Ann. Acad. Rom. Sci.
Ser. Math. Appl.
ISSN 2066-6594 Vol. 8, No. 2/2016

Coefficient Bounds For Certain Subclasses of Analytic and Bi-Univalent Functions *

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Abstract

In this paper, we introduce and investigate an interesting subclass of analytic and bi-univalent functions in the open unit disk $\mathbb{U}$. Furthermore, we find upper bounds for the second and third coefficients for functions in this subclass. The results presented in this paper would generalize and improve some recent works.

MSC: Primary: 30C45; Secondary: 30C50.

keywords: Analytic functions, Bi-univalent functions, Koebe one-quarter theorem, Coefficient estimates.

1 Introduction

Let $\mathcal{A}$ be a class of functions of the form

$$f(z) = z + \sum_{n=2}^{\infty} a_n z^n, \tag{1}$$

which are analytic in the open unit disk $\mathbb{U} = \{z \in \mathbb{C} : |z| < 1\}$. Also $\mathcal{S}$ denote the class of functions $f \in \mathcal{A}$ which are univalent in $\mathbb{U}$.

*Accepted for publication in revised form on September 19-th 2016

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The Koebe one-quarter theorem [5] ensures that the image of $\mathbb{U}$ under every univalent function $f \in \mathcal{S}$ contains a disk of radius $\frac{1}{4}$. So every function $f \in \mathcal{S}$ has an inverse f^{-1}, which is defined by

$$f^{-1}(f(z)) = z \qquad (z \in \mathbb{U}),$$

and

$$f(f^{-1}(w)) = w \qquad \left(|w| < r_0(f); r_0(f) \geq \frac{1}{4}\right),$$

where

$$g(w):=f^{-1}(w)=w - a_2w^2 + (2a_2^2 - a_3)w^3 - (5a_2^3 - 5a_2a_3 + a_4)w^4 + \cdots \quad (2)$$

A function $f \in \mathcal{A}$ is said to be bi-univalent in $\mathbb{U}$ if both f and f^{-1} are univalent in $\mathbb{U}$. Let Σ denote the class of bi-univalent functions in $\mathbb{U}$ given by (1). Examples of functions in the class Σ are

$$\frac{z}{1-z}, \qquad -\log(1-z), \qquad \frac{1}{2}\log(\frac{1+z}{1-z}),$$

and so on. However, the familiar Koebe function is not a member of Σ. Other common examples of functions in $\mathcal{S}$ such as

$$z - \frac{z^2}{2} \qquad \textit{and} \qquad \frac{z}{1-z^2}$$

are also not members of Σ.

Determination of the bounds for the coefficients a_n is an important problem in geometric function theory as they give information about the geometric properties of these functions. For example, the bound for the second coefficient a_2 of functions $f \in \mathcal{S}$ gives the growth and distortion bounds as well as covering theorems.

Lewin [11] investigated the class Σ of bi-univalent functions and showed that $|a_2| < 1.51$ for the functions belonging to Σ. Subsequently, Brannan and Clunie [3] conjectured that $|a_2| \leq \sqrt{2}$. Kedzierawski [10] proved this conjecture for a special case when the function f and f^{-1} are starlike functions. Tan [14] obtained the bound for $|a_2|$ namely $|a_2| \leq 1.485$ which is the best known estimate for functions in the class Σ. Recently there interest to study the bi-univalent functions class Σ (see [6, 8, 15, 16]) and obtain estimates on the first two Taylor-Maclaurin coefficients $|a_2|$ and $|a_3|$. The coefficient estimate problem i.e. bound of $|a_n|$ $(n \in \mathbb{N} - \{2, 3\})$ for each $f \in \Sigma$ formulated by [1] is still an open problem. In fact there is no direct way to get bound for coefficients greater than three. In special cases if $a_k = 0$ for $k = 2, \cdots, n-1$, there are some papers [2, 9, 17] which founded the bound for $|a_n|$, but in general case there is no direct way to get bound for coefficients $|a_n|$ for all n.

Recently Srivastava [12] introduced the following two subclasses of the bi-univalent function class Σ and obtained the following estimates on the first two Taylor-Maclaurin coefficients $|a_2|$ and $|a_3|$ of functions in each of these subclasses.

