USING AN IMPROVED HSDT-DEFORMATION THEORY IN ORDER TO BUILD UP A MATHEMATICAL MODEL FOR THE VIBRATIONS OF AN ORTHOTROPIC COMPOSITE BAR MAKING A SPATIAL MOTION

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Rezumat. *Acest articol arată cum se poate construi un model matematic pentru vibraţiile unei bare compozite având simetrie elastică (ortotropică) considerând mai întâi mişcarea sa ca un solid rigid şi, apoi, considerând o teorie a deformaţiilor HSDT de ordinul al treilea în acord cu condiţiile de compatibilitate Saint-Venant şi, într-un mod foarte original, a condiţiilor Gay pentru mişcarea barei ca un corp deformabil. În final este arătat un algoritm bazat pe diferenţe divizate pentru a rezolva modelul.*

Abstract. *This workpaper shows how to build up a mathematical model for the vibrations of a composite bar having elastic symmetry (orthotropic) considering first its motion as "rigid" body and, next, considering an HSDT-deformation theory of third order in full respect of Saint-Venant compatibility conditions and – in a very original way – of Gay conditions for the motion of the bar as a deformable body. Finally, a divided differences based algorithm designed to solve the mathematical model is shown.*

Keywords: HSDT-Deformation, Model, Composite, Vibrations

1. Introduction

There are a lot of attempts to model the elastic behavior and dynamics of composite materials [1], [2] and, especially the elastic behavior and dynamics of composite bars [3], [4].

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In [5] is shown how to build up a mathematical model for the vibrations of an orthotropic composite bar using the Hamilton's variation principle and assuming some new and original hypothesis concerning the calculus of the elastic coefficients and the elastic displacements in full respect of the well-known Saint-Venant compatibility conditions of the Gay conditions [6]. We'll show here how these results are to be applied in case of a composite bar making a particular motion. We'll also show how the mathematical model obtained in this specific case could be solved using a divided differences based algorythm.

A special focus is made on the motion of the bar as "rigid" body that turns out to be very important in order to establish the boundary conditions for the mathematical model.

2. The motion of the bar like "rigid" body

Let's consider a kinematic linkage having two bars as elements (fig.1). The couplings from O_2 and O_3 are considered to be spherical.

The linkage is rotating around the $O_0 x_3^0$ axis with a constant angular velocity $\omega = 104{,}72\ rad/sec$. The reference system $O_1 x_1^0 x_2^0 x_3^0$ is accelerated on the vertical direction $\vec{a}_T = -10{,}966\sin(104{,}72t)\,\vec{i}_3^{\,0}$ m/s^2, corresponding to a vertical motion described by the following equation: $\overrightarrow{O_0O_1} = 0{,}001\sin(104{,}72t)\,\vec{i}_3^{\,0}$ m.

What really concerns is kinematics of the element No.2, considered as being a deformable one. We'll show [7] that a motion that is analog to that when the element No.2 is considered as being one and the same with the element No.1 is fully compatible with dynamics of the linkage (considered for now as having rigid elements) despite the fact that couplings from O_2 and O_3 are spherical, so they are not introducing reaction moments.

The motion of the element No.1 is considered as being imposed the way that:

$$\alpha = \omega t; \tag{1}$$

where: $t \geq 0$ is time;

ω=const., $\omega > 0$ is the angular velocity (its absolute value) of the element No.1 (${}^0\vec{\omega}^1 = \omega\,\vec{i}_3^{\,0}$).

The angle δ has - obviously - a constant value.

We have established [7], [8] the motion like "rigid body" for the element No.2. In order to consider it as being deformable and to describe its vibrations we'll use the mathematical model established [8].

