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# ADAPTIVE-ROBUST CONTROL DESIGN METHODS

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**Rezumat.** *Articolul prezintă o procedură de dezvoltare a sistemelor de control implementate pe procese neliniare cu variabile dinamice. Strategia de control propusă este una adaptiv-robustă, ce ia în considerare atât avantajele controlului adaptiv cât si ale controlului robust şi foloseşte acelaşi criteriu integral pentru identificarea procesor şi pentru algoritmii de control. Un criteriu de optimalitate integrală şi o măsură corespunzătoare a degradării performanţelor sistemului, datorită variaţiilor modelului dinamic, sunt introduse. Acest criteriu integral este exprimat într-o formă directă, printr-o funcţie cost, definită în spaţiul parametrilor modelului şi controlerului. Pentru minimizarea funcţiei neliniare, este folosită o metodă numerică de programare neliniară. Abordarea teoretică prezentată in această lucrare este validată într-un sistem în buclă închisă, aplicaţia fiind dezvoltată în Visual C#5.*

**Abstract.** *This paper presents a design procedure for control systems implemented on dynamic variable and nonlinear processes. The proposed adaptive-robust control strategy is taking into account both adaptive control advantages and robust control benefits and is using the same integral criterion for the identification of the process and for the control algorithm design. An optimality integral criterion and an appropriate measure for degradation of the system performances due to variation of the dynamic model are introduced. This inteogral criterion is expressed in a direct form, through a cost function, defined in the model and the controller parameters' space. For the minimization of this nonlinear function, a numerical mathematic nonlinear programming method is used. The theoretical approach presented in this paper is validated on a close loop control system, the application being developed in Visual C#5.*

**Keywords:** control systems, adaptive control, robust control, system optimization

## 1. Introduction

The identification of the process and the control algorithm design are two important steps in the implementation of any control system solution.

The computer control system design consists in the effort of identifying the process P through the dynamic model M and in computing the control algorithm, C.

The pair (C, M) defines the nominal system (NS), illustrated in fig. 1.

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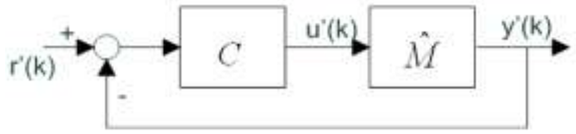


**Fig. 1.** Nominal System (*NS*).

The mathematical model *M* is obtained by solving the optimization problem:

$$min\ J_I\ (C,M) \tag{1}$$

with $J_I$, an identification criterion built with the help of the prediction error. The control algorithm *C*, based on the identified model, is obtained as the solution of a second optimization problem,

$$min\ J_C\ (C,\ M) \tag{2}$$

The optimal criterion $J_C$ is built with the help of the regulation error of the closed loop system. The design of the control algorithm for the nominal system (*NS*) may therefore endeavour to solve these two different optimization problems.

Our main objective is to ensure the real system (*RS*) represented by the pair *(μC,P)* from fig.2, with the realized performances (*RP*) close to the nominal performances (*NP*) obtained in simulation over the system (*NS*).

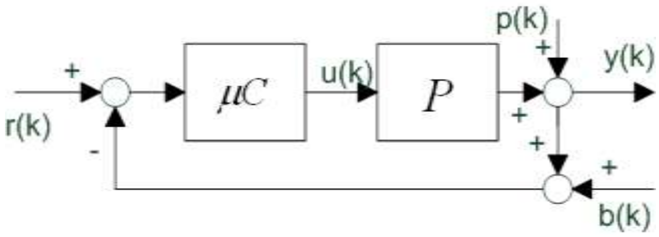


**Fig. 2.** *Real System (RS).*

For the system (RS) with variable evolution of P, estimated through a model M, it is recommended to be implemented either an adaptive strategy or a robust strategy.

