

## CONJUGATE GRADIENT WITH SUBSPACE MINIMIZATION BASED ON CUBIC REGULARIZATION MODEL OF THE MINIMIZING FUNCTION

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**Abstract.** *A new algorithm for unconstrained optimization based on the cubic regularization in two dimensional subspace is developed. Different strategies for search direction are also discussed. The stepsize is computed by means of the weak Wolfe line search. Under classical assumptions it is proved that the algorithm is convergent. Intensive numerical experiments with 800 unconstrained optimization test functions with the number of variables in the range [1000 - 10,000] show that the suggested algorithm is more efficient and more robust than the well established conjugate gradient algorithms CG-DESCENT, CONMIN and L-BFGS ( $m=5$ ). Comparisons of the suggested algorithm versus CG-DESCENT for solving five applications from MINPACK-2 collection, each of them with 40,000 variables, show that CUBIC is 3.35 times faster than CG-DESCENT.*

**Keywords:** unconstrained optimization, acceleration, regularization, conjugate gradient methods, applications

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### 1. Introduction

For solving the unconstrained optimization problem

$$\min f(x), \quad (1)$$

where  $f: \mathbb{R}^n \rightarrow \mathbb{R}$  is continuously differentiable and bounded from below, besides the well known line-search and trust-region methods, the  $p$ -regularization model is constructed by adding a  $p$ -th regularization term to the quadratic estimation of  $f$ . The idea is to construct and minimize a local quadratic approximation of the minimizing function with a weighted regularization term  $(\sigma_k / p) \|x\|^p$ ,  $p > 2$ . The most common choice to regularize the quadratic approximation is the  $p$ -regularization with  $p = 3$ , which is known as the *cubic regularization*. The idea of using the cubic regularization into the context of the Newton method first appeared in Griewank (1981) and was later

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