overset{(1)}{D}$, $\overset{(2)}{D}$ are ω-conjugated. We will write $\overset{(1)}{L} \overset{\omega}{\sim} \overset{(2)}{L}$ or $\overset{(1)}{D} \overset{\omega}{\sim} \overset{(2)}{D}$.

In [5] the first author gives, for the first time, a general theory of the geometries of the spaces with ω-conjugated linear connections. If we apply this theory to the above case, it results:

Proposition (2)([5]). The two-models $\overset{(1)}{L} = \left(E, \omega; \overset{(1)}{D}\right)$, $\overset{(2)}{L} = \left(E, \omega; \overset{(2)}{D}\right)$ are ω-compatible if and only if:

$$(7) \qquad \alpha(X)\omega(YZ) = \left(\overset{(1)}{D}_X \omega\right)(YZ) - \omega\left(Y, \overset{(21)}{\tau}(XZ)\right)$$

$$\forall XYZ \in \mathscr{X}(E); \alpha \in \Lambda_1(E)$$

where

$$(8) \qquad \overset{(21)}{\tau}(XZ) = \overset{(2)}{D}_X Z - \overset{(1)}{D}_X Z$$

It results:

Proposition (3). The two-models $\overset{(1)}{L}, \overset{(2)}{L}$ are ω-compatible if and only if:

$$(9) \qquad \alpha(X)\omega(YZ) = \left(\overset{(2)}{D}_X \omega\right)(YZ) - \omega\left(Y, \overset{(12)}{\tau}(XZ)\right)$$

Proposition (4). The two-models $\overset{(1)}{L}, \overset{(2)}{L}$ are ω-compatible if and only if we have:

$$(10) \qquad \alpha(X)\omega(YZ) = \left(\overset{(2)}{D}_X \omega\right)(YZ) + \omega\left(Y, \overset{(21)}{\tau}(XZ)\right)$$

Corollary (1). If $\overset{(1)}{L} \overset{\omega}{\sim} \overset{(2)}{L}$ then:

$$(11) \quad \overset{(1)}{D}_X \omega + \overset{(2)}{D}_X \omega = \alpha(X)\omega$$

We obtain:

Proposition (5). Two models $\overset{(1)}{L}, \overset{(2)}{L}$ cannot be ω-compatible if $\overset{(1)}{D}, \overset{(2)}{D}$ are ω-compatible $\left(\overset{(1)}{D}\omega = 0; \overset{(2)}{D}\omega = 0 \right)$.

Proof. If $\overset{(1)}{D}\omega = 0; \overset{(2)}{D}\omega = 0$ and we assume that $\overset{(1)}{L} \overset{\omega}{\sim} \overset{(2)}{L}$, it results $\alpha = 0$ and from (7) we obtain:

$$(12) \quad \overset{(21)}{\tau}(XZ) = 0 \text{ therefore } \overset{(1)}{D} = \overset{(2)}{D}, \text{ which is a contradiction.}$$

Proposition (6). Let us consider $\overset{(1)}{L} = \left(E, \omega; \overset{(1)}{D} \right)$, $\overset{(2)}{L} = \left(E, \omega; \overset{(2)}{D} \right)$ such that $\overset{(1)}{T} = \overset{(2)}{T}$.

Then $\overset{(1)}{L}, \overset{(2)}{L}$ are not ω-compatible if $\overset{(1)}{D}$ is ω-compatible or $\overset{(2)}{D}$ is ω-compatible.

Proof. From (7) it results:

$$(13) \quad \alpha(X)\omega(YZ) - \alpha(Z)\omega(YX) = -\omega\left(Y, \overset{(2)}{T}(XZ) - \overset{(1)}{T}(XZ) \right) = 0$$

if $\overset{(1)}{T} = \overset{(2)}{T}$ and $\overset{(1)}{D}\omega = 0$.

Therefore:

$$(14) \quad \omega\big(Y, \alpha(X)_\cdot Z - \alpha(Z)_\cdot X\big) = 0$$

Because ω is nondegenerate, we obtain:

$$(15) \quad \alpha(X)_\cdot Z - \alpha(Z)_\cdot X = 0$$

So $\alpha = 0$.

Because $\overset{(1)}{D}\omega = 0$ it results $\overset{(1)}{D} = \overset{(2)}{D}$, which is a contradiction.

As a corollary we obtain:

Proposition (7). Let us consider two Hamiltonian mechanical models, with the symplectic structure ω $(d\omega = 0)$. These two models are not ω-compatible if $\overset{(1)}{T} = \overset{(2)}{T} = 0$ and $\overset{(1)}{D}$ is ω-compatible.

Example (2). If $\omega = -d\beta$ where β is defined by the phase space, because $d\omega = 0$, then two models $\overset{(1)}{L}, \overset{(2)}{L}$ with symmetric connections are not ω-compatibles if $\overset{(1)}{D}$ is ω-compatible.

