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ON THE FRACTAL NATURE OF INTERNET

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Rezumat. *Lucrarea analizează două căi de a demonstra natura fractală a internetului. În primul rând, prezintă autosimilaritatea traficului pe Internet şi propune un model fractal pentru acesta. În al doilea rând, propune un model independent de scară a topologiei Internet-ului. În continuare, autorii demonstrează că structura fractală a topologiei influenţează comportarea fractală a traficului pe Internet. Această afirmaţie este susţinută prin câteva rezultate experimentale.*

Abstract. *The paper analyses two ways to demonstrate the fractal nature of Internet. First, it presents the self-similarity of the Internet traffic and proposes a fractal model of this traffic. Secondly, it proposes a scale-free model of the Internet topology. Furthermore, the authors demonstrate that the fractal structure of the topology influences the fractal behaviour of the Internet traffic. This assertion is sustained by some experimental results.*

Keywords: fractal, self-similarity, traffic model, free-scale networks

1. Introduction

Traffic flowing through the telecommunication networks in the pre-internet age was predominantly 'voice'. The number of calls arriving at a station, namely the counting process, approximated a Poisson or renewal process. In either case arrivals were memory less in the Poisson case, or memory less at renewal points, and interarrival intervals were exponentially distributed. The Poisson arrival model and exponentially distributed holding time model allowed analytically and computationally simple Markov chains to be used for much of the telephone traffic modeling. An M/M/1/K chain can be used to accurately model a single server finite queue system with exponential service and Poisson arrivals yielding closed form solutions for queue length distribution, waiting time distribution, blocking probability etc.

Internet traffic, which behave very differently from such simple Markovian models. Traffic measurements made at the Local Area Networks (LAN) and Wide Area Networks (WAN) suggest that traffic exhibits variability (traditionally called 'burstiness') over multiple time scales [1]. The second order properties of the counting process of the observed traffic displayed behavior that is associated with self-similarity, multi-fractals and/or long range dependence (LRD). This indicates that there is a certain level of dependence in the arrival process.

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Near-range and long-range dependencies often manifest themselves in a network by causing frequent and irremediable packet losses and other serious effects in the network.

Dependencies and burstiness in traffic hence brought in an enormous amount of attention from researchers. They attempted to develop mathematically-based models that would help explain the nature of the systems exhibiting such phenomena and provide critical insight into the actual mechanisms that led to this behavior. Models like fractional Brownian motion, chaotic maps etc. were suited to capture the second order self-similar behavior of traffic [2]. Their results were difficult to get and harder to apply, and such models did not provide insight into the actual mechanism of traffic generation. Many analytically simpler modeling attempts to capture the first and second order properties of counts did not predict the queuing behavior well enough. In the late 90s researchers discussed the impact of other properties of the self-similar process, such as marginal distributions, in accurately predicting the queuing behavior. A simpler, more accurate and analytically tractable model that provides more physical insight into why they are meaningful on physical grounds would help the network designers produce more effective and efficient designs.

Some ideas generated in the last decade offer promise towards crafting the model. Anderson and Nielsen [3] illustrated that continuous parameter Markov chains (cpMc) can model the dependencies in network traffic over multiple time scales; the advantage of such models is the availability of ready-made tools for analysis. Their model matched the second order properties of the self-similar process closely, but it was not sufficient for accurate prediction of queuing behavior. Grossglauser and Bolot [4] discussed both the importance of limiting the view to the finite range of time scales of interest, and the influence of marginal distributions in performance evaluation and prediction problems. From the above discussions, one can infer that both the second order and marginal properties of the process need to be matched for more accurate results. Salvador et al. [5] achieved some degree of success by using a fitting procedure that matched both the marginal distribution and auto covariance of the counting process, but a solution form that provides deep insight into the system was still missing.

The Internet is a prime example of a self-organizing complex system, having grown mostly in the absence of centralized control or direction. In this network, information is transferred in the form of packets from the sender to the receiver via routers, computers which are specialized to transfer packets to another router "closer" to the receiver. A router decides the route of the packet using only local information obtained from its interaction with neighboring routers, not by following instructions from a centralized server. A router stores packets in its finite queue and processes them sequentially.

