COMPLEXITY APPROACH OF OPTICAL COMMUNICATIONS SYSTEMS

Dan-Alexandru IORDACHE[1], Daniela RADU[2], Octavian RADU[3]

Rezumat. *Sistemele actuale de comunicaţii optice au fost studiate pentru a constata existenţa (sau absenţa) principalelor caracteristici ale sistemelor complexe pentru componentele lor, respectiv pentru înşişi aceste sisteme ca reţele. A fost studiată prezenţa: a) legilor de tip putere, b) posibilităţilor de ieşire din stările de haos (îndeosebi prin formarea pulsurilor solitonice), c) altor relaţii specifice transformărilor de fază (ex. a relaţiilor de tipul Arrhenius), d) relaţiilor specifice proceselor de creştere (extinderea reţelelor de comunicaţii), etc. Constatările rezultate pot fi utilizate pentru perfecţionarea acestor sisteme, spre ex. pentru mai buna menţinere a profilului pulsurilor solitonice în cursul propagării lor.*

Abstract. *The present optical communications systems were studied in order to find if their components and themselves as a network system have the basic features of the complex systems. There were studied the presence of the: a) power laws, b) specific exits from chaos states (as those corresponding to solitary pulses), c) other relations specific to phase transforms (as the Arrhenius' ones), d) the typical growth relations (for the communications systems development), etc. The resulted findings can be used for the improvement of these systems, e.g. for a better maintenance of the solitary pulses profiles during their propagation.*

Key words: Optical communications systems, Power laws, Solitons, Arrhenius' relations, Communications systems extension

1. Introduction

While the first modern studies of the Information transmission systems appeared in 1928 [1], the first syntheses of the mathematical theories of Communications, of the Information and Complexity, resp. were elaborated by Claude E. Shannon [2] and Warren Weaver [3].

The study of the main modern treatises [4] on the optical transmission of information points out that the: a) solitonic signals present the lowest energy losses (that could be theoretically null in some conditions), b) optical fibers ensure also reduced energy losses, as well as a high rate of the transmitted information, c) lasers with rare earths ions ensure optical signals compatible with the best present optical fibers (those using the silicon dioxide, or with fluorides, resp.), with

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[1]Professor, Physics Department, University "Politehnica" of Bucharest, Romania.
[2]Professor MSci, Power High-school College, Bucharest, Romania.
[3]Professor MSci, Theoretical High-School "Decebal", Bucharest, Romania.

multiple variants (Er^{3+}, Nb^{3+} ions, etc) in the optical range of interest of the optical fibers. For this reason, the communications systems using the transmission of some solitonic signals through optical fibers doped with rare earths ions, represent now the most efficient manner of information transmission. The principle scheme of a such type of optical communications is presented by Fig. 1.
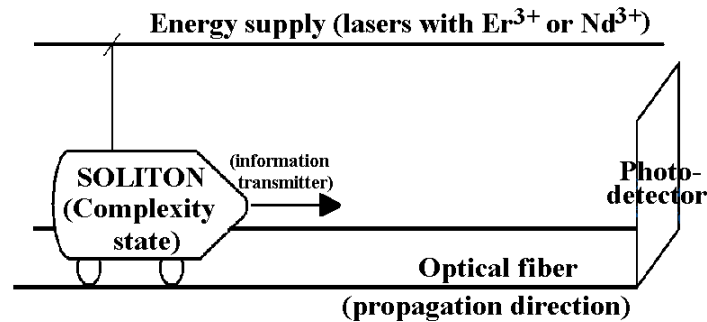
Fig. 1. Block scheme of a modern optical communications system