This model has the following form:

$$\left[P_j^1\right]\left\{\ddot{u}^j\right\}+\left[P_j^2\right]\left\{\dot{u}^j\right\}+\left[P_j^3\right]\left\{u^j\right\}+\left[P_j^4\right]\left\{\theta^j\right\}_{,1}+\left[P_j^5\right]\left\{u^j\right\}_{,11}=\left\{f_j\right\};$$

$$\left[V_j^1\right]\left\{\ddot{\theta}^j\right\}+\left[V_j^2\right]\left\{\dot{\theta}^j\right\}+\left[V_j^3\right]\left\{\theta^j\right\}+\left[V_j^4\right]\left\{u^j\right\}_{,1}+\left[V_j^5\right]\left\{\theta^j\right\}_{,11}=\left\{g_j\right\}; \tag{2}$$

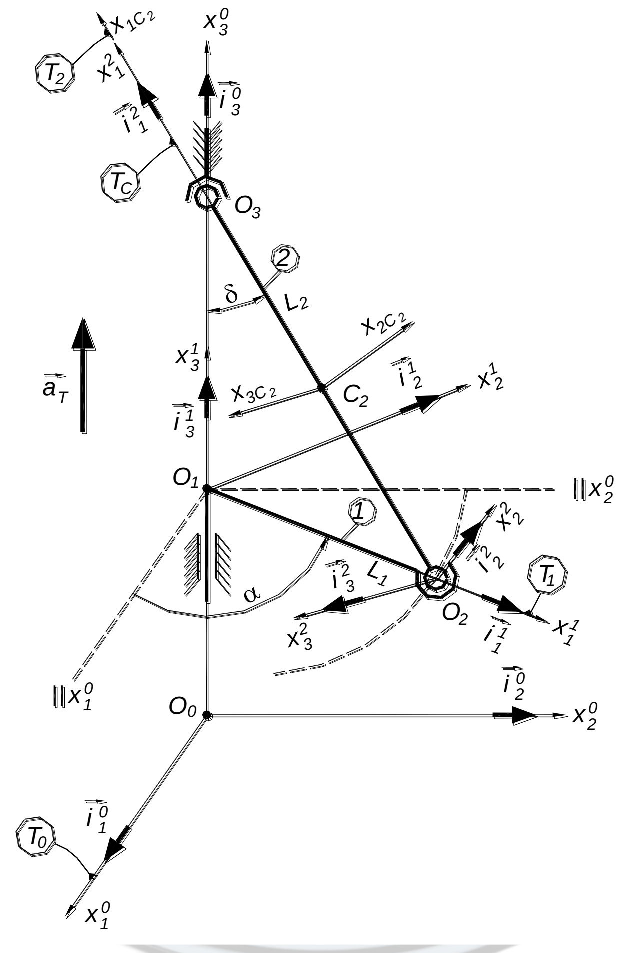

Fig. 1

Obviously we have:

$${}^0\omega_2^2 = \overset{0\,\bullet\,2}{\omega_2} = 0$$

$$\overset{0\,\bullet\,2}{\omega_1} = \overset{0\,\bullet\,2}{\omega_3} = 0$$

We'll focus on a specific case of a right composite bar having the next elastic characteristics:

- length: $L_2 = 0{,}45\ m$
- dimensions of the its section: $B = H = 0{,}01\ m$
- elastic coefficients:

$$E_{1111} = 2 \cdot 7 \cdot 10^{10}\ N/m^2$$
$$E_{1212} = E_{1221} = 3{,}18 \cdot 10^{9}\ N/m^2$$
$$E_{1313} = E_{1331} = 3{,}18 \cdot 10^{9}\ N/m^2$$

- the bar is considered to be an orthotropic one (even a transverse isotropic in order to make things more simple) and having elastic homogeneity:
- the bar presents also mass homogeneity having the specific mass:

$$\rho = 1188\ \ N/m^2 .$$

We'll consider the case of a velocity for the driving element of 1000 rot/min the way that:

$$\omega = \frac{\pi \cdot 1000}{30} \cong 104{,}72\ rad/sec .$$

Also we consider as known:

- length of the driving element: $L_1 = 0{,}225\ m$
- components of the acceleration of the origin O_2 with respect T_0 written in the T_2 reference system:

$$a^0_{0_2(2)1} = \frac{\omega_1^2 L_1^2}{L_2} + \frac{a_T \sqrt{L_2^2 - L_1^2}}{L_2};\ a^0_{0_2(2)2} = 0;$$

$$a^0_{0_2(2)3} = \frac{\omega^2 L_1 \sqrt{L_2^2 - L_1^2}}{L_2} - \frac{a_T L_1}{L_2}$$ exprimed in m/s^2