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However, if the queue overflows due to excess demand, the router will discard incoming packets, a situation corresponding to congestion. A number of studies have probed the topology of the Internet and its implications for traffic dynamics. [6,7].

To efficiently control and route the traffic on an exponentially expanding Internet, one must not only capture the structure of current Internet, but allow for long-term network design. Until recently all Internet topology generators provided versions of random graphs but in 1999 discovery of Faloutsos [8]: Internet is a scale-free network with a power-law degree distribution. Several contributors found that the Internet flow is strongly localized: most of the traffic takes place on a spanning network connecting a small number of routers which can be classified either as “active centers,” which are gathering information, or “databases,” which provide information. Experimental evidence for self-similarity in various types of data network traffic is already overwhelming and continues to grow. So far, simulations and analytical studies have shown that it may have a considerable impact on network performance that could not be predicted by the traditional short-range-dependent models. The most serious consequence of self-similar traffic concerns the size of bursts. Within a wide range of time-scales, the burst size is unpredictable, at least with traditional modeling methods.

This is the point from which the authors of this paper assume that the traffic behavior is strong influenced and depends of the network free-scale structure. We have also demonstrated that the scale-free Internet model displays a number of properties that distinguishes it from random graphs: wiring redundancy and clustering, non-trivial eigenvalue spectra of the connectivity matrix and a scale-free degree distribution.

2. Evidence of traffic self-similarity

2.1. *General considerations*

Using a number of experiments, the following results towards characterizing and quantifying the network traffic processes have been achieved:

First, self-similarity is an adaptability of traffic in networks. Many factors are involved in creating this characteristic. A new view of this self-similar traffic structure is provided. This view is an improvement over the theory used in most current literature, which assumes that the traffic self-similarity is solely based on the heavy-tailed file-size distribution.

Second, the scaling region for traffic self-similarity is divided into two timescale regimes: short-range dependence (SRD) and long-range dependence (LRD). Experimental results show that the network transmission delay (RTT time) separates the two scaling regions.

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This gives us a physical source of the periodicity in the observed traffic. Also, bandwidth, TCP window size, and packet size have impacts on SRD. The statistical heavy-tailedness (Pareto shape parameter) affects the structure of LRD. In addition, a formula to quantify traffic burstiness is derived from the self-similarity property.

Furthermore, studies of fractal traffic with multifractal analysis have given more interesting and applicable results. (1) At large timescales, increasing bandwidth does not improve throughput (or network performance). The two factors affecting traffic throughput are network delay and TCP window size. On the other hand, more simultaneous connections smooth traffic, which could result in an improvement of network efficiency. (2) At small timescales, traffic burstiness varies. In order to improve network efficiency, we need to control bandwidth, TCP window size, and network delay to reduce traffic burstiness. There are the tradeoffs from each other, but the effect is nonlinear. (3) In general, network traffic processes have a Hölder exponent α ranging between 0.7 and 1.3. Their statistics differ from Poisson processes. To apply this prior knowledge from traffic analysis and to improve network efficiency, a notion of the *efficient bandwidth*, EB, is derived to represent the fractal concentration set. Above that bandwidth, traffic appears bursty and cannot be reduced by multiplexing. But, below it, traffic is congested. An important finding is that the relationship between the bandwidth and the transfer delay is nonlinear.

The past few decades have seen an exponential growth in the amount of data being carried across packet switched networks, and particularly the Internet. This growth has brought packet switched networks to the point where the amount of traffic being carried on them is expected to exceed that carried on traditional circuit switched technology in the very near future. Packet switched networks are not new. They have been around for over 30 years. During that time, a number of models for the traffic carried across them have also been proposed. Early attempts at odeling network traffic odelin on Markovian models, such as the Markov-Modulated Poisson Process (MMPP) [9]. Markovian models were familiar to teletrafficists, due to their long association with the odeling of telephony traffic, and have the advantage of being generally tractable.