The detailed scheme of one of the first modern systems of optical communications using the propagation of some solitonic pulses through optical fibers [5] is presented by Fig. 2.
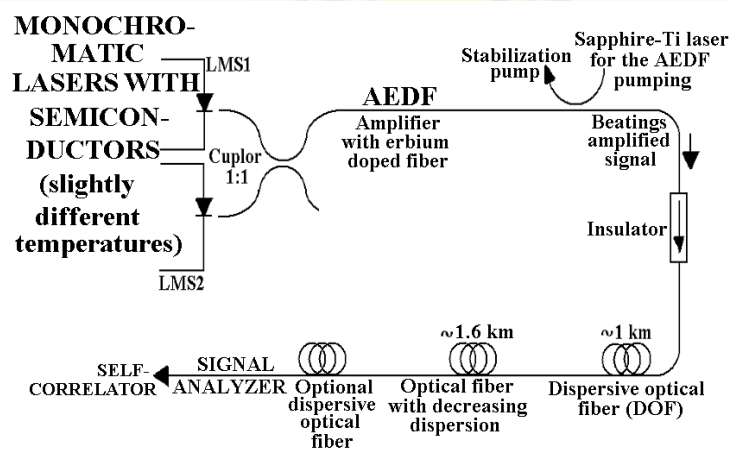
Fig. 2. Structure of an opto-electronic system used for the generation (with the frequency of 70 GHz) of solitonic pulses of 1.3 ps [5]

2. Basic Complexity features of physical systems

P. W. Anderson [6] explained the macroscopic behavior of the complex systems by means of *the appearance of some discontinuities* ("*seeds*", as the vortices from the turbulent flows, the genetic mutations, etc) *as a consequence of a Spontaneous Symmetry Breaking* [7]. *The stochastic auto-catalytic growth* of these "seeds" is correlated with other important features of the complex systems, as their *fractal scaling*, *the power laws*, etc. [7], [8], while another Complexity features referring

to the *self-organization in systems in non-equilibrium states, their chaotic behaviors*, *the transition towards the ordered la states* (to some solitonic states, particularly) etc. were studied by I. Prigogine [9].

The detailed quantitative description of the complex physical processes was achieved by K. G. Wilson[4], by means of his method of the "re-normalization group" [10], consisting in the successive integration of fluctuations, starting from the atomic level fluctuations and continuing for the higher levels of matter organization, up to fluctuations averaging for all matter organization levels.

It results that some notions, organized in universal sequences (i.e.: microscopic discontinuity ≡ "seed" → auto-catalytic growth → power laws → fractals, etc), allow the description of properties of complex systems of very different natures. This finding points out the existence of some *Universality features* [11] (described by numbers, as the so-called *similitude criteria* [12]) that govern the structures and evolutions of complex systems of arbitrary natures[5].

Among the numerous typical features of the complex systems, we consider as most characteristic: a) the preferential use of (similitude) numbers in the description of complex systems, which determines the fundamental role of *the power laws* in the description of such systems [13], b) *the phase transitions*, with their *associated Arrhenius' type relations*, etc. c) *the type* (fractal, statistical, solitonic, etc) *of the order installed at the exit from the chaos states* represents another essential feature of the complex systems, with very important implications for the modern optical communications systems, d) *the development (growth) features*, etc.

The above basic complexity features can be identified both for the components of the modern optical communications systems, as well as for the whole communication system as a (complex) network. Of course, we will examine in following only some typical complexity features of the basic components and of the entire network.

3. Typical power laws intervening in the photo-detectors functioning

a) Even the most usual photo-detector – *the human eye* presents a complex character, attested by Stanley's power law [14] , correlating the intensity I of the visual excitation and the produced senzation S: $S = k \cdot I^n$, (1)
where the exponent n is an irrational number, dependent on the excitation type.

b) Similarly, for very strong electrical fields ($\sim 10^8$ *V/m*), the width of the forbidden band corresponding to the *photo-diodes with avalanche* (used as photo-

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[4]Physics Nobel prize laureate in 1982, for "his theory of the **critical phenomena** in connection with the **phase transitions**".

