ANALYTICAL SIMULATION OF THE PULL-IN VOLTAGE TO EVALUATE THE PROCESS INDUCED STRESS AND YOUNG'S MODULUS INTO THE MICROMACHINING POLYSILICON LAYERS BY THE PULL-IN VOLTAGE METHOD

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Abstract. *A new form of the set of two equation applied to the test beam structure fabricated during the micromachining technology suitable to easy extract the material parameters in order to optimize the diffusion process and obtain a linear response of the polysilicon microelements like membranes for the silicon capacitive pressure sensors is presented. On this basis there were deduced simulating analytical solutions to describe the beam deformation and the pull-in voltage as a function of the beam length and other geometrical parameters, allowing to optimize the pull-in voltage structure and to easily extract the values of the stress and the young's modulus by a fitting procedure on the experimental data. A graphical method to evaluate directly the induced stress and a combined graphical method with an iteration procedure to determine both the stress and young's modulus are also presented.*

Keywords: Analytical simulation; polysilicon capacitive pressure sensors for biomedical applications: pull-in voltage method; process-induced stress and Young's modulus.

1. Introduction

Surface micromachining is one of the most used technology to achieve microsensor structures due to the capability to develop a wide range of sensitive elements [1, 2]. Reliable and reproductive sensor characteristics are obtained if low stress polysilicon layers are prepared, especially to determine homogeneous material parameters and a linear characteristic of the capacitive pressure sensors, which is closely related to the stress induced in the polysilicon membrane by the technological processes: compressive stress could produce the buckling of the free standing micromechanical elements and the deformation of the polysilicon membranes, while a high tensile stress can affect the sensitivity of the capacitive pressure sensors [3].

The membrane structures or suspended elements for microsystem applications can be obtained by bulk micromachining techniques [4, 5], where a strict control of a chemical etching process of the highly-doped boron layers should be applied

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[6 - 8], or by surface micromachining technique [4, 5], where the properties of the micro mechanical elements should be controlled during the chemical annealing of the phosphorus-doped polysilicon layers, restructured during the thermo-chemical process.

It is therefore of grate interest to develop suitable technical procedures for the material characterization, able to determine accurately, as more as possible, the values of the interest parameters like process induced stress σ and Young's modulus E into the polysilicon micromachined layers [9, 10]. The pull-in voltage technique [3, 9] it seems to be a very attractive one for this purpose because allows to determine the residual stress and the Young's modulus by using beam test structures submitted to the same fabrication process as the microsystem structures on the same silicon wafer. However, the extraction parameters appear to become a quite complicated procedure, necessitating multiple iterations steps to numerically fit the set of two equations on the experimental data.

In this paper are presented the analytical results allowing simulating by a single explicit relation the dependence of the pull-in voltage on the geometrical parameters of the test beam, the residual stress and Young's modulus, which permits the extraction of the suitable material parameters by applying the standard fitting procedure.

2. Problem statement and analytical solution

Within this section will be presented the conditions of the problem, the set equations, and the solutions. An approximate solution will be also derived, suitable for some simplified, but still convenient conditions for a rapid process evaluation.

2.1. A new form of set equations and analytical simulating solutions

Let's consider the suspended rectangular beam test structure as proposed in ref. [3], with the geometrical dimensions defined by the length l, the thickness h of the rectangular beam and the height d of the cavity formed between the free standing position of the beam and the substrate, useful to apply the so called pull-in voltage method (Fig.1).

This method consists in the application of a voltage across the structure of beam and the determination of the value of the voltage for which the deflection of the beam due to the electrostatic forces does not reach an equilibrium position, but will continue to increase until a physical contact is made with the bottom electrode substrate [3]. The critical value V is defined as the pull-in voltage.

As component parts of the integrated structure, the beam follows the same technological conditions as the capacitive pressure sensor structures.

Two equations were obtained earlier from the condition of the pull-in voltage [3], i.e. both first and second derivatives of the energy function to be zero (eq. (7) and (8) of that reference).