The adaptive control is efficient for the case where the processes have variable parameters and implies the (re)identification of the process model and the (re)design of the control algorithm at every sampling time; for a new model, only with a new controller will the performances of the system be conserved. The expression that is showing the adaptive control strategy from the moment (k) to the moment (k + 1) is as follows:

$$(C(k),M(k)) \to (C(k+1),M(k+1)) \tag{3}$$

The robust control is recommended for nonlinear processes with parametric or structural disturbances, and proposes a single controller that ensures the desired performances for a class of candidate models of the process, including process P itself. The robustness of the controller is expressed formally by equation (4), with the distance d, small enough:

$$|PC/(1+PC)-MC/(1+MC)|< d \tag{4}$$

Our research work proposes an integrated adaptive-robust design, built on the advantages of the two strategies that also eliminate their individual disadvantages.

Compared to the classical adaptive strategy, the adaptive-robust procedure preserves nominal performances for the implemented physical system by (re)identification, at the point where a significant degradation of the current model occurs and not at each sampling time. This is clearly reducing the numerical effort and the difficulties of the closed-loop identification procedure.

The robust strategy calculates a single (robust) control algorithm, tolerant to the nonlinearities and parametric or structural disturbances of the process; in this case, the performances required are for all the different operating points of the process (set of models).

The control algorithm remains unchanged as long as the robustness criterion that is measuring the degradation is respected. Only in the case of an observed excessive degradation of the model, will the controller be (re)calculated after the (re)identification of the dynamic model is finished. This procedure eliminates the relatively difficult calculation of the correction of the control algorithm, by means of imposing a reserve of robustness [9], [10].

## 2. Adaptive-Robust Control

We forget for instance the identification criterion and we consider a single optimal criterion (integral criteria for optimal control), thus we propose a recursive numerical procedure, where the two problems of identification and control are both integrated.

The two interconnected problems in a recursive way can be solved in a unified manner, based on their dual character.

For the process P, model M and the associated controller C, the next inequality is respected:

$$\begin{aligned} &\left\| J\ \ M,C\ \ M \ \right| - \left| J\ \ P\ ,C\ \ M \ \ - J\ \ M,C\ \ M \ \right\| \le \left| J\ \ P\ ,C\ \ M \ \right| \le \\ &\le \left| J\ \ M,C\ \ M \ \right| + \left| J\ \ P\ ,C\ \ M \ \ - J\ \ M,C\ \ M \ \right| \end{aligned} \tag{7}$$

The terms of the inequality (7) have the meaning:

$| J\ \ M,C\ \ M\ \ |$, the measurement of the nominal system’s performances (SN);

$|J(P,C(M))|$, the measurement of the real system's performances (SR);

$\left|J(P,C(M)) - J(M,C(M))\right|$, the measurement of the degradation of the performances due to changes in the model, or due to the measurement of the degradation of the system robustness.

With the help of the upper inequalities are established the conditions to be met for nominal performances to be reflected in the achieved performances by the physical (real) system, (RS).

The measurement of the degradation will be verified by the inequation:

$$\left|J(P,C(M)) - J(M,C(M))\right| \le \left|J(M,C(M))\right| \quad (8),$$

where $\left|J(M,C(M))\right|$ takes small enough values, imposed by a specified limit, $\delta$ .

The restriction (8) expresses the robust behaviour of the algorithm, since degradation is bordered by low values provided by $|J(M,C(M))|$.

So, the recursive mechanism may proceed or not according to the assessment of the previous inequation.

We consider the controller $C_i$ available for the actual model $M_i$ but if the robustness performances are unsatisfied, it is necessary to calculate the next model using the following relation (not the standard identification algorithm):

$$M_{i+1} = \arg\min_{M} \left|J(P,C_i) - J(M,C_i)\right| \quad (9)$$

For the model $M_{i+1}$ we build the next controller:

$$C_{i+1} = \arg\min_{C} \left|J(M_{i+1},C)\right| \quad (10)$$

For each step of calculation, we check the robustness constraints imposed by the size of the value δ.

$$\left|J(M_{i+1},C_{i+1})\right| < \delta, \quad (11)$$

and,

$$\left|J(P,\mathrm{C}_{i+1}) - J(M_{i+1},\mathrm{C}_{i+1})\right| \le \left|J(M_{i+1},C_{i+1})\right| < \delta \quad (12)$$

The next computation is not being performed at each sampling time; it becomes active when the degradation relationship (12) is not checked. The recursive computation ends with the fulfilment of the inequality (12) which measures performance (robustness) degradation and resume when inequality is violated.

