

## POLAR CHARACTERISTIC OF ENERGETIC INTENSITY EMITTED BY AN ANISOTROPIC THERMAL SOURCE IRREGULARLY SHAPED

Constantin DUMACHE<sup>1</sup>, Corina GRUESCU<sup>2</sup>, Ioan NICOARĂ<sup>3</sup>

**Rezumat.** *Lucrarea prezintă o metodă analitică de calcul al distribuției spațiale de intensitate energetică emisă de o sursă termică anizotropă. Problema diferitelor sisteme de localizare a țintei este calculul irradiației produse de țintă într-un punct situat pe o direcție de observație dată. Problema poate fi soluționată prin utilizarea caracteristicii  $I(\varphi, \theta)$ .*

**Abstract.** *The paper presents an analytical method to compute the spatial distribution of the energetic intensity emitted by an anisotropic thermal source. The essential problem of different target location systems is the calculus of the irradiance produced by the target in a point on the observation direction. The problem is solvable using the characteristic  $I(\varphi, \theta)$ .*

**Key words:** locating system, anisotropic source, energetic intensity distribution

### 1. Introduction

The calculus of any locating system begins with the evaluation of the energetic intensity  $I_e$ , emitted by the target along the receiver's direction [1], [2].

Generally, the energetic intensity calculus is a difficult problem, because the characteristics of the radiation depend on many parameters (nature and temperature of the target, roughness, degree of corrosion, orientation in respect with the receiver). Considering a spatial polar reference system (fig.1), the energetic intensity is a function depending on the coordinates  $(\varphi, \theta)$ :

$$I_e = I(\varphi, \theta) \quad (1)$$

and represents the spatial distribution of intensity.

As the graphical representation of the function  $I_e = I(\varphi, \theta)$  is difficult, a family of curves obtained for  $\theta = ct.$  or  $\varphi = ct.$ , is used.

<sup>1</sup>Dr. Eng., Optica S.A., Timișoara.

<sup>2</sup>Ass. Prof., Dr. Eng., University "Politehnica" Timișoara.

<sup>3</sup>Prof. Dr. Eng., University "Politehnica" Timișoara (inicoara@upt.ro).

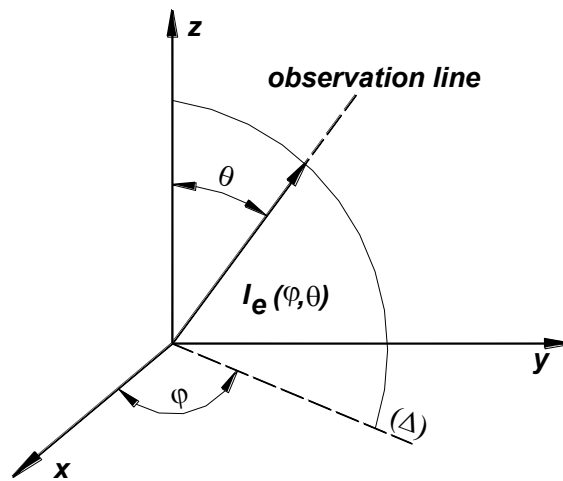


Fig.1. Polar reference system attached to the target

The curves  $I(\varphi, \theta)|_{\theta=\theta_0}$  and  $I(\varphi, \theta)|_{\varphi=\varphi_0}$  are defined as the polar characteristic of the intensity emitted by the object [3], [4], [5].

Some simplifying hypotheses are taken into account:

- the object is considered a gray body, with a total radiance

$$R_e = \varepsilon \sigma T^4 \quad (2)$$

where  $\varepsilon$  is the emission coefficient of the object surface,

$\sigma = 5.76 \cdot 10^{-12} \text{ Wcm}^{-2}\text{K}^{-4}$  – Stefan-Boltzman constant,

$T$  – absolute temperature of the object surface.

- the irradiance at the object surface is a constant size depending only on the temperature  $T$  and independent on the angles  $\varphi$  and  $\theta$ .

As most targets respect these conditions on certain spectral range, the emitted radiation obeys Lambert's Law. The energetic intensity along the observation direction is:

$$I(\alpha) = I_o \cos \alpha \quad (3)$$

where  $I_o$  is the energetic intensity along the normal direction to the surface,

$\alpha$  - the angle between the normal to the surface and the observation line.