For an easier analysis of the testing results, it is proposed a new form of the set of two equations describing the beam pull-in voltage effect as follows:

$$\sigma = \gamma V^2 \ell^2 / x(1-x)^{3/2} - (\beta x^2+\alpha)E/\ell^2 \qquad (1)$$

$$\sigma = 3\gamma V^2 \ell^2 / 2(1-x)^{5/2} - (3\beta x^2+\sigma)E/\ell^2 \qquad (2)$$

where $x = w/d$ (w representing the critical deflection of the beam), $\alpha=(\pi^2 h^2/3)$, $\beta=(\pi^2 d^2/4)$, $\gamma=(\varepsilon_o/2\pi^2 hd^3)$, ε_o representing the value of the vacuum electrical permittivity. It is important to note that γ could be expressed as a function of α and β by the relation: $\gamma=\varepsilon_o\pi^2/16\beta(3\alpha\beta)^{½}$.

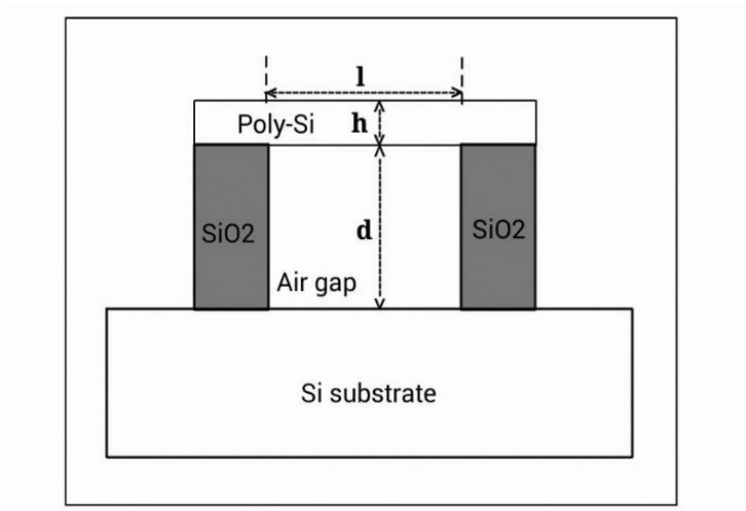

Fig. 1. Schematic representation of the cross-section of a test structure used to apply the pull-in voltage method, where the geometrical parameters are defined by: ℓ - beam length, h - beam thickness, d - depth of the gap cavity formed between the polysilicon layer and the surface of silicon.

The value V is measured by the maximum voltage across the beam-substrate structure, before its dropping when the substrate is touched by the deflected beam. σ is the residual stress into the polysilicon beam, l is the free stand length of the beam, E is the Young's modulus. The critical position w is associated to the pull-in voltage V, and is the deflected position when the value V is attained.

Eliminating the parameter σ between the two equations, the following compact relation, suitable for the calculation of V if x is known, is deduced:

$$V^2 = 4(\beta E/\gamma\ell^4)x^3(1-x)^{5/2}/(5x-2) \qquad (3)$$

Substituting the expression $\gamma V^2\ell^2/x(1-x)^{3/2}$ obtained from eq. (1) into eq. (2), it is obtained a new equation suitable to offer an explicit solutions for x as follows:

$$9x^3 - 6x^2 + 5Sx - 2S = 0 \quad (4)$$

where:

$$S = \sigma\ell^2/\beta E + \alpha/\beta \quad (5)$$

Relations (3) and (4) together with (5) form a set of equations allowing expressing the pull-in voltage V as a function of the interest material parameters, i.e. residual stress σ and Young's modulus E. As it can be seen, eq. (3) expresses the dependence of V on ℓ, as required by the experimental purpose, but also on x, a complex quantity depending also on ℓ in an implicit way.

To solve this point it is necessary to find the real solution of eq. (4), where S is explicitly expressed by rel. (5).

The real solution x_o of eq.(4) is expressed as x_{o+} when S is positive ($S > 0$) by the explicit analytical relation:

$$x_{o+} = 2/9 + (2/3(3)^{1/3})\{(S/2 + 1/9) + [1/3(5S/4-1/3)^3 + (S/2+1/9)^2]^{1/2}\}^{1/3} + (2/3(3)^{1/3})\{(S/2 + 1/9) - [1/3(5S/4-1/3)^3 + (S/2+1/9)^2]^{1/2}\}^{1/3} \quad (6)$$

and by x_{o-} when S is negative, getting values into the interval [-1, 0], as follows:

$$x_{o-} = 2/9 + (4/3(3)^{1/2})(5S/4+1/3)^{1/2}\cos\{1/3\arccos[(3)^{1/2}(S/2+1/9)/(5S/4-1/3)^{3/2}] \quad (7)$$

2.2 Simplified extraction procedure of material parameters

The variation of S with x derived from rel. (4) is shown in Fig. 2 (marked by the curve (1)), defining the domain where S could get useful values able to satisfy the requirements of the problem.