In recent analyses of traffic measurements, evidence of non-Markovian effects, such as burstiness across multiple time scales, long range dependence and self similarity; have been observed in a wide variety of traffic sources. As is clearly shown in [10, 11, 12], the performance of processes exhibiting these properties is radically different from that of the traditional models. Given the evidence of long range dependence and self-similarity in such a wide variety of sources, it is clear that any general model for data traffic must account for these properties. This has led to the development of a number of new models.

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2.2. *Testing a fractal traffic model*

Mandelbrot and his co-workers introduced an analogy between *self-similar* (SS) processes and *fractal* processes [8]. Referring directly to the incremental process $X_{s,t} = X_t - X_s$, he defines stochastic self-similarity as:

$$X_{t0,t0+rt} = r^H X_{t0,\, t0+t}, \qquad \forall t_0,\, t,\ \forall r>0 \tag{1}$$

Mandelbrot constructs his SS process (*fractional Brownian motion*, fBm) starting with two properties of the Brownian motion (Bm): it has independent increments and it is self-similar with Hurst parameter H= 0.5

Denoting Bm as $B(t)$ and fBm as $B_H(t)$, here is a simplified version of Mandelbrot's definition of the fBm: $B_H(0) = 0$, $H \in [0,1]$ and

$$B_H(t) = \frac{1}{\Gamma(H+0.5)} \left\{ \int_{-\infty}^{0} \left[(t-s)^{H-1/2} - (-s)^{H-1/2} \right] dB(s) + \int_{0}^{t} (t-s)^{H-1/2} dB(s) \right\} \tag{2}$$

An SS process is called a *long-range dependence* (LRD) process if there are constants $\alpha \in (0,1)$ and $C > 0$ such that

$$\lim_{k \to \infty} \frac{\rho(k)}{C k^{-\alpha}} = 1 \tag{3}$$

where $\rho(k)$ is the autocorrelation of lag k.

When represented in logarithmic coordinates, eq. (3) is called the *correlogram* of the process, and has an asymptote of slope $-\alpha$. It is to note that there are SS processes which are not LRD and, conversely, there are LRD processes which are not SS. However, the fBm with $H > 0.5$ is both SS and LRD type.

In his landmark paper [1], Leland at al. report the discovery of self-similarity in local area network (LAN) traffic, more precisely Ethernet traffic. To be precise, we note that all methods used in [1] (and in numerous papers that followed) detect and estimate LRD rather than SS. Indeed, the only "proof" offered for SS *per se* is the visual inspection of the time series at different time-scales. "Self-similarity" (actually LRD) has since been reported in various types of data traffic: LAN, WAN, Variable-Bit-Rate video, SS7 control, HTTP etc. Lack of access to high-speed, high-aggregation links, and lack of devices capable of measuring such links have until recently prevented similar studies from being performed on Internet *backbone* links. In principle, traffic on the backbone could be qualitatively different from the types enumerated above, due to factors such as much higher level of aggregation, traffic conditioning (policing and shaping) performed at the edge, and much larger round-trip-time (RTT) for TCP sessions. Actually, some researches have even claimed that aggregating Internet traffic causes convergence to a Poisson limit.

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For reasons presented in the next section and based on the remarks that on *shorter time scales*, effects due to the network transport protocols are believed to dominate traffic correlations and on *longer time scales*, non-stationary effects such as diurnal traffic load patterns become significant, we disagree.

In our tests we have simulated link speeds ranging from 10 Mbps to 622 Mbps, average bandwidths between 1.4 and 42 Mbps, minimum time-scale of 1ms (in only one instance – usually above 10-100ms), and at most 6 orders of magnitude for time-scales. The correlograms (see fig.1) shown that traffic considered specific for the Internet backbone is indeed *asymptotically* SS, and also reported a new autocorrelation structure for short lags. The autocorrelation function for short lags has the same power form as for long lags, i.e. $\rho(k) \sim k^{-\alpha}$, but the parameter α turns out to assume values which are significantly larger: $\alpha \in [0.55, 0.71]$ for $k \in [50\mu s, 10ms]$, compared to $\alpha \in [0.1, 0.18]$ for $k \in [100ms, 500s]$.