[5]**The category of complex systems** involve not only the physical and the technical ones, but also the biological, the social, the economic systems, etc.

detectors) reduces to very small values $w = \dfrac{E_g}{E} \sim 100$ Å (see Fig. 3), determining so high tunneling probabilities (of the potential barrier between the valence band and the conduction one), and the appearance of the breakdown current for rather small ($< \sim 5$ V) breakdown voltages (*the Zener effect*, see [14]).
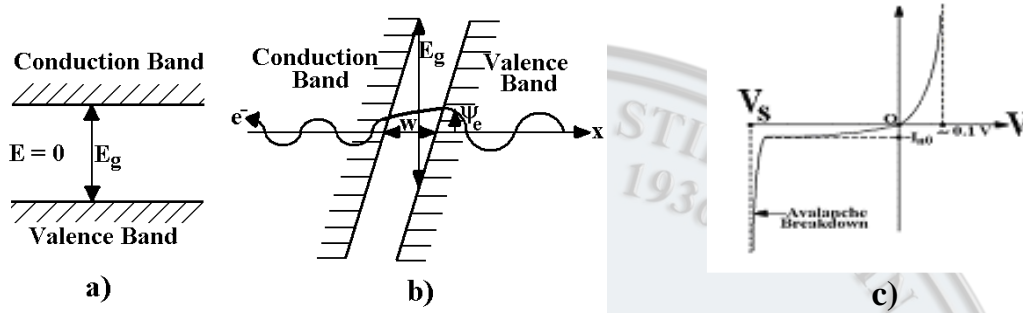

Fig. 3. Mechanism of the Zener's type tunnel effect and the associated avalanche breakdown.

For these photo-detectors (avalanche photo-diodes), the average number of pairs electrons-holes produced at each collision in avalanche (the multiplication factor, *M*), is given by the power law: $1 - \dfrac{1}{M} = \left(\dfrac{V}{V_s} \right)^n$, (2)

where V [V_b = the breaking (polarization) voltage] is the inverse bias (see fig. 4) and *n* is the irrational number associated to this power law.

c) *The display devices* (of received signals) with liquid crystals have also a complex character, which leads a certain power law correlating the inter-molecular potential u_i on the molecular volume V (see e.g. [15], pp. 92-96):

$$u_i \propto V^{-m} ,$$ (3)

where *m* is again an irrational number (different of the classical theoretical value: $m = 2$). This power law is corresponds to the transition between the nematic and the isotropic phases of the liquid crystal.

4. Arrhenius' type relations

As it is well-known [16], the rate *v* of a di-molecular reaction $A \rightarrow B$ is given by the expression: $v = -\dfrac{d[A]}{dt} = \dfrac{d[B]}{dt} = k \cdot [A] \cdot [B]$, where $[A], [B]$ are the concentrations of the 2 substances at time *t*, and *k* is the rate constant of the considered reaction. The temperature dependence of the reaction constant *k* is given by *the Arrhenius' relation*: $k(T) = A \cdot \exp\left(-\dfrac{E_a}{RT} \right)$, (4)

where E_a is the activation energy of the considered chemical reaction.

Relations of the Arrhenius' type are frequently met also in the description of some processes intervening in the manufacturing technologies of the optical guides, e.g. in the cases of the: a) *manufacturing technology of the optical guides with Er:Ti:LiNbO₃*, described by the Fick's law:

$$\frac{\partial}{\partial x}\left(D\frac{\partial C}{\partial x}\right) = \frac{\partial C}{\partial t} \,, \tag{5}$$

the temperature dependence of the diffusion coefficient D being given by the Arrhenius' relation [17]:

$$D(T) = D_0 \cdot \exp\left(-\frac{E_a}{RT}\right), \tag{6}$$

b) *the manufacturing technology of the optical guides by means of an ionic exchange between a salt involving the ion* 1 *and glass, involving the ion* 2, when the diffusion Fick's law is:

$$\frac{\partial}{\partial x}\left(\bar{D} \cdot \frac{\partial C_i}{\partial x}\right) = \frac{\partial C_i}{\partial t} \,, \tag{6}$$

with [17]:

$$\vec{D} = \frac{D_1 D_2 (C_1 + C_2)}{D_1 C_1 + D_2 C_2} \,, \text{ and: } D_i(T) = D_{i0} \cdot \exp\left(-\frac{E_a}{RT}\right). \tag{7}$$