From this graph and also from rel. (3) it is evident that x should be different of the value 2/5, which is a discontinuity point, determining both V and S to show a asymptotic variation near this value.

Also, although S could get negative values for $x < 2/5$, as it can be seen from Fig. 2 curve (1), this domain is prohibited by expression (3) of the pull-in voltage V as a function of x.

In the same figure it is represented curve (2) as $V = [x^3(1-x)^{5/2}]^{1/2}$, which is x-dependent part of the pull-in voltage expression deduced from rel. (3), showing a rapid, nearly a linear decreasing close to the value x=2/5 and a relatively slow variation when x goes to null value.

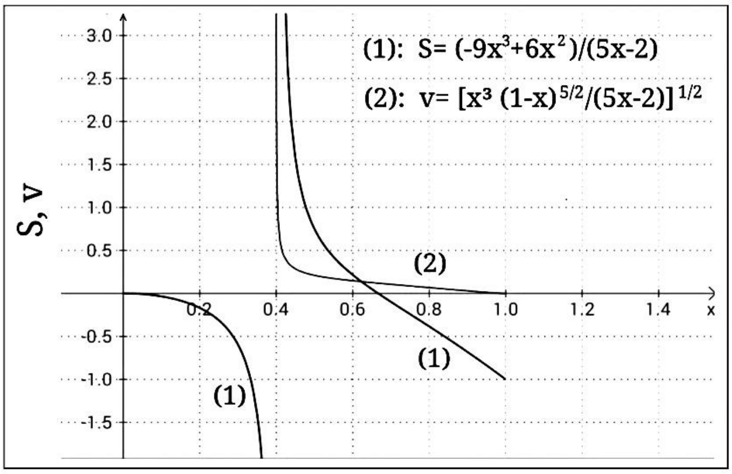

Fig. 2. Variation of the parameter *S* and *V* as a function of the critical normalized depth *x* as defined in the inserted relations.

This particular variation of *V* near the prohibited value x=2/5, which corresponds to $S \gg 1$, allows deducing an analytical expression useful for the extraction of both required parameters *E* and σ as follows:

$$Y = (0.186/\gamma)(\sigma X + \alpha E X^2) \quad (8)$$

where there were applied the substitutions $Y=1/V^2$ and $X=1/\ell^2$. Relation (8) allows extracting elegantly and accurately the parameters *E* and σ from a parabolic type expression. Relation (8) was deduced substituting *x* in eq. (4) with its value 2/5 and deducing therefore $(5x\text{-}2) = 0.384/S$, which was furthermore introduced into expression of *V* given by rel. (3), where also *x* was substituted by its relatively constant value 2/5. In order to explicitly obtain the dependence of the final parameters in rel. (8), it was taken into account rel. (5) expressing *S* as a function of σ_o and *E*.

According to the variation of *S* depicted in Fig. 2 and rel. (5), when the residual stress is positive, then $S > 0$ and decreases with the decrease of the positive value of σ.

When σ starts to become negative ($\sigma < 0$), *S* continues to decrease. For $S = 0$ the quantity within the right brackets in rel. (6) is null and then x=2/3, corresponding to the situation $\sigma\ell^2/\beta E = \alpha/\beta$. The limitation to the value -1 of *S* in the negative variation interval (0, -1] for $x > 2/3$, comes from the physical limitation of *x*, which can get the maximum value 1, as it can be seen also from Fig. 2 and in rel. (3).

When S = -1, from rel. (5) can be deduced that $-\sigma = \alpha/\beta - \beta E/\ell^2$, showing that the geometrical dimensions of the beam determines the limit of the negative values of σ which can be measured, i.e. small values of α/β and ℓ and high values of β are favorable to measure higher negative values of stress.