- components of the specific mass gravitation force distributed on length:

$$p_1 = -0{,}1188 \frac{\sqrt{L_2^2 - L_1^2}}{L_2};\ p_2 = 0;\ p_3 = 0{,}1188 \frac{L_1}{L_2}\ \text{[N/m]}$$

3. Study of the vibrations of the composite bar

We are now ready to consider the bar as being deformable. We'll consider the following initial conditions [8]:

$$\{u\}\Big|_{t=0} = \begin{Bmatrix} 0 \\ 0 \\ 0 \end{Bmatrix};\quad x_1 \in [0, L_2];\qquad \{\dot{u}\}\Big|_{t=0} = \begin{Bmatrix} 0 \\ 0 \\ 0 \end{Bmatrix};\quad x_1 \in [0, L_2]$$

$$\{\theta\}\Big|_{t=0} = \begin{Bmatrix} 0 \\ 0 \\ 0 \end{Bmatrix}; \quad x_1 \in [0, L_2]; \; \{\dot{\theta}\}\Big|_{t=0} = \begin{Bmatrix} 0 \\ 0 \\ 0 \end{Bmatrix}; \quad x_1 \in [0, L_2].$$

And the following boundary conditions:

$$\{u\}\Big|_{x_1=0} = \{u\}\Big|_{x_1=L_2} = \{0\}\Big|_{3\times 1}; \; (\forall)\, t \ge 0$$

$$\{\theta\}_{,1}\Big|_{x_1=0} = \{0\}\Big|_{3\times 1}; \; (\forall)\, t \ge 0$$

$$\theta_1(L_2, t) = 0; \; \theta_{2,1}(L_2, t) = 0; \; \theta_{3,1}(L_2, t) = 0$$

Having one single bar we'll drop down the "j" index:

$$\begin{aligned} &[P^1]\{\ddot{u}\} + [P^2]\{\dot{u}\} + [P^3]\{u\} + [P^4]\{\theta\}_{,1} + [P^5]\{u\}_{,11} = \{f\} \\ &[V^1]\{\ddot{\theta}\} + [V^2]\{\dot{\theta}\} + [V^3]\{\theta\} + [V^4]\{u\}_{,1} + [V^5]\{\theta\}_{,11} = \{g\}; \end{aligned} \tag{3}$$

and, of course: $x_1^j = x_1 = x$ the way that we have:

$$\begin{aligned} &\{u\} = \{u\}(x,t); && \{\theta\} = \{\theta\}(x,t) \\ &x \in [0, L_2]; && t \in [0, t_f] \end{aligned} \tag{4}$$

The nature of the motion like "rigid body" gives $\{f\}$ and $\{g\}$ the following forms [9]:

$$\{f\} = \{f\}(x,t); \; \{g\} = \{const.\} \; . \tag{5}$$

The matrix coefficients of the unknowns $\{u\}$ and $\{\theta\}$ are fully described in [8].

4. A divided differences algorithm designed to solve the mathematical model

We are looking at the relation (3). We'll divide [9] the interval [0, L_2] into n subintervals having the pace on the h axis the way that a x_i coordinate will be:

$$x_i = ih; \; h = \frac{L_2}{n}; \; 0 \le i \le n; \tag{6}$$

We'll divide [9] the time interval of our study $\left[0,t_f\right]$, t_f being a final value, into m subintervals the way that at a certain moment of time t_j:

$$t_j = j\tau;\ \tau = \frac{t_f}{m};\ 0 \le j \le m. \tag{7}$$

The procedure (central differences) leads firstly to the following expression:

$$\left[P^1\right]\cdot\frac{1}{\tau^2}\left\{\left\{u_{i\,j+1}\right\}-2\left\{u_{ij}\right\}+\left\{u_{i\,j-1}\right\}\right\}+\left[P^2\right]\cdot\frac{1}{2\tau}\left\{\left\{u_{i\,j+1}\right\}-\left\{u_{i\,j-1}\right\}\right\}+\left[P^3\right]\left\{u_{ij}\right\}+$$
$$+\left[P^4\right]\left\{\left\{\theta_{i+1\,j}\right\}-\left\{\theta_{i-1\,j}\right\}\right\}\frac{1}{2h}+\left[P^5\right]\frac{1}{h^2}\left\{\left\{u_{i+1\,j}\right\}-2\left\{u_{ij}\right\}+\left\{u_{i-1\,j}\right\}\right\}=\left\{f_{ij}\right\}. \tag{8}$$