The analogy of relations (4), (6) and (7) corresponds to the presence of phase transitions, hence of the complex character of the optical guides with "implant" of ions in their substrate (glass, lithium niobium, etc). We have to underline that the diffusion and the depletion dark currents in the metal-oxide-semiconductor (MOS) photo-sensors present also a temperature dependence described by Arrhenius' relations [18].

5. Specific features of the non-linear Schrödinger solitons used by some optical communications systems

Unlike the mechanical solitons, which are sometimes naturally obtained (in complete agreement with the corresponding propagation medium, e.g. [19]), the optical ones are usually obtained by the modulation of electromagnetic waves of high frequency [4e], hence they are "artificial" solitons, in incomplete agreement with the specific parameters of their propagation medium. For this reason, the optical solitons (non-linear Schrödinger, particularly) are subject to a certain (rather small, but non-negligible for large distances) attenuation, described by the equation [17]:

$$i\frac{\partial q}{\partial X} + \frac{1}{2}\cdot\frac{\partial^2 q}{\partial T^2} - |q|^2 \cdot q = -i\Gamma \cdot q, \tag{8}$$

where Γ is the non-dimensional losses coefficient.

The optical amplification processes (as those affecting the Stokes' components in frame of the Brillouin's and Raman's effects, or corresponding to lasers when the optical fibers are doped with Er^{3+} ions, etc) are described by a term of opposite sign to that corresponding to losses, hence the non-linear Schrödinger equation of

optical solitons through a real optical fiber (implying both attenuation and optical amplification processes of solitons) becoming:

$$i\frac{\partial q}{\partial X}+\frac{1}{2}\cdot\frac{\partial^2 q}{\partial T^2}-|q|^2\cdot q=-i\Gamma\cdot q+iG(X)\cdot q \, ,\tag{9}$$

where $G(X)$ is the non-dimensional coefficient of the optical gain (amplification), dependent on the non-dimensional distance X in the optical fiber.

The achievement of equilibrium of the losses and gain terms from the right part of equation (9) ensures the maintenance of the solitons shape and amplitude.

Additional aspects referring to: a) the computer simulations of the optical solitons propagation through different media and: b) the procedures intended to their parameters maintenance, were studied by our recent work [20].

6. Complex character of the modern optical communications networks

The usual theoretical models of the solid samples take into consideration the crystalline lattices formed by a limited number (small, usually) of micro-particles types (atoms, ions) among whom are exerted local interactions (with the nearest neighbors). When both the interactions at small distance, as well as those at rather large distances are taken into consideration, the specific statistical models (Néel, Ising, Heisenberg) lead to some power laws, e.g. of the Domb-Fisher's type for ferri-magnetic materials [21].

A high interest corresponds also to the complex networks formed by a large number of different elements, presenting specific interactions at distance (described by topological networks whose vertices represent the network elements, while their sides correspond to the interactions among these elements). Such networks involve the complex systems from table 1 [22a], the modern optical communications systems, inclusively.

The basic theoretical models of the complex networks (random graphs) can be classified as: a) *static models* (the Erdös-Renyi's (*ER*) models [23] and the Watts-Strogatz's (*WS*) one [24], mainly), b) *the linearly growing models* (the Barabási-Albert's (*BA*) model [22a-c]), c) *the generalized growing models* [22d].

Conclusions

The accomplished study pointed out the complex character both of the components of the modern optical communications systems, as well as of the whole network.