From rel. (5) it can be seen that the condition $S \gg 1$ can be fulfilled generally if $\sigma \gg 0$. However, for small values of the induced stress (even for $\sigma = 0$), S could be still sufficiently high so that an approximated expression like (8) would continue to work. This happens when the ratio α/β would be quite high, higher that a few units. In this case have to calculate the value x_o from eq. (4) knowing that $S = \alpha/\beta$ from (5) (established by the construction) and therefore the parameter E and σ are to be easily extracted from the parabolic type relation, which must be represented as $V^2 = f(1/\ell^2)$:

$$V^2 = 4\, x_o\, (1- x_o\,)^{5/2}/3\gamma\, (2\text{-}3x_o)\, [\sigma(1/\ell^2\,) + \alpha E\, (1/\ell^4\,)] \qquad (9)$$

Such kind of procedure allows developing around a fixed convenient point $x_o < 2/3$ an approximated solution to easily extract both interest parameters E and σ from a parabolic type relation described by (9).

3. Simulation results and discussion

From Fig. 2 it is possible to evaluate rapidly the order of magnitude of σ considering E known, for a test structure with a single beam (ℓ, h, d and then α, β, and γ are known). Indeed, from curve (2) of that figure it is obtained the corresponding x and by using this particular value a corresponding S and then σ could be obtained from the curve (1) and rel. (5). In the case when v and therefore S is high, it is difficult to obtain a certain value of S from this graph, but in this case rel. (8) could be successfully used to extract accurately the interest parameter σ. If the test structure consists of two beams, the same procedure will be performed as follows: the evaluation will be started supposing an E known in one equation $V_1 = 2\, v_1[(\beta E)]^{1/2}/(\ell_1^{\,2}\, (\gamma)^{1/2})$ derived from the definition of v and rel. (3), and then the set of values (E, σ) obtained from this equation and from that of S (rel. (5)) will be improved by iterations, checking how this set fulfils the second one in V and S.

The same way could be also followed by calculations: introducing the experimental obtained value of V in rel. (3), the corresponding value of v is obtained and consequently a particular value of x, which is used to calculate a corresponding value of S and then of that of σ, according to the relations inserted in that figure, corroborated with rel. (5). If the test structure consists of two beams with different lengths, then a system of two equations obtained from rel. (3) will be used to adequately calculate both E and σ parameters, taking into account the explicit value of x from rels. (6) and (7).

In Fig. 3 and Fig. 4 are represented the theoretical values of the pull-in voltage as a function of the beam lengths l for various values of σ (both positive and negative) as indicated in each figure, and the beam thickness as parameter (h = 2 μm and h = 1 μm respectively), maintaining the same the gap of the test structure (d = 1 μm) and the value of the Young's modulus $E=1.7\times10^{11}$ Pa. From these figures it can be seen that for a given beam length, the necessary value of the pull-in voltage to attain the critical conditions is lower as the stress decreases from positive large values (1×10^{8}Pa) to negative stress.