Definition 1.1. ([12]) A function $f(z)$ given by (1) is said to be in the class H_{Σ}^{α}, $(0 < \alpha \leq 1)$ if the following conditions are satisfied:

$$f \in \Sigma \quad and \quad |arg(f'(z)| < \frac{\alpha\pi}{2} \quad (z \in \mathbb{U}), \quad |arg(g'(w)| < \frac{\alpha\pi}{2} \qquad (w \in \mathbb{U}),$$

where the function g is given by (2).

Theorem 1.2. *([12]) Let $f(z)$ given by (1) be in the class H_{Σ}^{α}, $(0 < \alpha \leq 1)$. Then*

$$|a_2| \leq \alpha\sqrt{\frac{2}{\alpha+2}}, \qquad |a_3| \leq \frac{\alpha(3\alpha+2)}{3}.$$

Definition 1.3. ([12]) A function $f(z)$ given by (1) is said to be in the class $H_{\Sigma}(\beta)$, $(0 \leq \beta < 1)$ if the following conditions are satisfied:

$$f \in \Sigma \qquad and \qquad \mathfrak{Re}(f'(z)) > \beta \quad (z \in \mathbb{U}), \qquad \mathfrak{Re}(g'(w)) > \beta \qquad (w \in \mathbb{U}),$$

where the function g is given by (2).

Theorem 1.4. *([12]) Let $f(z)$ given by (1) be in the class $H_{\Sigma}(\beta)$, $(0 \leq \beta < 1)$. Then*

$$|a_2| \leq \sqrt{\frac{2(1-\beta)}{3}}, \qquad |a_3| \leq \frac{(1-\beta)(5-3\beta)}{3}.$$

As a generalization of above classes, Frasin [7] introduced the following two subclasses of the bi-univalent function class Σ and obtained the following estimates on the first two Taylor-Maclaurin coefficients $|a_2|$ and $|a_3|$ of functions in each of these subclasses.

Definition 1.5. ([7]) Let $0 < \alpha \leq 1$ and $0 \leq \eta < 1$. A function $f(z) \in \Sigma$ given by (1) is said to be in the class $H_\Sigma(\alpha, \eta)$ if the following conditions are satisfied:

$$\left|arg(f'(z) + \eta z f''(z))\right| < \frac{\alpha\pi}{2} \qquad (z \in \mathbb{U}),$$
$$\left|arg(g'(w) + \eta w g''(w))\right| < \frac{\alpha\pi}{2} \qquad (w \in \mathbb{U}),$$

where the function g is given by (2).

Theorem 1.6. *([7]) Let $f(z)$ given by* (1) *be in the class $H_\Sigma(\alpha, \eta)$. Then*

$$|a_2| \leq \frac{2\alpha}{\sqrt{2(\alpha+2) + 4\eta(\alpha + \eta + 2 - \alpha\eta)}}, \qquad |a_3| \leq \frac{\alpha^2}{(1+\eta)^2} + \frac{2\alpha}{3(1+2\eta)}.$$

Definition 1.7. ([7]) Let $0 \leq \beta < 1$ and $0 \leq \eta < 1$. A function $f(z) \in \Sigma$ given by (1) is said to be in the class $H_\Sigma(\beta, \eta)$ if the following conditions are satisfied:

$$\mathfrak{Re}\left(f'(z) + \eta z f''(z)\right) > \beta \qquad (z \in \mathbb{U}),$$
$$\mathfrak{Re}\left(g'(w) + \eta w g''(w)\right) > \beta \qquad (w \in \mathbb{U}),$$

where the function g is given by (2).

Theorem 1.8 ([7]). *Let $f(z)$ given by* (1) *be in the class $H_\Sigma(\beta, \eta)$. Then*

$$|a_2| \leq \sqrt{\frac{2(1-\beta)}{3(1+2\eta)}}, \qquad |a_3| \leq \frac{(1-\beta)^2}{(1+\eta)^2} + \frac{2(1-\beta)}{3(1+2\eta)}.$$

Motivated and stimulated especially by the work of Frasin [7], we propose to investigate the bi-univalent function class $R_\Sigma^{h,p}(\eta, \gamma)$ introduced here in Definition 2.1 and derive coefficient estimates on the first two Taylor-Maclaurin coefficient $|a_2|$ and $|a_3|$ for a function $f \in R_\Sigma^{h,p}(\eta, \gamma)$ given by (1). Our results would generalize and improve the related works of Frasin [7] and Srivastava [12].

