





















The unit vector  $\vec{n}$  to the separation surface will have the  $Ox_3$  direction,  $\vec{t}$  will have the  $Ox_1$  direction and  $\vec{\tau}$  will have the  $Ox_2$  direction, so:

$$\vec{n} = \vec{i}_3; (\alpha_1=0; \beta_1=0; \delta_1=1); \vec{t} = \vec{i}_1; (\alpha_2=1; \beta_2=0; \delta_2=0); \vec{\tau} = \vec{i}_2; (\alpha_3=0; \beta_3=1; \delta_3=0). \quad (59)$$

$$[M] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}; [N] = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}; [P] = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}; [R] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}; \quad (60)$$

$$[E] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}; [F] = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}; [G] = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}; [H] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}; \quad (61)$$

With all that, we'll have:

$$\begin{aligned} \sum_{k=1}^2 [[M_k] + [N_k] \cdot [D_k]] &= [I]; \sum_{k=1}^2 [G_k] \cdot [T_k] = [0]; \\ \sum_{k=1}^2 [E_k] \cdot [T_k] &= v_1 [A_1]^{-1} + v_2 [A_2]^{-1} = [\varphi]; \sum_{k=1}^2 [N_k] \cdot [L_k] = [0], \\ \sum_{k=1}^2 [[F_k] + [G_k] \cdot [U_k]] &= -[v_1 [A_1]^{-1} [B_1] + v_2 [A_2]^{-1} [B_2]]^{not.} = -[\psi]; \\ \sum_{k=1}^2 [[H_k] + [G_k] \cdot [U_k]] &= [I]; \end{aligned} \quad (62)$$

$$\begin{aligned} \sum_{k=1}^2 [[P_k] + [R_k] \cdot [D_k]] &= v_1 [B_1]^t [A_1]^{-1} + v_2 [B_2]^t [A_2]^{-1} = [\psi]^t; \\ \sum_{k=1}^2 [R_k] \cdot [L_k] &= v_1 [[C_1] - [B_1]^t [A_1]^{-1} [B_1]] + v_2 [[C_2] - [B_2]^t [A_2]^{-1} [B_2]]^{not.} = [\xi]. \end{aligned}$$

This way (51); (52); (53); (54) takes the form:

$$[I] = [A][\varphi]; [0] = -[A][\psi] + [B]; [\psi]^t = [B'][\varphi]; [\xi] = -[B'][\psi] + [C]. \quad (63)$$

$$\text{So: } [A] = [\varphi]^{-1} = [v_1 [A_1]^{-1} + v_2 [A_2]^{-1}]^{-1}; \quad (64)$$

$$[B] = [\varphi]^{-1} [\psi] = [v_1 [A_1]^{-1} + v_2 [A_2]^{-1}]^{-1} \cdot [v_1 [A_1]^{-1} [B_1] + v_2 [A_2]^{-1} [B_2]]^{-1}; \quad (65)$$

$$[B'] = [\psi]^t [\varphi]^{-1} = [v_1 [B_1]^t [A_1]^{-1} + v_2 [B_2]^t [A_2]^{-1}] \cdot [v_1 [A_1]^{-1} + v_2 [A_2]^{-1}]^{-1}; \quad (66)$$

$$\begin{aligned} [C] &= [\xi] + [\psi]^t [\varphi]^{-1} [\psi] = v_1 [[C_1] - [B_1]^t [A_1]^{-1} [B_1]] + \\ &+ v_2 [[C_2] - [B_2]^t [A_2]^{-1} [B_2]] + [v_1 [B_1]^t [A_1]^{-1} + v_2 [B_2]^t [A_2]^{-1}] \cdot \\ &\cdot [v_1 [A_1]^{-1} + v_2 [A_2]^{-1}]^{-1} \cdot [v_1 [A_1]^{-1} [B_1] + v_2 [A_2]^{-1} [B_2]]. \end{aligned} \quad (67)$$

Where  $v_1$  and  $v_2$  are the volumic ratios for both phases. The matrices  $[A_1]$  and  $[A_2]$  are symmetric. It's easy to verify that:

$$[A]^t = [A]; [B]^t = [B]; [C]^t = [C]. \quad (68)$$

So, the rigidity matrix of the composite material is symmetric and that confirms once more the validity of the hypothesis adopted.

So, we have:

$$E_1^{(c)} = E_2^{(c)} = \frac{v_1^2 E_1^2 (1 - v_2^2) + v_2^2 E_2^2 (1 - v_1^2) + 2v_1 v_2 E_1 E_2 (1 - v_1 v_2)}{v_1 E_1 (1 - v_2^2) + v_2 E_2 (1 - v_1^2)}; \quad (69)$$

$$\nu_{12}^{(c)} = \nu_{21}^{(c)} = \frac{v_1 v_1 E_1 (1 - v_2^2) + v_2 v_2 E_2 (1 - v_1^2)}{v_1 E_1 (1 - v_2^2) + v_2 E_2 (1 - v_1^2)}; \quad (70)$$