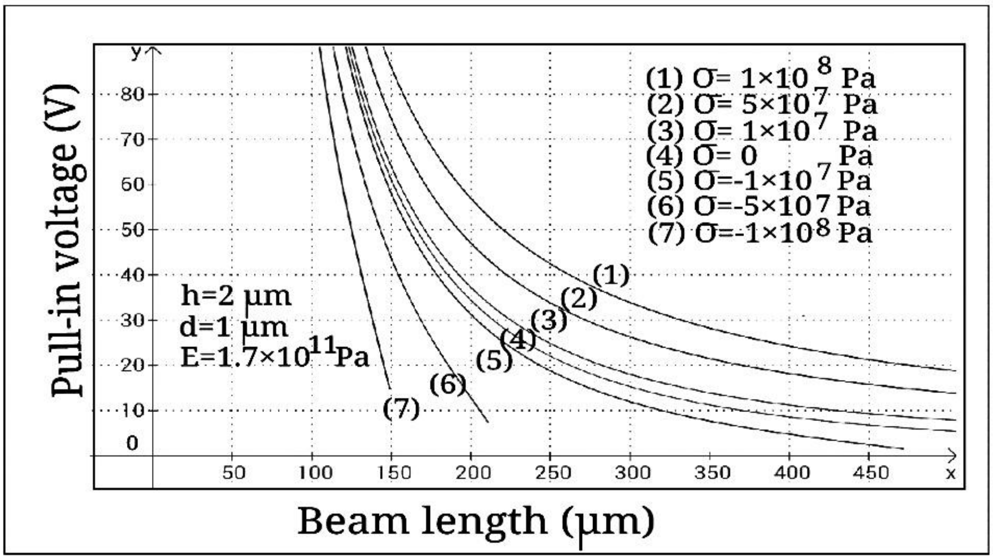

Fig. 3. Simulation of the pull-in voltage V as a function of the beam length ℓ for thick beams (h = 2 μm), d =1 μm, $E=1.7\times10^{11}$ Pa and various positive (tensile) and negative (compressing) residual stress inside of the polysilicon layer, getting values on the range $\sigma=-1\times10^{8}$ Pa ÷ 1×10^{8} Pa.

As it have to be expected, from these figures it can be seen also that it is necessary a smaller value of the pull-in voltage to attain the critical conditions as lower is the thickness of the beam, so that the test structures with a higher thickness will be useful to detect a higher value both of tensile or compressive stress, as it can be seen from Fig. 3 with respect to Fig. 4. From these figures appears that the highly negative values of stress ($\sigma < -1\times10^{8}$ Pa) are difficultly detectable (Fig. 4) or even undetectable for smaller values of h. For a given value of σ, the useful range of ℓ is lower as the value of h decreases.

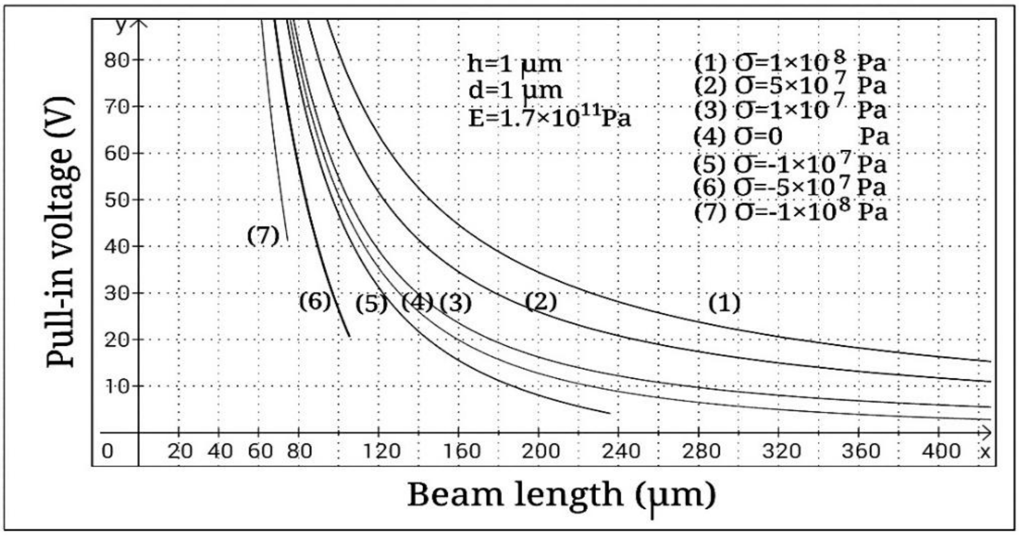

Fig. 4. Simulation of the pull-in voltage V as a function of beam length ℓ for a medium thickness value of the beam $h = 1$ μm, maintaining the other parameters as follows: $d = 1$ μm, $E = 1.7\times10^{11}$ Pa, and the residual stress varying on the range $\sigma = -1\times10^8$ Pa $\div 1\times10^8$ Pa.

This behavior is more evident from Fig. 5, where it is represented the pull-in voltage as a function of ℓ for $\sigma = 1\times10^7$ Pa, $d = 1$ μm and $h = 2$ μm, 1 μm and 0.5 μm taken as parameter. To attain the same pull-in voltage value, longer beams are necessary as the thickness of the beam is higher. From a practical point of view, according to the results represented in Figs. 3-5, it is better to use a higher beam thickness in order to be able to detect both positive (tensile) and negative (compressive) high level stress, maintaining a reasonable range of pull-in voltage variation. In Fig. 6 is represented the variation of the pull-in voltage with the beam length for various values of the induced stress into the beams under the following conditions: $E = 1.7\times10^{11}$ Pa, $d = 1$ μm and $h = 2$ μm, showing that around the zero value of σ (no stress), the pull-in voltage is slightly related to the beam length variation. However, in the case of the situation described in this figure, it is possible to still detect with sufficient precision the values of the stress $\sigma = \pm1\times10^6$ Pa, or even smaller.