Example (3). Let us consider the natural, almost symplectic structure ω defined by (9) §1. Generally speaking, we have $d\omega \neq 0$. So it does not exist a symmetric, ω-compatible connection D, but there exist ω-compatible connections D. We apply Proposition (5) and Proposition (6).

Proposition (8). Let us consider $\overset{(1)}{L} = \left(E, \omega; \overset{(1)}{D} \right)$, $\overset{(2)}{L} = \left(E, \omega; \overset{(2)}{D} \right)$ two models with $\overset{(1)}{T} = 0; \overset{(2)}{T} = 0$. They can be ω-compatible and $\overset{(1)}{D}, \overset{(2)}{D}$ are mutually determined.

Proof. From (7) it results:

$$(16) \qquad \omega\big(Y, \alpha(X)_{\bullet}Z - \alpha(Z)_{\bullet}X\big) = \left(\overset{(1)}{D}_X \omega\right)(YZ) - \left(\overset{(1)}{D}_Z \omega\right)(YX)$$

Because ω is nondegenerate we obtain that α is well-determined and therefore:

$$(17) \qquad \omega\left(Y, \overset{(2)}{D}_X Z\right) = \omega\left(Y, \overset{(1)}{D}_X Z\right) - \left(\overset{(1)}{D}_X \omega\right)(YZ) - \alpha(X)\omega(YZ)$$

where α is given by (16).

In the same way we obtain the invariant:

$$(18) \qquad \omega\big(Y, \alpha(X)_{\bullet}Z - \alpha(Z)_{\bullet}X\big) = \left(\overset{(1)}{D}_X \omega\right)(YZ) - \left(\overset{(1)}{D}_Z \omega\right)(YX) =$$

$$= \left(\overset{(2)}{D}_X \omega\right)(YZ) - \left(\overset{(2)}{D}_Z \omega\right)(YX)$$

and therefore

$$(19) \qquad \omega\left(Y, \overset{(1)}{D}_X Z\right) = \omega\left(Y, \overset{(2)}{D}_X Z\right) + \left(\overset{(2)}{D}_X \omega\right)(YZ) - \alpha(X)\omega(YZ)$$

Based on the above propositions we would can obtain a partition of all the ω-compatible models $\{L = (E, \omega; D)\}$ using the ω-conjugation criterion.

4. The classification criterion of the Hamiltonian models

Let us consider the set of the Hamiltonian models $\underset{(\omega)}{L(D)}$.

The relation „~" is not an equivalence one. It is symmetric one but, generally speaking, it is not a reflexive or transitive one.

Definition (1) ([5]). The model $\underset{(\omega)}{L}\left(D\right)$ is called ω-selfconjugated if D is ω-selfconjugated.

Let us consider the set of the Hamiltonian ω-selfconjugated models $\overset{(1)}{\underset{(\omega)}{L}}$ $\left(\overset{(1)}{\underset{(\omega)}{L}} \subset \underset{(\omega)}{L}(D)\right)$ and the model $\underset{(\omega)}{\overline{L}}\left(\overline{D}\right)$, where $\overline{D}$ is an ω-compatible connection $\left(\overline{D}\omega = 0\right)$.

Proposition (1). If $D \in \overset{(1)}{\underset{(\omega)}{L}}$, noting by $\overline{D}$ the ω-compatibilization of D, it results $\underset{(\omega)}{\overset{(1)}{L}}(D) \overset{\omega}{\sim} \underset{(\omega)}{\overline{L}}\left(\overline{D}\right)$ and conversely.

Proposition (2). On the set

$$(1) \qquad \underset{(\omega)}{C}\left(\overline{D}\right) = \left\{ \overset{(1)}{\underset{(\omega)}{L}}(D) \mid \overset{(1)}{\underset{(\omega)}{L}}(D) \overset{\omega}{\sim} \underset{(\omega)}{\overline{L}}\left(\overline{D}\right) \right\}$$

the relation „$\overset{\omega}{\sim}$" is an equivalence one. It results:

Proposition (3). The set $\overset{(1)}{\underset{(\omega)}{L}}$ admit the partition

$$(2) \qquad \overset{(1)}{\underset{(\omega)}{L}} = \cup \underset{(\omega)}{C}\left(\overline{D}\right)$$

$$(3) \qquad \underset{(\omega)}{C}\left(\overset{(1)}{\overline{D}}\right) \cap \underset{(\omega)}{C}\left(\overset{(2)}{\overline{D}}\right) = \Phi$$

for any ω-compatible linear connections $\overset{(1)}{\overline{D}}, \overset{(2)}{\overline{D}} \mid$.

It results:

Proposition (4). All the Hamiltonian models which are ω-compatible with the model $\left(E, \omega, \overline{D} \mid \overline{D}\omega = 0\right)$ are ω-compatible models (comparable models).