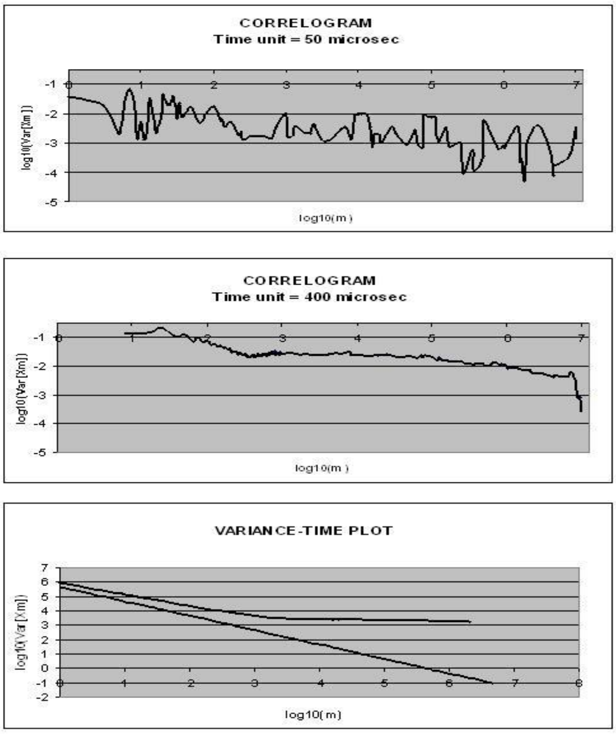

Fig. 1. Graphic representation of a network with 200 nodes

The first plot in fig.1 shows the correlogram for the shortest time unit used in our analysis. Although the linear trend is clearly present, the dependence is too chaotic to be of much use. For the second plot, the bytes arrived are aggregated in 0.4 ms time intervals, and the two slopes corresponding to the two values of α are easily seen.

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The third is a variance-time plot – just another way of looking at LRD. The straight line corresponds to a Hurst parameter $H = 0.5$, so clearly the asymptote of the function represented has a larger slope (between 0.84 and 0.96, to be precise). This being an arrival process with an average speed of about 700 Mbps, the hypothesis that at high speeds the traffic becomes Poissonian ($H \rightarrow 0.5$) is rejected.

3. A scale-free internet model

3.1. *Scale free topology*

To model a distributed network environment like the Internet, it is necessary to integrate data collected from multiple points in a network in order to get a complete picture of network-wide view of the traffic. Knowledge of dynamic characteristics is essential to network management (e.g., detection of failures/congestion, provisioning, and traffic engineering like QoS routing or server selections). However, because of a huge scale and access rights, it is expensive (sometime impossible) to measure such characteristics directly. To solve this, methods and tools for inferencing of unobservable network performance characteristics are used in large scale networking environment. A model where inference based on self similarity and fractal behavior can be applied is the scale free network.

Scale-free networks are complex networks in which some nodes are very well connected while most nodes have a very small number of connections. An important characteristic of scale-free networks is that they are size independent, that is they preserve the same characteristics regardless of the network size N. Scale-free networks have a degree distribution that follows a power relationship, $P(k) = k$^$(-\lambda)$, where the coefficient λ may vary approximately from 2 to 3 for most real networks. Many real networks have a scale-free degree distribution, including the Internet. The algorithm used for the generation of the scale-free network topology is generating networks with a cyclical degree that can be controlled, in our case, approximately 4% of the added nodes form a cycle.

The generated topology consists of three types of nodes:

- *Routers*, defined as nodes with one or several links. Routers do not initiate traffic and do not accept connections.
- *Servers* are defined as nodes with one connection but sometimes could have two or even three connections. Servers only accept traffic connections but do not initiate traffic.
- *Customers* (end-users) defined as nodes that have only one connection, very seldom two connections. Customers initiate traffic connections towards servers at random moments but usually in a time succession. For our proposed model, we chose a 20:80 customers to servers ratio.