2 The subclass $R_\Sigma^{h,p}(\eta, \gamma)$

In this section, we introduce and investigate the general subclass $R_\Sigma^{h,p}(\eta, \gamma)$.

Definition 2.1. Let $h, p : \mathbb{U} \to \mathbb{C}$ be analytic functions and

$$\min\{\mathfrak{Re}(h(z)), \mathfrak{Re}(p(z))\} > 0 \quad (z \in \mathbb{U}) \quad \textit{and} \quad h(0) = p(0) = 1.$$

Let $0 \leq \eta < 1$ and $\gamma \in \mathbb{C} \setminus \{0\}$. A function $f \in \mathcal{A}$ given by (1) is said to be in the class $R_{\Sigma}^{h,p}(\eta, \gamma)$ if the following conditions are satisfied:

$$1 + \frac{1}{\gamma}[f'(z) + \eta z f''(z) - 1] \in h(\mathbb{U}) \qquad (z \in \mathbb{U}), \tag{3}$$

and

$$1 + \frac{1}{\gamma}[g'(w) + \eta w g''(w) - 1] \in p(\mathbb{U}) \qquad (w \in \mathbb{U}), \tag{4}$$

where the function g is defined by (2).

Remark 2.2. This class introduced in this paper is motivated by the corresponding class investigated in [13].

Remark 2.3. There are many choices of h and p which would provide interesting subclasses of class $R_{\Sigma}^{h,p}(\eta, \gamma)$. For example,

1. For $h(z) = p(z) = \left(\frac{1+z}{1-z}\right)^{\alpha}$, where $0 < \alpha \leq 1$, it is easy to verify that the functions $h(z)$ and $p(z)$ satisfy the hypotheses of Definition 2.1. Now if $f \in R_{\Sigma}^{h,p}(\eta, \gamma)$, then

$$f \in \Sigma \quad \textit{and} \quad \left| arg(1 + \frac{1}{\gamma}[f'(z) + \eta z f''(z) - 1]) \right| < \frac{\alpha\pi}{2} \qquad (z \in \mathbb{U}),$$

and

$$\left| arg(1 + \frac{1}{\gamma}[g'(w) + \eta w g''(w) - 1]) \right| < \frac{\alpha\pi}{2} \qquad (w \in \mathbb{U}),$$

where the function g is given by (2).
Therefore in this case, if we take $\gamma = 1$ it reduce to class in Definition 1.5 and if we take $\gamma = 1$ and $\eta = 0$ it reduce to class in Definition 1.1.

2. For $h(z) = p(z) = \frac{1+(1-2\beta)z}{1-z}$, $0 \leq \beta < 1$ the functions $h(z)$ and $p(z)$ satisfy the hypotheses of Definition 2.1. Now if $f \in R_{\Sigma}^{h,p}(\eta, \gamma)$, then

$$f \in \Sigma \quad \textit{and} \quad \mathfrak{Re}\left(1 + \frac{1}{\gamma}[f'(z) + \eta z f''(z) - 1]\right) > \beta \qquad (z \in \mathbb{U}),$$

and

$$\mathfrak{Re}\left(1+\frac{1}{\gamma}[g'(w)+\eta w g''(w)-1]\right)>\beta \qquad (w\in\mathbb{U}),$$

where the function g is given by (2).
Therefore in this case, if we take $\gamma=1$ it reduce to class in Definition 1.7 and if we take $\gamma=1$ and $\eta=0$ it reduce to class in Definition 1.3.

2.1 Coefficient Estimates

Now, we obtain the estimates on the coefficients $|a_2|$ and $|a_3|$ for subclass $R_{\Sigma}^{h,p}(\eta,\gamma)$.