and:

$$\left[V^1\right]\cdot\frac{1}{\tau^2}\left\{\left\{\theta_{i\,j+1}\right\}-2\left\{\theta_{ij}\right\}+\left\{\theta_{i\,j-1}\right\}\right\}+\left[V^2\right]\frac{1}{2\tau}\left\{\left\{\theta_{i\,j+1}\right\}-\left\{\theta_{i\,j-1}\right\}\right\}+$$
$$+\left[V^3\right]\left\{\theta_{ij}\right\}+\left[V^4\right]\cdot\frac{1}{2h}\left\{\left\{u_{i+1\,j}\right\}-\left\{u_{i-1\,j}\right\}\right\}+\left[V^5\right]\frac{1}{h^2}\cdot$$
$$\cdot\left\{\left\{\theta_{i+1\,j}\right\}-2\left\{\theta_{ij}\right\}+\left\{\theta_{i-1\,j}\right\}\right\}=\left\{g\right\}, \tag{9}$$

where:

$$u_{ij}=u\left(x_i,t_j\right);\ \theta_{ij}=\theta\left(x_i,t_j\right);\ f_{ij}=f\left(x_i,t_j\right),\left\{g\right\}=\left\{const.\right\},$$
$$1\le i\le n-1;\ j\ge 0.$$

Or, more:

$$\left[\frac{1}{\tau^2}\left[P^1\right]+\frac{1}{2\tau}\left[P^2\right]\right]\left\{u_{i\,j+1}\right\}=\left[-\frac{1}{\tau^2}\left[P^1\right]+\frac{1}{2\tau}\left[P^2\right]\right]\left\{u_{i\,j-1}\right\}+\left[\frac{2}{\tau^2}\left[P^1\right]-\left[P^3\right]+\right.$$
$$\left.+\frac{2}{h^2}\left[P^5\right]\right]\left\{u_{ij}\right\}-\frac{1}{h^2}\left[P^5\right]\left\{u_{i+1\,j}\right\}-\frac{1}{h^2}\left[P^5\right]\left\{u_{i-1\,j}\right\}-\frac{1}{2h}\left[P^4\right]\left\{\theta_{i+1\,j}\right\}+$$
$$\frac{1}{2h}\left[P^4\right]\left\{\theta_{i-1\,j}\right\}+\left\{f_{ij}\right\};$$
$$\left[\frac{1}{\tau^2}\left[V^1\right]+\frac{1}{2\tau}\left[V^2\right]\right]\left\{\theta_{i\,j+1}\right\}=\left[-\frac{1}{\tau^2}\left[V^1\right]+\frac{1}{2\tau}\left[V^2\right]\right]\left\{\theta_{i\,j-1}\right\}+\left[\frac{2}{\tau^2}\left[V^1\right]-\left[V^3\right]+\right.$$
$$\left.+\frac{2}{h^2}\left[V^5\right]\right]\left\{\theta_{ij}\right\}-\frac{1}{h^2}\left[V^5\right]\left\{\theta_{i+1\,j}\right\}-\frac{1}{h^2}\left[V^5\right]\left\{\theta_{i-1\,j}\right\}-\frac{1}{2h}\left[V^4\right]\left\{u_{i+1\,j}\right\}+$$
$$+\frac{1}{2h}\left[V^4\right]\left\{u_{i-1\,j}\right\}+\left\{g\right\}; \tag{10}$$