The obtained findings could contribute to a better knowledge of the components of the modern optical communications systems, as well as of the basic features of the corresponding complex networks development.

Type of the complex network	Value of the power law exponent	Correspondence topology – network specific nature	
		Vertices nature (typical examples)	Network sides nature (typical examples)
Biological networks (systems)		Proteins	Chemical interactions between proteins
Nerves system		Nerves cells	Axons
Social networks		a) private people, b) organisations, c) countries	Interactions between them
Economy networks		Companies	Business relations
Transport networks	$\gamma_{electr.\ power} \cong 4$	Cities	Vehicles or electric lines among them
Internet (web pages)	$\gamma_{exits} \cong 2.45$ $\gamma_{entries} \cong 2.1$	HTML documents	Links between some web pages
Scientific networks	$\gamma_{citations} \cong 3$	Scientific researchers	Common works or citations (oriented graphs)
Artistic networks	$\gamma_{actors} \cong 2.4$	Actors	Common movies of actors

Table 1. Features of the main types of complex networks

R E F E R E N C E S

[1] R.V.L. Hartley, *Transmission of Information*, in Bell System Techn. J., July 1928.

[2] a) C.E. Shannon, W. Weaver, *The mathematical theory of communications*, Urbana, Univ. of Illinois Press, 1949; b) C.E. Shannon, W. Weaver, *The Mathematical Theory of Information*, Urbana, University Illinois Press, 1963.

[3] W. Weaver, *Science and Complexity*, in American Scientist, **vol. 36**, 1968, p. 536.

[4] a) P. Sterian, *Transmisia optică a informaţiei*, Editura tehnică, Bucureşti, 2 vol., 1981; b) John M. Senior, *Optical Fiber Communications*, Prentice Hall International Series in Opto-electronics, 1985; c) Harry J. R. Dutton, *Understanding Optical Communications*, Prentice Hall International Technical Support Organization, 1998; d) G. Keiser, *Optical Fiber Communications*, McGraw Hill, 2001; e) P. Sterian, *Bazele optoelectronicii*, Editura Printech, Bucureşti, 2002; f) S. Şişianu, T. Şişianu, O. Lupan, *Comunicaţii prin fibre optice*, Editura Tehnica-Info, Chişinău, 2003; g) K.S. Schneider, *Fiber Optic Data Communications for the Premises Environment*, chap. 2, http://www.telebyteusa.com/foprimer/foch2.htm#2.2

[5] S.V. Cernikov, D.J. Richardson et al, in Electronic Letters, **vol. 28**, no. 13, 1992, p. 210.

[6] a) *P. W. Anderson,* "More is different", in Science, **vol. 177,** 1972, p. 293; b) *P. W. Anderson*, Proc. Natl. Acad. Science (USA), **vol. 92,** 1995, pp. 6653-6654.

[7] a) S. Solomon, E. Shir, Europhysics News, **vol. 34**, no. 2, 2003, pp. 54-57; b) S. Solomon, Annual Reviews of Comp. Physics II, pp. 243-294, D. Stauffer ed., World Scientific, 1995.

[8] a) A.N. Kolmogorov, C.R. Acad. Sci. USSR, **vol. 31,** 1941, p. 538 (translated in S. K. Friedlander, L. Topper, eds., Turbulence Classic Papers on Statistical Theory, Interscience Publ., New York, 1961); b) A.N.Kolmogorov, *J. Fluid Mechanics*, **vol. 13**, 1962, p. 82.

[9] a) I. Prigogine, G. Nicolis, *Self-organization in Non-equilibrium systems: from dissipative structures to order through fluctuations*, J. Wiley and Sons, New York, 1977; b) I. Prigogine, G. Dewel, D. Kondepudi, *Chemistry Far from Equilibrium: Thermodynamics, Order and Chaos*, in New Chemistry (Hall N., editor), Cambridge University Press, Cambridge, 2001.