It results :

Proposition (5). Let us consider

$$(4) \qquad \overset{(2)}{\underset{(\omega)}{L}} = \underset{(\omega)}{L}(D) \setminus \overset{(1)}{\underset{(\omega)}{L}}$$

If $\left(E, \omega, \overset{(1)}{D} \right) = \underset{(\omega)}{L_1}\left(\overset{(1)}{D} \right) \overset{\omega}{\sim} \left(E, \omega, \overset{(2)}{D} \right) = \underset{(\omega)}{L_2}\left(\overset{(2)}{D} \right)$ and $\underset{(\omega)}{L_1}\left(\overset{(1)}{D} \right) \subset \overset{(2)}{\underset{(\omega)}{L}}$

then $\underset{(\omega)}{L_2}\left(\overset{(2)}{D} \right) \subset \overset{(2)}{\underset{(\omega)}{L}}$.

Proposition (6). If $\underset{(\omega)}{L_1}\left(\overset{(1)}{D} \right) \overset{\omega}{\sim} \underset{(\omega)}{L_2}\left(\overset{(2)}{D} \right)$ and $\underset{(\omega)}{L_1}\left(\overset{(1)}{D} \right) \subset \overset{(1)}{\underset{(\omega)}{L}}$,

then $\underset{(\omega)}{L_1}\left(\overset{(1)}{D} \right) \equiv \underset{(\omega)}{L_2}\left(\overset{(2)}{D} \right) = \underset{(\omega)}{C}\left(\overline{D} \right),$

where $\overline{D}$ is the common ω-compatibilisation of the linear connections $\overset{(1)}{D}, \overset{(2)}{D}$.

Proposition (7). Let us consider the mean connection $\overset{(m)}{D}$ of two linear connections $\overset{(1)}{D}, \overset{(2)}{D}$.

If

$$(5) \qquad \underset{(\omega)}{L_1}\left(\overset{(1)}{D} \right) \overset{\omega}{\sim} \underset{(\omega)}{L_2}\left(\overset{(2)}{D} \right)$$

then

$$(6) \qquad \underset{(\omega)}{L_1}\left(\overset{(m)}{D} \right) \subset \underset{(\omega)}{C}\left(\overline{D} \right)$$

where $\overline{D}$ is the common c-compatibilisation of the connections $\overset{(1)}{D}, \overset{(2)}{D}$.

Namely the mean connection of any two ω-conjugated linear connection is ω-selfconjugated one.

The mean Hamiltonian model $\underset{(\omega)}{L}\left(\overset{(m)}{D} \right) = \left(E, \omega, \overset{m}{D} \right)$ is always ω-compatible with any model from the class $\underset{(\omega)}{C}\left(\overline{D} \right)$ where $\overline{D}$ is the ω-compatibilisation of the mean connection and also of those connections from respective class.

Generally speaking the mean connection $\overset{(m)}{D}$ is different from $\overline{D}$. An extended study of other situations can be founded in References.

As a summary we have:

Proposition (8). The set of the Hamiltonian models admits the partition:

$$(7) \qquad \underset{(\omega)}{L} = \underset{(\omega)}{\overset{(1)}{L}} \cup \underset{(\omega)}{\overset{(2)}{L}}; \underset{(\omega)}{\overset{(1)}{L}} \cap \underset{(\omega)}{\overset{(2)}{L}} = \Phi$$

where $\underset{(\omega)}{\overset{(1)}{L}}$ admits the partition in equivalence classes (2) (3).

Therefore we have a criterion to compare two Hamiltonian models , given by Proposition (8).

This criterion is a natural one and a very general one because it is related only with the conservation of the ω-conjugation of the directions at the natural parallel transport with respect to these two linear connections.

Some applications of these results to the cases of the tangent bundle TM and cotangent bundle T^*M will guide us to new results in the Hamiltonian theory.

Starting from the general theory which was elaborated by the second author the first author obtain for the natural structure ω on the almost hermitian model of a Generalized Lagrange Space [3] a concrete form of the ω-conjugation and also give simple expressions of the relations between the tensorial d-components of the curvature tensors defined by $\overset{(1)}{D}, \overset{(2)}{D}$, if $\overset{(1)}{D}, \overset{(2)}{D}$ are normal linear d-connections on $E = TM$.

Using these results and the above classification we will obtain new results in the Hamiltonian mechanical systems theory.

We are working on this. The relation between the pseudo-Riemannian conjugations [6] and the ω-conjugations in the cases TM and T^*M is also studied by the authors.

We will highlight the relations between the relativistic models and the Hamiltonian models

A very important problem is to obtain the linear connection transformations $\tau_1 : \overset{(1)}{D} \to \overset{\underline{(1)}}{D}; \tau_2 : \overset{(2)}{D} \to \overset{\underline{(2)}}{D}$ which preserve the ω-compatibility criterion of the Hamiltonian models.

General curvature invariants will corresponds to these transformations.

The authors are working on this and they will present these results into a future paper.

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