We'll note:

$$\frac{1}{\tau^2}\left[P^1\right]+\frac{1}{2\tau}\left[P^2\right]=\left[A\right];$$

$$-\frac{1}{\tau^2}\left[P^1\right]+\frac{1}{2\tau}\left[P^2\right]=\left[B_1\right];$$

$$\frac{2}{\tau^2}\left[P^1\right]-\left[P^3\right]+\frac{2}{h^2}\left[P^5\right]=\left[B_2\right]; \qquad (11)$$

and:

$$\frac{1}{\tau^2}\left[V^1\right]+\frac{1}{2\tau}\left[V^2\right]=\left[C\right];$$

$$-\frac{1}{\tau^2}\left[V^1\right]+\frac{1}{2\tau}\left[V^2\right]=\left[D_1\right];$$

$$\frac{2}{\tau^2}\left[V^1\right]-\left[V^3\right]+\frac{2}{h^2}\left[V^5\right]=\left[D_2\right]; \qquad (12)$$

With (12) and (11) the system (10) becomes:

$$\left\{u_{i\ j+1}\right\}=\left[A\right]^{-1}\left[B_1\right]\left\{u_{i\ j-1}\right\}+\left[A\right]^{-1}\left[B_2\right]\left\{u_{ij}\right\}-\frac{1}{h^2}\left[A\right]^{-1}\left[P^5\right]\left\{u_{i+1\ j}\right\}-$$

$$-\frac{1}{h^2}\left[A\right]^{-1}\left[P^5\right]\left\{u_{i-1\ j}\right\}-\frac{1}{2h}\left[A\right]^{-1}\left[P^4\right]\left\{\theta_{i+1\ j}\right\}+\frac{1}{2h}\left[A\right]^{-1}\left[P^4\right]\left\{\theta_{i-1\ j}\right\}+\left[A\right]^{-1}\left\{f_{ij}\right\}$$

$$\left\{\theta_{i\ j+1}\right\}=\left[C\right]^{-1}\left[D_1\right]\left\{\theta_{i\ j-1}\right\}+\left[C\right]^{-1}\left[D_2\right]\left\{\theta_{ij}\right\}-\frac{1}{h^2}\left[C\right]^{-1}\left[V^5\right]\left\{\theta_{i+1\ j}\right\}-$$

$$-\frac{1}{h^2}\left[C\right]^{-1}\left[V^5\right]\left\{\theta_{i-1\ j}\right\}-\frac{1}{2h}\left[C\right]^{-1}\left[V^4\right]\left\{u_{i+1\ j}\right\}+\frac{1}{2h}\left[C\right]^{-1}\left[V^4\right]\left\{u_{i-1\ j}\right\}+\left[C\right]^{-1}\left\{g\right\};$$

$$1\le i\le n-1;\ j\ge 0. \qquad (13)$$

We have seen that:

$$\left\{u\right\}=\left\{u_1;u_2;u_3\right\}^t$$

$$\left\{\theta\right\}=\left\{\theta_1;\theta_2;\theta_3\right\}^t; \qquad (14)$$

The boundary and the initial conditions leads us to the following conclusions:

$$\left\{u_{i0}\right\}=\left\{0\right\},(\forall)\,i; \qquad (15)$$

$$\left\{u_{0j}\right\}=\left\{0\right\},\ (\forall)\ j; \qquad (16)$$

$$\left\{u_{nj}\right\}=\left\{0\right\},\ (\forall)\ j; \qquad (17)$$

$$\frac{1}{2\tau}\left\{\left\{u_{i\ j+1}\right\}-\left\{u_{i\ j-1}\right\}\right\}\Big|_{j=0}=\left\{0\right\} \qquad (18)$$

So that:

$$\{u_{i,-1}\}=\{u_{i1}\},\ (\forall)\ \mathrm{i}$$

$$\{\theta_{i0}\}=\{0\},\ (\forall)\ i \tag{19}$$

$$\frac{1}{2\tau}\left\{\{\theta_{i\ j+1}\}-\{\theta_{i\ j-1}\}\right\}\Big|_{j=0}=\{0\}$$

And more:

$$\{\theta_{i,-1}\}=\{\theta_{i1}\},\ (\forall)\ i \tag{20}$$

$$\frac{1}{h}\left\{\{\theta_{i+1\ j}\}-\{\theta_{i\ j}\}\right\}\Big|_{i=0}=\{0\}$$

And more

$$\theta_{0\,j}=\theta_{1\,j},\ (\forall)\ j \tag{21}$$

in (46) having, obviously, ascendant differences.