Theorem 2.4. *Let $f(z)$ given by* (1) *be in the class $R_{\Sigma}^{h,p}(\eta,\gamma)$. Then*

$$|a_2|\leq \min\left\{\sqrt{\frac{|\gamma|^2(|h'(0)|^2+|p'(0)|^2)}{8(1+\eta)^2}},\sqrt{\frac{|\gamma|(|h''(0)|+|p''(0)|)}{12(1+2\eta)}}\right\}, \tag{5}$$

and

$$|a_3|\leq \min\left\{\frac{|\gamma|^2(|h'(0)|^2+|p'(0)|^2)}{8(1+\eta)^2}+\frac{|\gamma|(|h''(0)|+|p''(0)|)}{12(1+2\eta)},\frac{|\gamma||h''(0)|}{6(1+2\eta)}\right\}. \tag{6}$$

Proof. First of all, we write the argument inequalities in (3) and (4) in their equivalent forms as follows:

$$1+\frac{1}{\gamma}[f'(z)+\eta z f''(z)-1]=h(z) \qquad (z\in\mathbb{U}), \tag{7}$$

and

$$1+\frac{1}{\gamma}[g'(w)+\eta w g''(w)-1]=p(w) \qquad (w\in\mathbb{U}), \tag{8}$$

respectively, where functions h and p satisfy the conditions of Definition 2.1. Also, the functions h and p have the following Taylor-Maclaurin series expansions:

$$h(z)=1+h_1z+h_2z^2+h_3z^3+\cdots, \tag{9}$$

and

$$p(w)=1+p_1w+p_2w^2+p_3w^3+\cdots. \tag{10}$$

Now, upon substituting from (9) and (10) into (7) and (8), respectively, and equating the coefficients, we get

$$2(1+\eta)a_2 = \gamma h_1, \tag{11}$$
$$3(1+2\eta)a_3 = \gamma h_2, \tag{12}$$
$$-2(1+\eta)a_2 = \gamma p_1, \tag{13}$$

and

$$6(1+2\eta)a_2^2 - 3(1+2\eta)a_3 = \gamma p_2. \tag{14}$$

From (11) and (13), we get

$$h_1 = -p_1, \tag{15}$$

and

$$8(1+\eta)^2 a_2^2 = \gamma^2(h_1^2 + p_1^2). \tag{16}$$

Adding (12) and (14), we get

$$6(1+2\eta)a_2^2 = \gamma(p_2 + h_2). \tag{17}$$

Therefore, from (16) and (17), we have

$$a_2^2 = \frac{\gamma^2(h_1^2 + p_1^2)}{8(1+\eta)^2}, \tag{18}$$

and

$$a_2^2 = \frac{\gamma(p_2 + h_2)}{6(1+2\eta)}, \tag{19}$$

respectively. Therefore, we find from the equations (18) and (19), that

$$|a_2|^2 \leq \frac{|\gamma|^2(|h'(0)|^2 + |p'(0)|^2)}{8(1+\eta)^2},$$

and

$$|a_2|^2 \leq \frac{|\gamma|(|h''(0)| + |p''(0)|)}{12(1+2\eta)},$$

respectively. So we get the desired estimate on the coefficient $|a_2|$ as asserted in (5).

Next, in order to find the bound on the coefficient $|a_3|$, by subtracting (14) from (12), we get

$$6(1+2\eta)a_3 - 6(1+2\eta)a_2^2 = \gamma(h_2 - p_2). \tag{20}$$

Upon substituting the value of a_2^2 from (18) into (20), it follows that

$$a_3 = \frac{\gamma^2(h_1^2 + p_1^2)}{8(1+\eta)^2} + \frac{\gamma(h_2 - p_2)}{6(1+2\eta)},$$

Therefore, we get

$$|a_3| \leq \frac{|\gamma|^2(|h'(0)|^2 + |p'(0)|^2)}{8(1+\eta)^2} + \frac{|\gamma|(|h''(0)| + |p''(0)|)}{12(1+2\eta)}, \tag{21}$$

On the other hand, upon substituting the value of a_2^2 from (19) into (20), it follows that

$$a_3 = \frac{\gamma(p_2 + h_2)}{6(1+2\eta)} + \frac{\gamma(h_2 - p_2)}{6(1+2\eta)} = \frac{\gamma h_2}{3(1+2\eta)},$$

Therefore, we get

$$|a_3| \leq \frac{|\gamma||h''(0)|}{6(1+2\eta)}. \tag{22}$$

So we obtain from (21) and (22) the desired estimate on the coefficient $|a_3|$ as asserted in (6). This completes the proof. □

3 Conclusions

If we take

$$h(z) = p(z) = \left(\frac{1+z}{1-z}\right)^{\alpha} \qquad (0 < \alpha \leq 1,\ z \in \mathbb{U}),$$

in Theorem 2.4, we conclude the following result.