If the irradiance  $B$  of a surface is constant, the energetic intensity along the normal direction is:

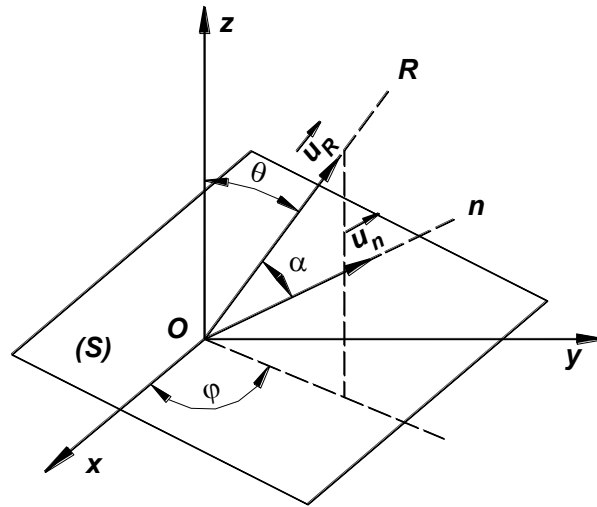
$$I_o = BS \quad (4)$$

or

$$I = BS \cos \alpha = BS_a \quad (5)$$

where  $S_a$  is the apparent surface equal to  $S \cos \alpha$ . The apparent surface is the projection of the surface  $S$  on a plane normal to the observation line.

The present study proposes an analytical method to determine the angle  $\alpha$  as a function of observation direction, for an object cubic shaped.



**Fig.2.** Definition of the angle  $\alpha$

-  $\vec{u}_R$  - a unit vector belonging to the observation line ( $OR$ ), oriented towards the receiver. In an orthogonal reference system  $\vec{u}_R$  is defined by the relation:

$$\vec{u}_R = \cos \varphi \sin \theta \vec{i} + \sin \varphi \sin \theta \vec{j} + \cos \theta \vec{k}, \quad (6)$$

where  $\vec{i}, \vec{j}$  and  $\vec{k}$  are the vectors of module 1 for the reference axis.

-  $\vec{u}_n$  - a unit vector, normal to the surface ( $S$ ) in point  $O$ , oriented along the propagation direction of the radiation.  $\vec{u}_n$  is described by the relation:

$$\vec{u}_n = u_x \vec{i} + u_y \vec{j} + u_z \vec{k}, \quad (7)$$

where  $u_x, u_y$  and  $u_z$  are the components of the vector  $\vec{u}_n$  and depend on surface's ( $S$ ) orientation.

The scalar product is:

$$\vec{u}_R \cdot \vec{u}_n = \cos \alpha = \cos \varphi \sin \theta u_x + \sin \varphi \sin \theta u_y + \cos \theta u_z. \quad (8)$$

The relations (5) and (8) lead to the expression for the energetic intensity:

$$I(\varphi, \theta) = BS(\cos \varphi \sin \theta u_x + \sin \varphi \sin \theta u_y + \cos \theta u_z). \quad (9)$$

If  $k$  surfaces of the object are visible from the point  $R$ , the polar characteristic is:

$$I(\varphi, \theta) = \sum_{i=1}^k B_i S_i (\cos \varphi \sin \theta u_{xi} + \sin \varphi \sin \theta u_{yi} + \cos \theta u_{zi}), \quad (10)$$

Where  $S_i$  represents the area of the order  $i$  surface,  
 $B_i$  – the correspondent irradiance.

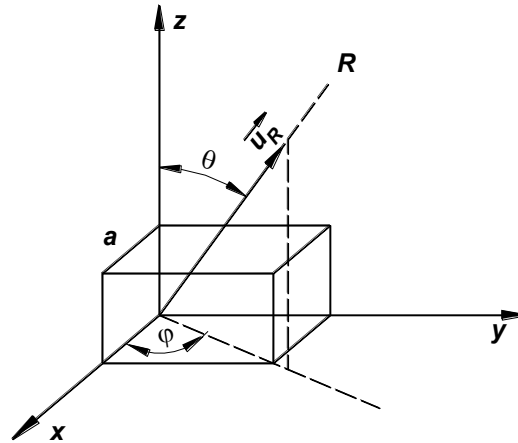


Fig.3. Cub shaped object

For  $0 \leq \varphi \leq 90^\circ$  and  $0 \leq \theta \leq 90^\circ$ ,  $u_x = u_y = u_z = 1$  and the relation (9) becomes:

$$I(\theta, \varphi) = a^2 B (\cos \varphi \sin \theta + \sin \varphi \sin \theta + \cos \theta). \quad (11)$$

The relation (11) can be written in terms of intensities:

$$I(\theta, \varphi) = I_{ox} \cos \varphi \sin \theta + I_{oy} \sin \varphi \sin \theta + I_{oz} \cos \theta, \quad (12)$$

where  $I_{ox} = I_{oy} = I_{oz} = a^2 B$  is the energetic intensity along the directions  $Ox$ ,  $Oy$  and  $Oz$ .