However, an improvement of the parameter extraction could be obtained by using rel. (9), which allows to deduce in a rapid manner both E and σ from a parabolic type dependence. An example of this situation is given in Fig. 7, where rel. (9) is used to extract $E = 1.7\times10^7$ Pa and $\sigma = -3.2\times10^4$ Pa from a set of experimental data used as a structure test for the fabrication of the capacitive sensors for biomedical applications [10].

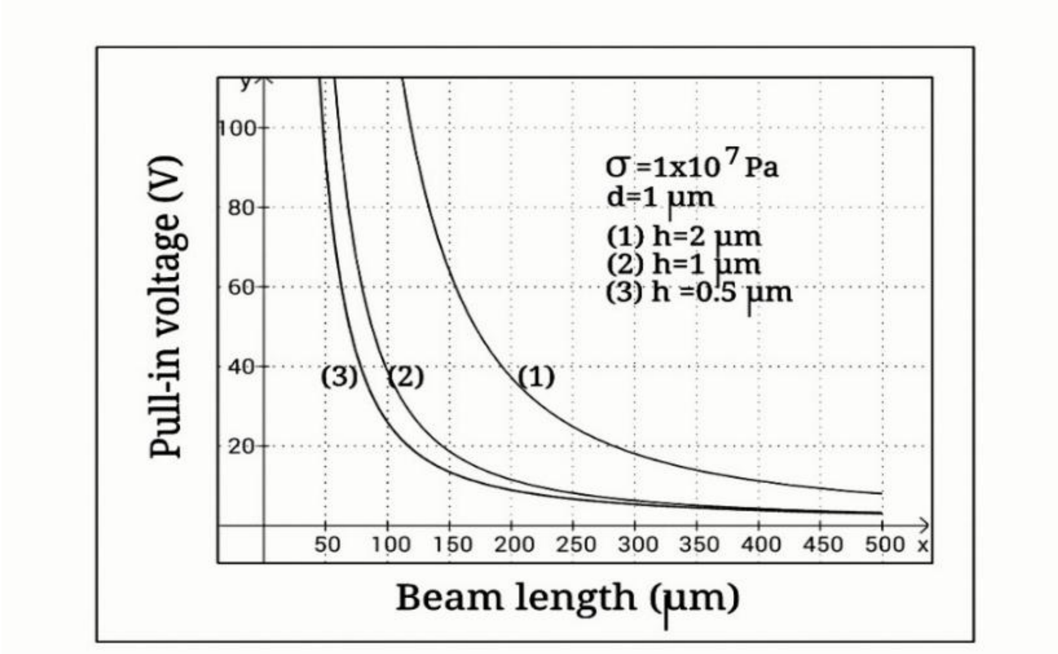

Fig. 5. Comparison between V(l) simulated graphs for $\sigma = 1\times10^7$ Pa, $E = 1.7\times10^{11}$ Pa, $d = 1$ μm corresponding to various thickness values $h = 2$ μm, 1 μm and 0.5 μm.