Corollary 3.1. *Let the function $f(z)$ given by (1) be in the class $R_{\Sigma}^{h,p}(\eta,\gamma)$. Then*

$$|a_2| \leq \min\left\{\frac{|\gamma|\alpha}{1+\eta}, \frac{\sqrt{2|\gamma|}\alpha}{\sqrt{3(1+2\eta)}}\right\},$$

and

$$|a_3| \leq \frac{2|\gamma|\alpha^2}{3(1+2\eta)}.$$

By setting $\gamma = 1$ in Corollary 3.1, we obtain the following result which is an improvement of the Theorem 1.6.

Corollary 3.2. *Let the function f given by* (1) *be in the class $H_\Sigma(\alpha,\eta)$. Then*

$$|a_2| \le \frac{\sqrt{2}\alpha}{\sqrt{3(1+2\eta)}},$$

and

$$|a_3| \le \frac{2\alpha^2}{3(1+2\eta)}.$$

Remark 3.3. It is easy to see that

$$\frac{\sqrt{2}\alpha}{\sqrt{3(1+2\eta)}} \le \frac{2\alpha}{\sqrt{2(\alpha+2)+4\eta(\alpha+\eta+2-\alpha\eta)}},$$

and

$$\frac{2\alpha^2}{3(1+2\eta)} \le \frac{\alpha^2}{(1+\eta)^2} + \frac{2\alpha}{3(1+2\eta)},$$

which, in conjunction with Corollary 3.2, would obviously yield an improvement of Theorem 1.6.

If we take $\eta = 0$ in Corollary 3.2, then we get the following result which is an refinement of Theorem 1.2.

Corollary 3.4. *Let the function f given by* (1) *be in the class H_Σ^α. Then*

$$|a_2| \le \sqrt{\frac{2}{3}}\alpha,$$

and

$$|a_3| \le \frac{2\alpha^2}{3}.$$

Remark 3.5. Since

$$\sqrt{\frac{2}{3}}\alpha \le \alpha\sqrt{\frac{2}{\alpha+2}}, \tag{23}$$

and

$$\frac{2}{3}\alpha^2 \le \frac{\alpha(3\alpha+2)}{3}, \tag{24}$$

Corollary 3.4 is an refinement of Theorem 1.2.

By setting

$$h(z) = p(z) = \frac{1 + (1 - 2\beta)z}{1 - z} \qquad (0 \leq \beta < 1,\ z \in \mathbb{U}),$$

in Theorem 2.4, we deduce the following result.

Corollary 3.6. *Let the function f given by* (1) *be in the class $R_{\Sigma}^{h,p}(\eta, \gamma)$. Then*

$$|a_2| \leq \min\left\{ \frac{|\gamma|(1-\beta)}{1+\eta}, \sqrt{\frac{2|\gamma|(1-\beta)}{3(1+2\eta)}} \right\},$$

and

$$|a_3| \leq \frac{2|\gamma|(1-\beta)}{3(1+2\eta)}.$$

If we take $\gamma = 1$ in Corollary 3.6, we obtain the following result which is an improvement of the estimates obtained by Frasin in Theorem 1.8.

Corollary 3.7. *Let the function f given by* (1) *be in the class $H_{\Sigma}(\beta, \eta)$. Then*

$$|a_2| \leq \min\left\{ \frac{(1-\beta)}{1+\eta}, \sqrt{\frac{2(1-\beta)}{3(1+2\eta)}} \right\},$$

and

$$|a_3| \leq \frac{2(1-\beta)}{3(1+2\eta)}.$$

Remark 3.8. Corollary 3.7 is an improvement of the following estimates obtained by Frasin in Theorem 1.8. Because, for the coefficient $|a_2|$, if $\eta > \frac{3\delta - 2 + \sqrt{3\delta(3\delta - 2)}}{2}$ and $\frac{2}{3} < \delta < \frac{8}{9}$ where $\delta = 1 - \beta$. Then

$$\frac{1-\beta}{1+\eta} < \sqrt{\frac{2(1-\beta)}{3(1+2\eta)}}.$$

Also for the coefficient $|a_3|$, we have

$$\frac{2(1-\beta)}{3(1+2\eta)} \leq \frac{(1-\beta)^2}{(1+\eta)^2} + \frac{2(1-\beta)}{3(1+2\eta)}.$$

If we take $\eta = 0$ in Corollary 3.7, then we obtain the following consequence which is an improvement of the estimates obtained by Frasin in Theorem 1.4.