For  $\theta = 90^\circ$  results the characteristic in the plane  $xy$ :

$$I(\varphi) = I_{ox} \cos \varphi + I_{oy} \sin \varphi = \sqrt{I_{ox}^2 + I_{oy}^2} \cos(\varphi - \alpha), \quad (13)$$

Where

$$\alpha = \operatorname{arctg} \frac{I_{oy}}{I_{ox}} = \frac{\pi}{4}.$$

Relation (13) represents the equation of a circle, passing through the point  $O$ , of radius  $r = \frac{1}{2} \sqrt{I_{ox}^2 + I_{oy}^2} = \frac{\sqrt{2}}{2} a^2 B$  and center  $C\left(\frac{\sqrt{2}}{2} a^2 B, \frac{\sqrt{2}}{2} a^2 B\right)$ , belonging to

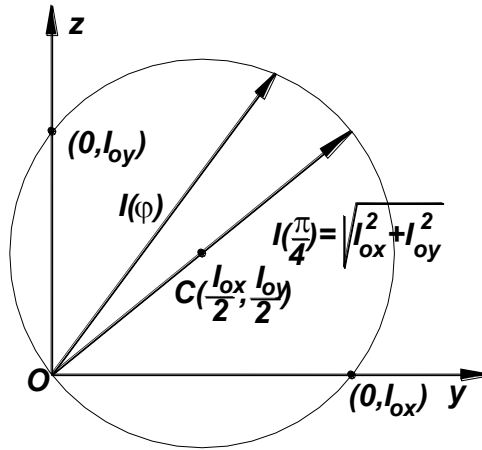


Fig.4. Graphic representation of the function  $I(\varphi)$ .

Similarly, the characteristics  $I(\varphi, \theta)$  in planes  $yz$  ( $\varphi = \pi/2$ ) and  $xz$  ( $\varphi = 0$ ) are also circles, described by the equations:

$$\begin{aligned} I_y^2 + I_z^2 - I_{oy}I_y - I_{oz}I_z &= 0, \\ I_x^2 + I_z^2 - I_{ox}I_x - I_{oz}I_z &= 0, \end{aligned} \quad (15)$$

identical to equation (14). The three circles belong to a sphere centered in the origin of the reference system. Its expression is:

$$I_x^2 + I_y^2 + I_z^2 - I_{ox}I_x - I_{oy}I_y - I_{oz}I_z = 0. \quad (16)$$

The size of the sphere radius is:

$$R = \frac{1}{2} \sqrt{I_{ox}^2 + I_{oy}^2 + I_{oz}^2} = \frac{\sqrt{3}}{2} a^2 B. \quad (17)$$

The center of the sphere situated on the main diagonal of the cube has the following coordinates:

$$x_C = \frac{I_{ox}}{2}, y_C = \frac{I_{oy}}{2}, z_C = \frac{I_{oz}}{2}. \quad (18)$$

The main diagonal of the cube follows the direction on which the energetic intensity maximum is ( $I_{max} = \sqrt{3}a^2B$ ).

In order to demonstrate this assumption, the angles  $\varphi$  and  $\theta$  are determined from the condition  $I(\varphi, \theta) = \max$ . The results are:

$$\frac{\partial I(\varphi, \theta)}{\partial \varphi} = 0 \text{ for } \varphi = \frac{\pi}{4} \text{ and } \frac{\partial I(\varphi, \theta)}{\partial \theta} \Big|_{\varphi = \frac{\pi}{4}} = 0 \text{ for } \theta = \arctg \sqrt{2}, \quad (19)$$

meaning the main diagonal direction.

Similarly, for the angles  $\pi/2 < \varphi < 2\pi$ , the characteristic is a sphere passing through point  $O$  and having the center situated on the main diagonal.