[10] K.G. Wilson, *Renormalization group and critical phenomena*, in Phys. Rev. B, **vol. 4,** 1971, pp. 3174, 3184.

[11] G. B. West, J. H. Brown, Physics Today, **vol. 57**, no. 9, 2004, p. 26.

[12] a) A. A. Gukhman, *Introduction to the Theory of Similarity*, Academic Press, New York, 1965; b) G.I. Barenblatt, *Dimensional Analysis*, Gordon and Breach, New York, 1987; c) G.I. Barenblatt, *Scaling, Self-Similarity and Intermediate Asymptotics*, Cambridge Texts in Applied Mathematics, 1996.

[13] D. Iordache, P.P. Delsanto, *Could be active in the nature sciences – the Complexity theory*?, in Annals of the Academy of Romanian scientists, Series on Science and Technology of Information, in print.

[14] P. Sterian, *Transmisia optică a informaţiei*, Editura tehnică, Bucureşti, vol. 2, 1981.

[15] C. Moţoc, I. Muşcutariu, *Introducere în fizica cristalelor lichide*, Edit. Facla, Timişoara, 1986.

[16] Linus Pauling, *General Chemistry*, 3rd edition, W. H. Freeman, San Francisco, 1970.

[17] N. Puşcaş, *Sisteme de comunicaţii optice*, Editura Matrix Rom, Bucureşti, 2006.

[18] R. Widenhorn, M.M. Blouke, A. Weber, A. Rest, E. Bodegom, *Temperature dependence of dark current in a CCD*, in Proc. SPIE, **vol. 4669**, 2002, pp. 193-201.

[19] a) J.S. Russell, *Report on waves*, in Report of the 14th Meeting of the British Association for the Advancement of Science, John Murray, London, 1844; b) V. Chiroiu, L. Munteanu, *Solitons in mechanics of solids, part I: The elastic rod,* in Rev. Roum. Sci. Techn. – Méc. Appl., **vol. 38,** no. 6, 1993, pp. 587–600; part II: *A one-dimensional wave problem,* ibid., **vol. 39**, no. 1, 1994, pp. 3–10; c) L. Munteanu, Şt. Donescu, *Introduction to Soliton Theory: Applications to Mechanics* (Book Series Fundamental Theories of Physics), vol. 143, Kluwer Academic Publishers, 2004.

[20] O. Radu, D. Radu, D. A. Iordache, *Studiul sistemelor de comunicaţii optice folosind semnale solitonice*, în Evrika, **vol. 19**, nr. 10(218), 2008, pag. 51-57.

[21] a) C. Domb, M. F. Sykes, Proc. Roy. Soc., A, **vol. 240,** 957, p. 214; b) M.E. Fisher, *Physica*, **vol. 25**, 1959, p. 521; c) C. Domb, M. F. Sykes, J. Math. Phys., **vol. 2**, 1961, p. 63; d) C. Domb, *M. F. Sykes*, Phys. Rev., **vol. 128,** 1962, p. 168.

[22] a) A. L. Barabási, R. Albert, H. Jeong, *Mean-field theory for scale-free random networks*, in Physics A, **vol. 272**, 1999, pp. 173-187; b) A.L. Barabási, R. Albert, Science, **vol. 286**, 1999, p. 509; c) R. Albert, H. Jeong, A. L. Barabási, Nature, **vol. 401**, 1999, p. 130; d) R. Albert, A.L. Barabási, *Topology of complex networks: local events and universality*, Physical Review Letters, **vol. 85**, 2000, pp. 5234-5237.

[23] a) P. Erdös, A. Rényi, Pub. Math. Inst. Hung. Acad. Sci., **vol. 5**, 1960, p. 17; b) B. Bollobás, Random graphs, Academic Press, London, 1985.

[24] D.J. Watts, S.H. Strogatz, Nature, **vol. 393**, 1998, p. 440.