Corollary 3.9. *Let the function f given by* (1) *be in the class $H_\Sigma(\beta)$. Then*

$$|a_2| \leq \begin{cases} \sqrt{\dfrac{2(1-\beta)}{3}}, & 0 \leq \beta \leq \frac{1}{3} \\ (1-\beta), & \frac{1}{3} \leq \beta < 1 \end{cases}$$

and

$$|a_3| \leq \frac{2(1-\beta)}{3}.$$

Acknowledgments

The authors wish to thank the referee, for the careful reading of the paper and for the helpful suggestions and comments.

References

[1] R. M. Ali, S. K. Lee, V. Ravichandran, S. Subramaniam: Coefficient estimates for bi-univalent Ma-Minda starlike and convex functions, Appl. Math. Lett., 25, pp. 344 – 351, 2012.

[2] S. Altinkaya, S. Yalcin: Faber polynomial coefficient bounds for a subclass of bi-univalent functions, C. R. Math. Acad. Sci. Paris, 353, pp. 1075 – 1080, 2015.

[3] D. A. Brannan, J. Clunie, W. E. Kirwan: Coefficient estimates for a class of starlike functions, Canad. J. Math., 22, pp. 476 – 485, 1970.

[4] D. A. Brannan, T. S. Taha: On some classes of bi-univalent functions, in Mathematical Analysis and Its Applications, S. M. Mazhar, A. Hamoui, and N. S. Faour, Eds., 3 of KFAS Proceedings Series, pp. 53 – 60, Pergamon Press (Elsevier Science), 1988, Studia Universitatis Babes-Bolyai, Series Mathematica, 31, no. 2, pp. 70 – 77, 1986.

[5] P. L. Duren: Univalent Functions, Grundlehren der Mathematischen Wissenschaften, Band 259, Springer-Verlag, New York, Berlin, Heidelberg and Tokyo, 1983.

[6] T. Hayami, S. Owa: Coefficient bounds for bi-univalent functions, Pan-American Mathematical Journalm 22, pp. 15 – 26, 2012.

[7] B. A. Frasin: Coefficient bounds for certain classes of bi-univalent functions, Hacet. J. Math. Stat., 43, 3, pp. 383 – 389, 2014.

[8] B. A. Frasin, M. K. Aouf: New subclasses of bi-univalent functions, Appl. Math. Lett., 24, pp. 1569 – 1573, 2011.

[9] J. M . Jahangiri, S .G. Hamidi: Faber polynomial coefficient estimates for analytic bi-Bazilevič functions, Matematicki Vesnik., 67, pp. 123 – 129, 2015.

[10] A. W. Kedzierawski: Some remarks on bi-univalent functions, Ann. Univ. Mariae Curie-Sklodowska Sect. A., 39, pp. 77 – 81, 1985.

[11] M. Lewin: On a coefficient problem for bi-univalent functions, Proc. Amer. Math. Soc., 18, pp. 63 – 68, 1967.

[12] H. M. Srivastava, A. K. Mishra and P. Gochhayat: Certain subclasses of analytic and biunivalent functions, Appl. Math. Lett., 23, pp. 1188 – 1192, 2010.

[13] A. Swaminathan: Sufficient conditions for hypergeometric functions be in a certain class of analytic functions, Comput. Math. Appl., 59, pp. 1578 – 1583, 2010.

[14] D. L. Tan: Coefficient estimates for bi-univalent functions, Chinese Ann. Math. Ser. A., 5, pp. 559 – 568, 1984.

[15] Q. H. Xu, Y. -C. Gui, H. M. Srivastava: Coefficient estimates for a Certain subclass of analytic and bi-univalent functions, Appl. Math. Lett., 25, pp. 990 – 994, 2012.

[16] Q. H. Xu, H. -G. Xiao, H. M. Srivastava: A certain general subclass of analytic and bi-univalent functions and associated coefficient estimate problems, Appl. Math. Comput., 218, 23, pp. 11461 – 11465, 2012.

[17] P. G. Todorov: On the Faber polynomials of the univalent functions of class Σ, J. Math. Anal. Appl., 162, pp. 268 – 276, 1991.