In order to design the characteristic  $I(\varphi, \theta)$ , vector radii are traced at lengths proportional to  $I_e$  computed with relation (11), at different values of  $\varphi$  and  $\theta$ .

The evolut of radii end provides the characteristic. In figure 5 is shown a characteristic drawn for  $\varphi = 45^\circ$  and  $\theta = 0^\circ, 15^\circ, 30^\circ, 45^\circ, 60^\circ, 75^\circ$  and  $90^\circ$ .

As the energetic intensity is known, the total energetic flow, emitted by the cube into a hemisphere can be calculated:

$$\Phi_e = \int_{\Omega} I(\varphi, \theta) d\omega = \int_{\varphi} \int_{\theta} I(\varphi, \theta) \sin \theta d\theta d\varphi \quad (20)$$

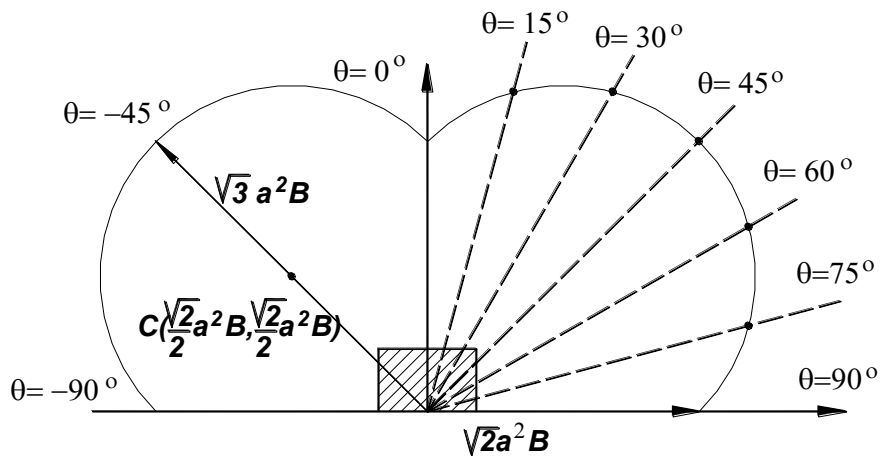


Fig. 5. Plot of characteristic  $I(\varphi, \theta)$ .

The radiance produced by the cube at the surface of a receiver which is placed on the observation line, at the distance  $d$ , is:

$$E_R = \frac{I(\varphi_o, \theta_o)}{d^2}, \quad (21)$$

where  $(\varphi_o, \theta_o)$  are the angles which define the observation line.

### Conclusions

For the objects on the field, situated at a long distance to the receiver, is difficult to find the characteristic  $I(\varphi, \theta)$ , from several reasons:

- ❑ the object surfaces have various shapes and unknown orientation in relation to the receiver
- ❑ the temperature gradient and emission coefficient of these surfaces are also unknown.

The solving of the problem is possible by means of a simplified object model.

The model is shared in regular geometrical shapes.

Only significant surfaces of the object should be taken into account.

Each surface is treated as a gray body, at known, constant temperature.

Using equation (10) results the function  $I(\varphi, \theta)$ .

Relation (21) allows determination of radiance onto the pupil entrance of the locating system, which represents the basic element for the energetic calculus of the system.

The entire demonstration above refers to a cube shape.

Any other geometrical shape can be studied in a similar manner in order to get the analytical equation in energetic intensity.

## REFERENCES

- [1] M. Baas, *Handbook of Optics*, vol. I, II, McGraw Hill Inc., NY, 1988.
- [2] E. Hecht, *Optics*, 3<sup>rd</sup> Edition, A. W. Longman, 1998.
- [3] Constantin Dumache, *Contribuții privind determinarea poziției unei surse de radiație în infraroșu folosind modulația fluxului radiant incident*, PhD Thesis, Universitatea Politehnica Timișoara, 2005.
- [4] C. Dumache, I. Nicoară, *Utilizarea transformatorilor Fourier la determinarea caracteristicii statice a traductorului optic de poziție utilizat la măsurarea mărimilor reglate*, a XXIX-a Sesiune de Comunicări Științifice cu participare internațională, Academia Tehnică Militară, București, 2001.
- [5] C. Gruescu, I. Nicoară, *Aparate optice. Analiza și sinteza sistemelor optice lenticulare*, Editura Politehnica, Timișoara, 2004.