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3.2. *Scale-free network design algorithm*

Several models have been presented for the evolution of scale-free networks, each of which may lead to a different ensemble. The first suggestion was the *preferential attachment* model by Barabasi and Albert, which came to be known as the “Barabasi-Albert (BA)” model [13]. Several variants have been suggested to this model. One of them, known as the “Molloy-Reed construction” [14], which ignores the evolution and assumes only the degree distribution and no correlations between nodes, will be considered in the following. Thus, the site reached by following a link is independent of the origin. We designed and implemented an algorithm that generates those subsets of the scale-free networks that are close to a real computer network such as the Internet. Our application is able to handle very large collections of nodes, to control the generation of network cycles, and the number of isolated nodes. The application was written in Python being, as such, portable. It runs very fast on a decent machine (less than 5 minutes for 100.000 nodes model).

Network generation algorithm:

1. set node_count and λ
2. compute the optimal number of nodes per degree
3. create manually a small network of 3 nodes
4. for each node from 4 to node_count
  4.1. call add_node procedure
  4.2. while adding was not successful
    4.2.1. call recompute procedure
    4.2.2. call add_node procedure
5. save network description file

add_node procedure

1. according to the preferential attachment, compute the degree of the parent node
2. if degree could be chosen then exit procedure
3. compute the number of links that the new node shall establish with descendants of its future parent, according to copy model
4. chose randomly a parent from the nodes having the degree as computed above
5. compute the descendant_list, the list of descendants of the newly chosen parent
6. create the new node and links
7. for each descendant of the descendant_list create the corresponding links
8. exit procedure with success code

recompute procedure

1. for each degree category
  1.1. calculate the factor needed to increase the optimal count of nodes per degree
  1.2. if necessary increase the optimal number of nodes per degree
2. exit procedure

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The algorithm starts with a manually created network of several nodes, then using preferential attachment and growth algorithms, new nodes are added.

We introduced an original component, the computation in advance of the number of nodes on each degree-level. The preferential attachment rule is followed by obeying to the restriction of having the optimal number of nodes per degree. Fig. 2 presents an example of a network with 128 nodes, the initial number of nodes being $m_0 = 5$ and an incremental growth of one link per step.