The connection between the two problems, of identification and control, is illustrated by the recursive mechanism expressed by the above figure 3.

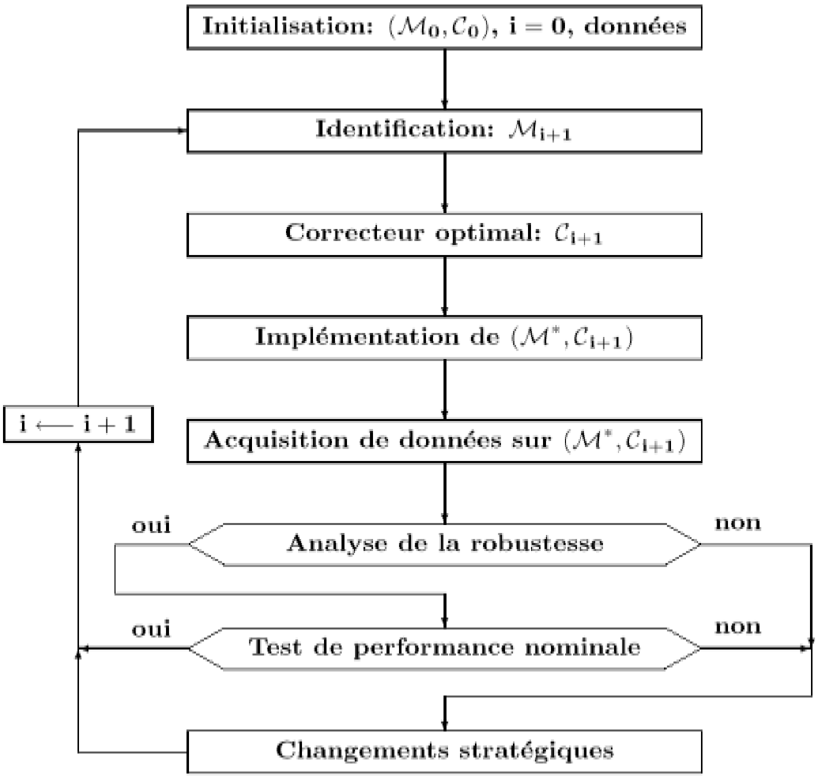


**Fig.3** Flow diagram for adaptive-robust control strategy.

## 3. Validation of the theoretical approach and implementation aspects

To validate the adaptive-robust mechanism, a simple numerical example is considered by the 1st order process,

$$P = \frac{K}{Ts + 1} \tag{5}$$

and by a standard PI controller:

$$C = K_r \left(1 + \frac{1}{T_i s}\right) \tag{6}$$

Let the integral criterion usually used in the optimal control theory:

$$J = \int_0^\infty \varepsilon^2(t) + T_\varepsilon^2 \dot{\varepsilon}^2(t) dt \tag{7}$$

where the first part of the criterion is used to minimize the response time, while the second one is needed to control the speed of the closed loop dynamic system.

The computational effort to minimize this integral criterion is very difficult. Therefore, we shall rewrite it in a direct form, using the algorithm from [8].

Firstly, we separate the two terms of the integral,

$$J = \int_0^{\infty} \varepsilon^2(t) + T_\varepsilon^2 \dot{\varepsilon}^2(t) dt = J_1 + T_\varepsilon^2 J_2 \tag{8}$$

and we shall separately compute each term of the sum, in the direct form.