$$\theta_1(x_n,t_j)=0 \tag{22}$$

$$\frac{1}{h}\left[\theta_2(x_n,t_j)-\theta_2(x_{n-1},t_j)\right]=0$$

And, still more

$$\theta_2(x_n,t_j)=\theta_2(x_{n-1},t_j) \tag{23}$$

$$\frac{1}{h}\left[\theta_3(x_n,t_j)-\theta_3(x_{n-1},t_j)\right]=0$$

$$\text{or: }\theta_3(x_n,t_j)=\theta_3(x_{n-1},t_j) \tag{24}$$

in (24) and (23) utilizing descending differences.

Let's put in (13) $\mathrm{j}=0$, and taking into account (18), (15), (20), (19) we have:

$$\{u_{i1}\}=[A]^{-1}[B_1]\{u_{i1}\}+[A]^{-1}\{f_{i0}\}$$

$$\{\theta_{i1}\}=[C]^{-1}[D_1]\{\theta_{i1}\}+[\mathrm{C}]^{-1}\{\mathrm{g}\};\ 1\le i\le n-1 \tag{25}$$

And because $t=\tau$:

$$\{u_{i1}\}=\left[[I]_{3\times3}-[A]^{-1}[B_1]\right]^{-1}[A]^{-1}\{f_{i0}\}$$

$$\{\theta_{i1}\}=\left[[I]_{3\times3}-[C]^{-1}[D_1]\right]^{-1}[C]^{-1}\{g\};\ 1\le \mathrm{i}\le \mathrm{n}-1 \tag{26}$$

It becomes useful to make $j=1$, taking into account (19) and (15):

$$\{u_{i2}\}=[A]^{-1}[B_2]\{u_{i1}\}-\frac{1}{h^2}[A]^{-1}[P^5]\{u_{i+1\,1}\}-\frac{1}{h^2}[A]^{-1}[P^5]\{u_{i-1\,1}\}-$$

$$-\frac{1}{2h}[A]^{-1}[P^4]\{\theta_{i+1\,1}\}+\frac{1}{2h}[A]^{-1}[P^4]\{\theta_{i-1\,1}\}+[A]^{-1}\{f_{i1}\}$$

$$\{\theta_{i2}\}=[C]^{-1}[D_2]\{\theta_{i1}\}-\frac{1}{h^2}[C]^{-1}[V^5]\{\theta_{i+1\,1}\}-\frac{1}{h^2}[C]^{-1}[V^5]\{\theta_{i-1\,1}\}-$$

$$-\frac{1}{2h}[C]^{-1}[V^4]\{u_{i+1\,1}\}+\frac{1}{2h}[C]^{-1}[V^4]\{u_{i-1\,1}\}+[C]^{-1}\{g\};\ 1\le i\le n-1 \tag{27}$$

We'll note: $\{w_{n1}\} = \{0; \theta_{2,n-1\,1}; \theta_{3,n-1\,1}\}^t$;

$$\{w_{nj}\} = \{0; \theta_{2,n-1\,j}; \theta_{3,n-1\,j}\}^t; \; j = \overline{2, m-1} \tag{28}$$

Taking into account (28), (21) and (15):

$$\{u_{12}\} = [A]^{-1}[B_2]\{u_{11}\} - \frac{1}{h^2}[A]^{-1}[P^5]\{u_{21}\} - \frac{1}{2h}[A]^{-1}[P^4]\{\theta_{21}\} +$$
$$+ \frac{1}{2h}[A]^{-1}[P^4]\{\theta_{11}\} + [A]^{-1}\{f_{11}\}$$
$$\{\theta_{12}\} = [C]^{-1}[D_2]\{\theta_{11}\} - \frac{1}{h^2}[C]^{-1}[V^5]\{\theta_{21}\} - \frac{1}{h^2}[C]^{-1}[V^5]\{\theta_{11}\} -$$
$$- \frac{1}{2h}[C]^{-1}[V^4]\{u_{21}\} + [C]^{-1}\{g\}; \; 2 \le i \le n-2 \tag{29}$$

The next values $\{u_{i2}\}$ and $\{\theta_{i2}\}$ continuing the algorithm as (27) imposes.