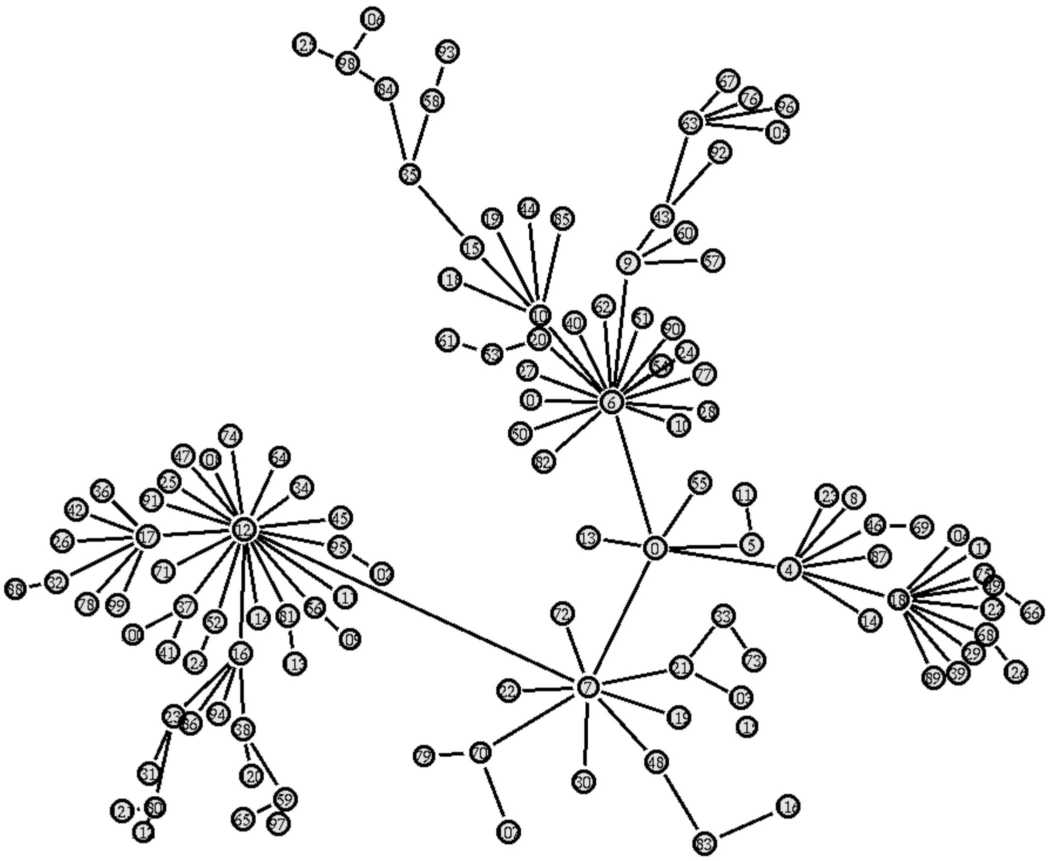

Fig. 2. Graphic representation of a network with 128 nodes

One can see that poor connected nodes have smaller chances of getting new connections. Besides following the repartition law mentioned above, some other restrictions (for example those related to cycles and long chains) had to be applied in order to make the generated model more realistic and similar to the Internet. A more subtle restriction is related to the TTL (Time-to-living) which is a way to avoid routing loops in a real Internet. This translates in a restriction for our topology – there can be no more that 30 nodes to get from any node to any other node.

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4. Influence of network topology on traffic behavior

The fractal nature of both traffic and topology of an Internet network and their reciprocal influence was tested considering simulated web traffic on the Internet SFN based model. After the generation of a huge network model, we have split it in several sub-networks (federations) and then we have verified the traffic similarities investigating measurement series having fractal properties. Self-similarity is a rigorous statistical property. Let assume we have (very long) time series data with finite mean and variance (i.e., covariance stationary stochastic process). Self-similarity implies a “fractal-like” behavior: no matter what time scale is used to examine the data, similar patterns are obtained. The main features deduced from self-similarity are: slowly decaying variance, long range dependence and non-degenerate autocorrelations. The “variance-time plot” is one of the means to test for the slowly decaying variance property. For example, if we plot the variance of the sample versus the sample size, on a log-log plot, it results for most processes a straight line with slope -1; for self-similar, the line is much flatter. Furthermore, the autocorrelation function for the aggregated process is indistinguishable from that of the original process. For the simulation of self-similar traffic it was used a superposition of ON-OFF sources after a Pareto distribution, with $1<\alpha<2$. The Pareto distribution has two parameters, the parameter of shape α and the low-cutting parameter β. The Cumulate Distribution Function (CDF) Pareto is $F(x)=1-\left(\frac{\beta}{x}\right)^{\alpha}$, and the function of the probability density is $f(x)=\frac{\alpha}{\beta}\left(\frac{\beta}{x}\right)^{\alpha+1}$ for $x>\beta$ and $\alpha>0$. Moreover, the parameter α is relationated with the Hurst parameter H as $H=\frac{3-\alpha}{2}$.

In the simulation the whole network was splitted in subnetworks with at most 40 nodes. The parameter for simulation of such a subset were the total number of nodes/subnetwork is N = 40, the number of the initial nodes is m_0 = 5 and a value m = 2 (i.e. at each incremental step one add two links in order to maintain a non-zero grouping coefficient). For the simulation we have used 32 associated traffic sources randomly associated to TCP traffic agents. The value of the shape coefficient α was 1,4 which lead to an expected value of H = 0,8. In fig. 3 are shown the diagrams of the aggregate number of packets on three time units: 1 second, 100 milliseconds and 10 milliseconds. The gray color represents the zoom. The strong burstiness of the traffic in all three diagrams confirms the presence of the self-similarity phenomenon.