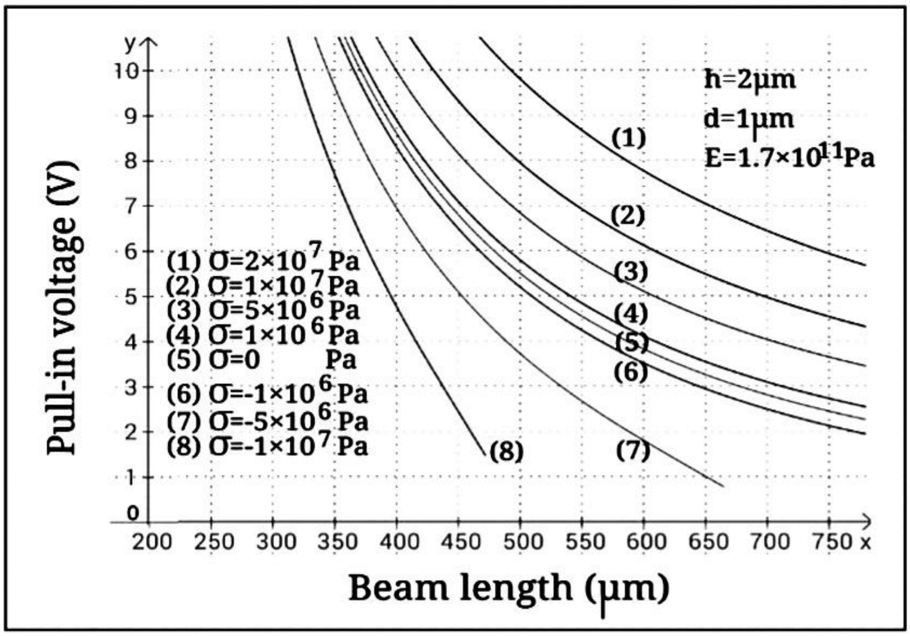

Fig. 6. Simulation of V(l) in a low range ($V < 10$ V), useful to detect the small values of the residual stress on a test polysilicon structure with $h = 2$ μm, $d = 1$ μm, $E = 1.7\times10^{11}$ Pa.

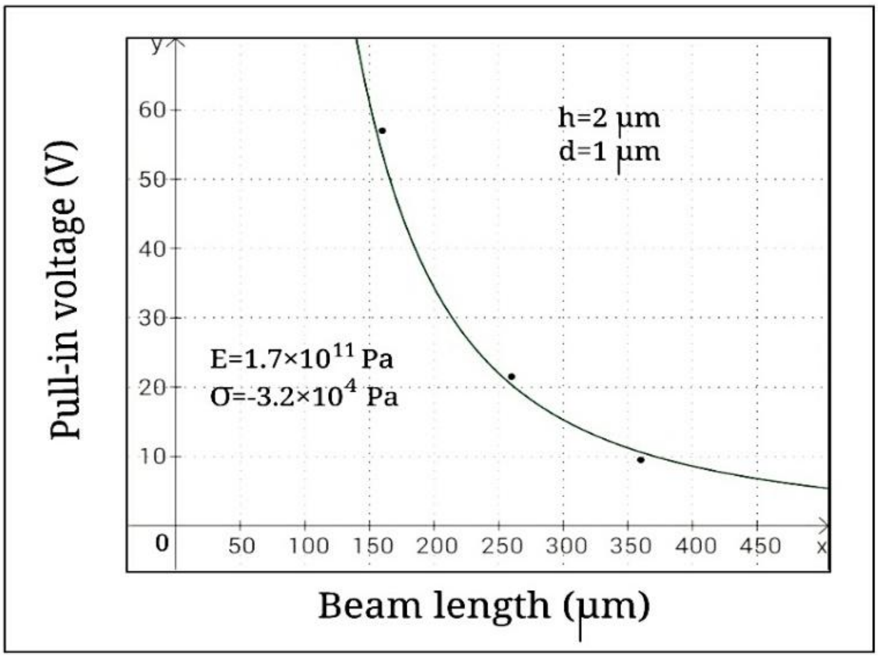

Fig. 7. Parabolic type representation of the pull-in voltage parameters as defined by rel. (8), allowing to accurately extract the interest parameters σ and *E*.