$$G_{12}^{(c)} = v_1 G_1 + v_2 G_2; \quad (71)$$

$$\nu_{13}^{(c)} = \nu_{31}^{(c)} = \frac{[v_1 v_1 (1 - v_2) + v_2 v_2 (1 - v_1)] \cdot [v_1 E_1 (1 + v_2) + v_2 E_2 (1 + v_1)]}{v_1 E_1 (1 - v_2^2) + v_2 E_2 (1 - v_1^2)}; \quad (72)$$

$$G_{13}^{(c)} = G_{23}^{(c)} = \frac{G_1 G_2}{G_1 v_2 + G_2 v_1}; \quad (73)$$

$$\frac{1}{E_3^{(c)}} = \frac{2[v_1 v_1 (1 - v_2) + v_2 v_2 (1 - v_1)]^2 \cdot [v_1 E_1 (1 + v_2) + v_2 E_2 (1 + v_1)]}{(1 - v_1)(1 - v_2)[v_1^2 E_1^2 (1 - v_2^2) + v_2^2 E_2^2 (1 - v_1^2) + 2v_1 v_2 E_1 E_2 (1 - v_1 v_2)]} + \frac{v_1 (1 + v_1)(1 - 2v_1)}{(1 - v_1)E_1} + \frac{v_2 (1 + v_2)(1 - 2v_2)}{(1 - v_2)E_2}; \quad (74)$$

$$\begin{Bmatrix} \varepsilon_{11} \\ \varepsilon_{22} \\ \gamma_{12} \\ \gamma_{31} \\ \gamma_{32} \\ \varepsilon_{33} \end{Bmatrix} = \begin{bmatrix} \frac{1}{E_1^{(c)}} & -\frac{\nu_{12}^{(c)}}{E_1^{(c)}} & 0 & 0 & 0 & -\frac{\nu_{13}^{(c)}}{E_1^{(c)}} \\ -\frac{\nu_{21}^{(c)}}{E_2^{(c)}} & \frac{1}{E_2^{(c)}} & 0 & 0 & 0 & -\frac{\nu_{23}^{(c)}}{E_2^{(c)}} \\ 0 & 0 & \frac{1}{G_{12}^{(c)}} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{G_{13}^{(c)}} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{G_{23}^{(c)}} & 0 \\ -\frac{\nu_{13}^{(c)}}{E_1^{(c)}} & -\frac{\nu_{23}^{(c)}}{E_2^{(c)}} & 0 & 0 & 0 & \frac{1}{E_3^{(c)}} \end{bmatrix} \begin{Bmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{12} \\ \sigma_{31} \\ \sigma_{32} \\ \sigma_{33} \end{Bmatrix}; \quad (75)$$

We have to specify that the "(c)" index refers to the entire composite.

Finally, we have, writing (75) in its usual form:

$$\begin{Bmatrix} \varepsilon_{11} \\ \varepsilon_{22} \\ \varepsilon_{33} \\ \gamma_{32} \\ \gamma_{31} \\ \gamma_{21} \end{Bmatrix} = \begin{bmatrix} \frac{1}{E_1^{(c)}} & -\frac{\nu_{12}^{(c)}}{E_1^{(c)}} & -\frac{\nu_{13}^{(c)}}{E_1^{(c)}} & 0 & 0 & 0 \\ \frac{\nu_{21}^{(c)}}{E_2^{(c)}} & \frac{1}{E_2^{(c)}} & -\frac{\nu_{23}^{(c)}}{E_2^{(c)}} & 0 & 0 & 0 \\ -\frac{\nu_{13}^{(c)}}{E_1^{(c)}} & -\frac{\nu_{23}^{(c)}}{E_2^{(c)}} & \frac{1}{E_3^{(c)}} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{G_{23}^{(c)}} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{G_{13}^{(c)}} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{G_{12}^{(c)}} \end{bmatrix} \begin{Bmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{33} \\ \sigma_{32} \\ \sigma_{31} \\ \sigma_{21} \end{Bmatrix}; \quad (76)$$

### Conclusions

The conclusion is that in this case the composite material can be considered as being homogenous and orthotropic having the planes  $Ox_1x_2$ ;  $Ox_1x_3$ ;  $Ox_2x_3$  as planes of elastic symmetry.

### Acknowledgements

The authors acknowledge the Romanian Ministry of Education and Research for financial support under the PNII-CAPACITATI program, Project 126/14.09.2007 and Project 180/3.09.2008.

### REFERENCES

- [1] [1] Gay, D., *Materiaux composites*, Edition Hermes, Paris, 1989.
- [2] [2] Barrau, J.J., Laroze, S., *Calcul des structures en materiaux composites*, Tome 4, Ecole Nationale de l'aéronautique et de l'espace, Toulouse, 1987.
- [3] [3] Rizescu, S., *Structuri spațiale cu elemente compozite*, Editura Universitaria, Craiova, 2004.
- [4] [4] Bolcu, D., Stănescu, G., Ursache, M., *Theoretical and Experimental Study on the Determination of the Elastic Properties of composite Materials*, Romanian Reports in Physics, 57, 3-13, 2004.