The Hurst parameter, H, for a given sequence was calculated using a number of three different estimation methods: the diagram of the rescaled domain, the diagram dispersion-time and the periodogram. In theory, the expected value of

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Hurst is 0.8, but the real results are, in the order in which the methods were presented: 0.8115, 0.9761 and 1.1325. Quite the coefficients for the last two methods are over evaluated, one can conclude that the tested model presents statistical self-similarity.

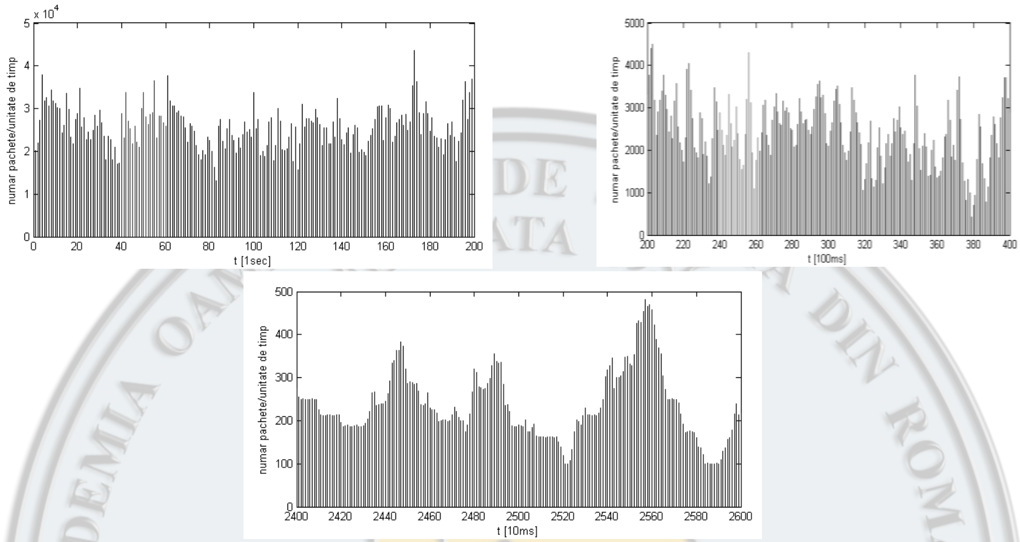

Fig.3. Number of packets/s for three time scales: 1, 0.1 and 0.01 seconds.

Conclusions

The advantage of the model proposed here is its flexibility: it offers an universally acceptable skeleton for potential Internet models, on which one can build features that could lead to further improvements. The model introduced here offers a realistic starting point for a general class of network topologies that combine the scale-free structure with a precise spatial layout.

Although the traffic processes in high-speed Internet links exhibit *asymptotic* self-similarity, their correlation structure at short time-scales makes their modeling as *exact* self-similar processes (like the fractional Brownian motion) inaccurate. Based on simulations made on the SFN based Internet model we conclude that Internet traffic retains its self-similar properties even under high aggregation.

The experiments have let to the following results: **1)** self-similarity is an adaptability of traffic in the network and is not based only on the heavy-tailed file-size distribution; **2)** the scaling region on traffic self-similarity is divided into two timescale regimes: short range dependencies (SRD), determined by bandwidth, TCP window size and packet size, and long range dependencies (LRD), determined by the statistical heavy-tails; **3)** in LRD, increasing the bandwidth does not improve throughput (or network performance; **4)** there is a significant advantage in using fractal analysis methods to solve the problem of anomaly detection.

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An accurate estimation of the Hurst parameter for the MIB variables offers a valuable abnormality indicator obtained for the bursty variables. Thus, by improving the capability of predicting impending network failures, it is possible to reduce network downtime and increase network reliability.

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