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SOME CONSIDERATIONS ON DISLOCATION FOR THERMOELASTIC MICROSTRETCH MATERIALS

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Rezumat. *Scopul studiului nostru este de a obţine o relaţie de tip De Hoop-Knopoff pentru deplasarea câmpurilor în contextul corpurilor termoelastice cu microstructură. Apoi, ca o consecinţă, se obţine o expresie explicită a sarcinii corpului echivalentă cu o dislocare seismică. Rezultatele sunt extensii ale celor din teoria clasică a corpurilor elastice.*

Abstract. *The aim of our study is to derive a relation of De Hoop-Knopoff type for displacement fields within the context of thermoelastic microstretch bodies. Then, as a consequence, an explicit expression of the body loadings equivalent to a seismic dislocation, is obtained. The results are extensions of those from the classical theory of elastic bodies.*

Keywords: microstretch; seismic; dislocation; thermoelastic body

1. Introduction

The theory of thermo-microstretch elastic solids was first elaborated by Eringen, [4], and, briefly, this is a theory of thermoelasticity with microstructure that includes intrinsic rotations and microstructural expansion and contractions.

The purpose of this theory is to eliminate discrepancies between classical elasticity and experiments, since the classical elasticity failed to present acceptable results when the effects of material microstructure were known to contribute significantly to the body's overall deformations, for example, in the case of granular bodies with large molecules (e.g. polymers), graphite or human bones.

These cases are becoming increasingly important in the design and manufacture of modern day advanced materials, as small-scale effects become paramount in the prediction of the overall mechanical behavior of these materials.

Other intended applications of this theory are to composite materials reinforced with chopped fibers and various porous materials.

This theory can be useful in the applications which deal with porous materials as geological materials, solid packed granular materials and many others.

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On the other hand, materials which operate at elevated temperatures will invariably be subjected to heat flow for some time during normal use. Such heat flow will involve a non-linear temperature distribution which will inevitable give rise to thermal stresses. For these reasons, the development, design and selection of materials for high temperature applications require a great deal of care. The role of the pertinent material properties and other variables which can affect the magnitude of thermal stress must be considered.

The main difficulty of the thermo-microstretch materials is the large number of the thermoelastic coefficients and, as such, the problem of their determination in the laboratory. Yet many authors consider that this problem will be solved in the future.

Already, in the isotropic case, when strongly decreases the number of coefficients, they are calculated as can be seen in many works due to Eringen or Iesan, see [4]-[6].

The present paper must be considered as a first step to a better understanding of microstretch and thermal stress in the study of above enumerated materials.

The reciprocity and representation relations that appear in our study constitute powerful theoretical tools in the assessment of the theory of seismic-sources mechanism, in the studies connected with seismic wave propagation.

Also, we think that this paper is a good help to understanding the application of microstretch mechanism to earthquake problems.

There are many results regarding the mechanism of earthquake, as in the papers [1]-[3] and [5]-[8].

For instance, in the paper [6] the authors establish a reciprocity relation which forms the basis for a uniqueness result, a continuous dependence of solutions upon initial data and body loads and a variational characterization of solutions. The effects of a concentrated heat supply and of a concentrated heat volume charge density in an unbounded homogeneous and isotropic electromagnetic body are investigated.

We find another results regarding thermoelasticity of microstretch materials, as in the papers [4], [9], [10]. The main results of our study are extensions of some similar results in the classical elasticity in order to cover the thermoelasticity of microstretch bodies.

First expressions of the body loadings equivalent to a seismic dislocation where obtained by De Hoop [2] and Knopoff [7].

2. Basic equations

For convenience the notations and terminology chosen are almost identical to those of our studies [9], [10]. Our present paper is concerned with an anisotropic and homogeneous material.

Let the body occupy, at time $t = 0$, a properly regular region B of the three-dimensional Euclidian space, bounded by the piece-wise smooth surface ∂B and we denote the closure of B by $\bar{B}$. We refer the motion of the body to a fixed system of rectangular Cartesian axes $Ox_i, i = 1,2,3$ and adopt the Cartesian tensor notation. Points in B are denoted by x_j and $t \in [0, \infty)$ is temporal variable. Throughout this work the Einstein summation convention over repeated indices is used. The subscript j after comma indicates partial differentiation with respect to the spatial argument x_j. All Latin subscripts are understood to range over the integers $(1,2,3)$, while the Greek indices have the range $(1,2)$. A superposed dot denotes the derivatives with respect to the $t-$ time variable. Also, the spatial argument and the time argument of a function will be omitted when there is no likehood of confusion.

Let us denote by u_i the components of the displacement vector and by φ_i the components of the microrotation vector. Also, we denote by ω the microstretch function and by θ the temperature measured from the constant absolute temperature T_0 of the body in its reference state.