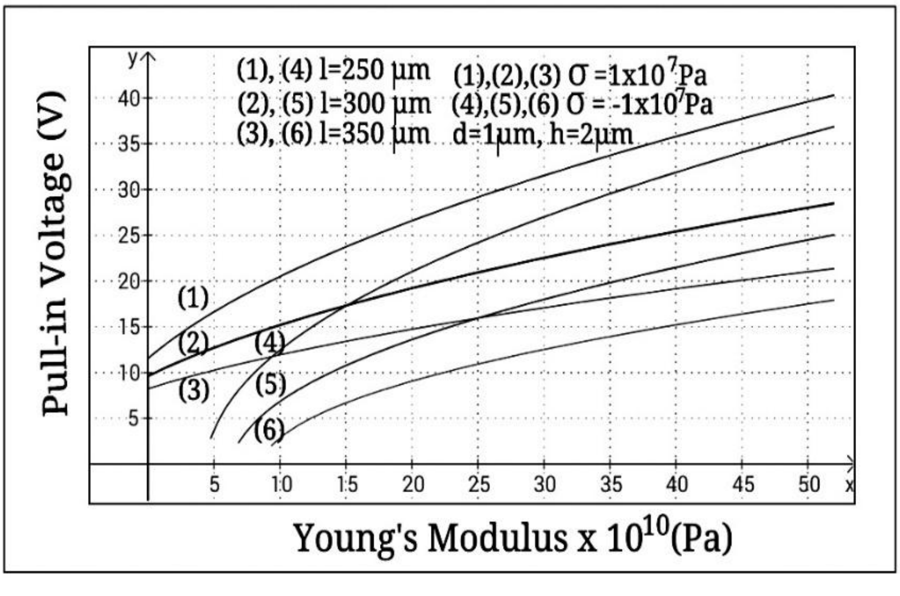

Fig. 8. Simulation of the pull-in voltage *V* as a function of the Young's modulus on the range $E > 5\times10^{10}$ Pa, by using a test structure defined by $h = 2$ μm, $d = 1$ μm and supposing two extreme values of the stress: $\sigma = 1\times10^7$ Pa and $\sigma = -1\times10^7$ Pa for comparison.

In Fig. 8 is represented the variation of the pull-in voltage with *E* on the range $5\times10^{10}\div5\times10^{11}$ Pa for various beam length (ℓ = 250 μm, 300 μm and 350 μm) and two distinct values of the induced stress ($\sigma = 1\times10^{7}$ Pa and $\sigma = -1\times10^{7}$ Pa), maintaining as constant all other geometrical parameters (h = 2 μm, d = 1 μm). From this figure it can be seen that the pull-in voltage increases with the increase of *E* and is higher for positive stress values. The variation of *V* on this range is relatively linear and the higher slope is associated with the increasing values of the stress and is higher as smaller are the values of the beam length. An important conclusion derived also from Fig. 8 is that the pull-in voltage could be significantly affected by relatively large variations of *E*. However, the characteristic variation of *E* obtained from experiment is relatively low, around 1.7×10^{11} Pa [9], in agreement with the results reported also in refs. [10 - 12].

Conclusions

In order to simplify the extraction procedure of the material parameters σ (induced stress) and *E* (Young's modulus) of the polysilicon layers used for the fabrication of the micromechanical elements by micromachining technology, it was reduced the set of two equations specific for the pull-in voltage method to a compact set, one of them depending only on a unique parameter $S = S(\alpha, E)$ and the second one as a product of a function $v(x)$ and an expression depending on *E*. On this basis was deduced the permitted variation range of *x* and *S*, allowing to obtain real values of the defined quantities.

Such a form allowed to represent $x = x(S)$ and $v = x(x)$ and on this to deduce graphically the value of σ if *E* is supposed known, and by the iteration method both *E* and σ parameters if the test structure consists at least of two beams of different lengths. It is also commented the possibility to obtain these values by direct calculations.

The first equation depending on the parameter *S* was solved analytically and where obtained the suitable explicit solutions both for positive and negative values of *S*.

By using these results, it was possible to obtain an analytical way to extract directly σ if *E* is known, in the case when only one beam is available, or even both parameters *E* and σ when the test structure consists at least of two beams.

A significant simplification is deduced, allowing to extract directly both σ and E from a parabolic type expression in a range close to the prohibited value $x = 2/5$ or near null value of *x*, reducing on this way the extraction procedure at one of standard type. This parabolic expression also permits the extraction when the application of the graphical method indicates a range of x close to $x = 2/5$. Such an expression was also successfully used to fit same experimental data when the values of σ are very low ($|\sigma| < 10^{6}$ Pa).

From the graphic representation of *V* as a function of ℓ with σ as parameter, useful conclusions for practical application were deduced, showing that in the range of the negative values of σ it is necessary to dispose of beam of sufficiently high value, our recommended used data being *h* = 2 µm, *h* = 1 µm. The variation of *V* with *E* is also commented in terms of ℓ and σ range.

R E F E R E N C E S

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