With (17):

$$\{u_{n-1\,2}\} = [A]^{-1}[B_2]\{u_{n-1\,1}\} - \frac{1}{h^2}[A]^{-1}[P^5]\{u_{n-2\,\,1}\} - \frac{1}{2h}[A]^{-1}[P^4]\{w_{n1}\} +$$
$$+ \frac{1}{2h}[A]^{-1}[P^4]\{\theta_{n-2\,\,1}\} + [A]^{-1}\{f_{n-1\,1}\}$$
$$\{\theta_{n-1\,2}\} = [C]^{-1}[D_2]\{\theta_{n-1\,1}\} - \frac{1}{h^2}[C]^{-1}[V^5]\{w_{n1}\} - \frac{1}{h^2}[C]^{-1}[V^5]\{\theta_{n-2\,\,1}\} +$$
$$+ \frac{1}{2h}[C]^{-1}[V^4]\{u_{n-2\,\,1}\} + [C]^{-1}\{g\}; \tag{30}$$

And for $2 \le i \le n-2$ we have at our disposal (27) putting $j = 1$:

On the other hand for $i = 1$ the system (13) becomes with (21) and (16):

$$\{u_{1\,\,j+1}\} = [A]^{-1}[B_1]\{u_{1\,j-1}\} + [A]^{-1}[B_2]\{u_{1j}\} - \frac{1}{h^2}[A]^{-1}[P^5]\{u_{2j}\} -$$
$$- \frac{1}{2h}[A]^{-1}[P^4]\{\theta_{2j}\} + \frac{1}{2h}[A]^{-1}[P^4]\{\theta_{1j}\} + [A]^{-1}\{f_{1j}\}$$
$$\{\theta_{1\,\,j+1}\} = [C]^{-1}[D_1]\{\theta_{1\,j-1}\} + [C]^{-1}[D_2]\{\theta_{1j}\} - \frac{1}{h^2}[C]^{-1}[V^5]\{\theta_{2j}\} -$$
$$- \frac{1}{h^2}[C]^{-1}[V^5]\{\theta_{1j}\} - \frac{1}{2h}[C]^{-1}[V^4]\{u_{2j}\} + [C]^{-1}\{g\}; \tag{31}$$

For $2 \le i \le n-2$ the relations (13) remain available in the algorithm.

For $i = n-1$, with (28) and (17) we have:

$$\{u_{n-1\,\,j+1}\} = [A]^{-1}[B_1]\{u_{n-1\,\,j-1}\} + [A]^{-1}[B_2]\{u_{n-1\,\,j}\} - \frac{1}{h^2}[A]^{-1}[P^5]\{u_{n-2\,\,j}\} -$$

$$-\frac{1}{2h}[A]^{-1}\left[P^4\right]\left\{w_{nj}\right\}+\frac{1}{2h}[A]^{-1}\left[P^4\right]\left\{\theta_{n-2\ j}\right\}+[A]^{-1}\left\{f_{n-1\ j}\right\}$$

$$\left\{\theta_{n-1\ j+1}\right\}=[C]^{-1}\left[D_1\right]\left\{\theta_{n-1\ j-1}\right\}+[C]^{-1}\left[D_2\right]\left\{\theta_{n-1\ j}\right\}-\frac{1}{h^2}[C]^{-1}\left[V^5\right]\left\{w_{n\ j}\right\}-$$

$$-\frac{1}{h^2}[C]^{-1}\left[V^5\right]\left\{\theta_{n-2\ j}\right\}-\frac{1}{2h}[C]^{-1}\left[V^4\right]\left\{u_{n-2\ j}\right\}+[C]^{-1}\{g\} \tag{32}$$

for $2 \le i \le n-2$ and using the system (13).
This way we can determine the values for $\left\{u_{ij}\right\}$ and $\left\{\theta_{ij}\right\}$.
The algorithm fits nicely to work it using programs like TurboPascal or C++.

Conclusions

We have shown how to build up and to solve a mathematical model for the vibrations of an orthotropic composite bar. More specific, the mathematical model will provide the evolutions in function of time for the displacement of the current point of the medium fiber “u” and for the rotation of the current right section “θ”.

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