We continue by applying the Laplace transform for $J_1$

$$J_1 = \int_0^{\infty} \varepsilon^2(t) dt = \int_0^{\infty} \varepsilon(t) \{L^{-1}[\varepsilon(s)]\} dt \tag{9}$$

$$J_1 = \int_0^{\infty} \varepsilon(t) \left[ \frac{1}{2\pi j} \int_{-j\infty}^{j\infty} \varepsilon(s) e^{st} ds \right] dt \tag{10}$$

$$J_1 = \frac{1}{2\pi j} \int_{-j\infty}^{j\infty} \varepsilon(s) \left[ \int_0^{\infty} \varepsilon(t) e^{st} dt \right] ds = \frac{1}{2\pi j} \int_{-j\infty}^{j\infty} \varepsilon(s) \varepsilon(-s) ds \tag{11}$$

The direct form result for $J_1$ is,

$$J_1 = \frac{T_i + K_r K T}{2 K_r K (1 + K_r K)} \tag{12}$$

Based on the same methodology, we obtained the direct expression for the second part of the integral criterion *J*,

$$J_2 = \frac{K_r K \left(1 + K_r K \frac{T_i}{T}\right)}{2 T_i (1 + K_r K)} \tag{13}$$

In the end, the integral criterion *J* is equal to:

$$J = J_1 + J_2 = \frac{T_i + K_r K T}{2 K_r K (1 + K_r K)} + T_\varepsilon^2 \frac{K_r K \left(1 + K_r K \frac{T_i}{T}\right)}{2 T_i (1 + K_r K)} \tag{14}$$

The optimization problem will be:

$$\min \{J(K, T, Kr, Ti)\} \tag{15}$$

in the space of the model and controller parameters, eventually with some constraints in the parameters.

When the degradation condition is violated, the control algorithm parameters are preserved and an (re)identification procedure estimates the process model; a new controller will be computed corresponding to the new model, and the degradation performances condition will be respected.

The minimization of the nonlinear criterion (24) is accomplished using an appropriate nonlinear mathematical programming method. We chose the Nelder – Mead method, which is based on the SIMPLEX research algorithm [2], [3], this algorithm offering good performances for this class of problems. The SIMPLEX algorithm may converge to the minimum-point, solution for the nonlinear optimization problem (15).

To test the adaptive – robust control system, a real time simulation mode was implemented based on the dedicated software. With a given input data: sampling period and the first identified model of the process, the application will simulate the evolution of the given process, starting from the initial process *P* (parameters).

The first time the – mechanism runs with the controller *Ci* implemented on the process *P* , the controller *Ci* beeing calculated for the identified model *Mi* and the following steps are introduced:

- Generate new parameters for the process, by using a random number generator.
- With the above generated values, and using the controller *Ci*, the value of the integral criterion, *J* is calculated for *Mi* model and for the process *P*.
- The robustness test is run. This test consists in verifying the robustness degradation inequality.

  - if the inequality is respected, the test is passed and the current controller *Ci* is used to control the process.

  - if the pair (*Ci*, *Mi*) fails the robustness test, the algorithm will estimate a new model *Mi+1* which will be used to calculate the new controller, *Ci+1* which is used to control the process.

It is obvious that the computational effort for the implementation of this algorithm is significantly reduced when compared to the adaptive control or the robust control, respectively.

**Conclusions**

We proposed an adaptive – robust strategy for the design of a control system, combining the adaptive control and robust control theories taking into account the advantages of both adaptive and robust strategies. The standard model based control design procedure is ignored and a new strategy is developed, based on the measurement of the performance degradation for closed loop system. The process identification problem is treated implicitly, using an indicator that measures the degradation of the performances of the system, due to the dynamic changes noted in the process evolution. The control algorithm design is based on an integral criterion of optimality expressed in the direct form in order to facilitate the computational effort. The optimal control algorithm's parameters are obtained by minimizing the criterion function, using the SIMPLEX method. The theoretical approach presented in this work is validated by the implementation of the adaptive – robust algorithm for a closed loop control system, simulated in an application developed in C programming language. The mechanism of the adaptive - robust control system strategy can be implemented in different real time industrial applications.

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