As usual, we denote by t_{ij} the components of the stress tensor and by m_{ij} the components of the couple stress tensor over B. Also, we denote by λ_i the components of the microstress vector.

For clarity and simplification in presentation, the regularity hypotheses on the considered functions will be omitted.

On these grounds, the field equations in the dynamic theory of thermoelasticity of microstretch bodies are, (see, [4], [10]):

- the equations of motion

$$\begin{aligned} &t_{ji,j} + \rho F_i = \rho \ddot{u}_i, \\ &m_{ji,j} + \varepsilon_{ijk} t_{jk} + \rho G_i = I_{ij} \ddot{\varphi}_j; \end{aligned} \tag{1}$$

- the balance of the equilibrated forces

$$\lambda_{i,i} - s + \rho L = J\ddot{\omega}. \tag{2}$$

The equation of energy is given by

$$\rho T_0 \dot{\eta} = q_{i,i} + \rho r. \tag{3}$$

In the above equations we have used the following notations:

- F_i the components of body force;

- G_i the components of body couple;

- $L=$ the generalized external body load;
- s is the generalized internal body load;
- λ_i are the components of the internal hypertraction vector;
- ρ is the reference constant mass density;
- J and $I_{ij}=I_{ji}$ are the coefficients of microinertia;
- t_{ij}, m_{ij} are the respective components of the stress and couple stress;
- η is the entropy per unit mass;
- r is the heat supply per unit mass;
- q_i are the components of the heat flux vector.

For an anisotropic and homogeneous microstretch thermoelastic material, the consti-tutive equations have the form:

$$\begin{aligned}
&t_{ij} = A_{ijrs}\,\varepsilon_{rs} + B_{rsij}\mu_{rs} + D_{ijr}\gamma_r + a_{ij}\omega - E_{ij}\theta,\\
&m_{ij} = B_{ijrs}\,\varepsilon_{rs} + C_{ijrs}\mu_{rs} + E_{ijr}\gamma_r + b_{ij}\omega - D_{ij}\theta,\\
&\lambda_i = D_{rsi}\,\varepsilon_{rs} + E_{rsi}\mu_{rs} + C_{ij}\gamma_j + d_i\omega - L_i\theta,\\
&s = a_{ij}\,\varepsilon_{ij} + b_{ij}\mu_{ij} + d_i\gamma_i + m\omega - \alpha\theta,\\
&\eta = \eta_0 + E_{ij}\,\varepsilon_{ij} + D_{ij}\mu_{ij} + L_i\gamma_i + \alpha\omega + \frac{a}{T_0}\theta,\\
&q_i = k_{ij}\theta_{,j}.
\end{aligned} \tag{4}$$

where A_{ijrs}, B_{ijrs}, C_{ijrs}, D_{ijr}, E_{ijr}, a_{ij}, b_{ij}, c_{ij}, d_i, E_{ij}, D_{ij}, L_i, m, α, a and k_{ij} are the characteristic constitutive coefficients.

The components of the strain tensors ε_{ij}, μ_{ij} and γ_i are defined by means of the geometric equations:

$$\varepsilon_{ij} = u_{j,i} + \varepsilon_{jik}\varphi_k,\ \mu_{ij=}\varphi_{j,i},\ \gamma_i = \omega_{,i}, \tag{5}$$

where ε_{ijk} is the alternating symbol.

The constitutive coefficients obey the following symmetry relations

$$A_{ijrs} = A_{rsij}, C_{ijrs} = C_{rsij}, C_{ij} = C_{ji}, k_{ij} = k_{ji}. \tag{6}$$

One can assume that a positive constant λ_0 exists such that $I_{ij}\xi_i\xi_j \ge \lambda_0\xi_i\xi_i, \forall\xi_i$.

Also, the Second Law of Thermodynamics implies that:

$$k_{ij}\xi_i\xi_j \ge 0, \forall\xi_i. \tag{7}$$

We denote by t_i the components of surface traction, m_i the components of surface couple, p the microsurface traction and q the heat flux. These quantities are defined by $t_i = t_{ji}n_j$, $m_i = m_{ji}n_j$, $p = \lambda_i n_i$, $q = q_i n_i$, at regular points of the surface ∂B.

Here, n_i are the components of the outward unit normal of the surface ∂B.

Along with the system of field equations (1-5) we consider the following initial conditions:

$$\begin{aligned} &u_i(x,0) = u_i^0(x), \;\; \dot{u}_i(x,0) = u_i^1(x), \\ &\varphi_i(x,0) = \varphi_i^0(x), \;\; \dot{\varphi}_i(x,0) = \varphi_i^1(x), \; x \in \overline{B} \\ &\omega(x,0) = \omega^0(x), \;\; \dot{\omega}(x,0) = \omega^1(x) \\ &\theta(x,0) = \theta^0(x), \end{aligned} \tag{8}$$

and the following prescribed boundary conditions

$$\begin{aligned} &u_i = \overline{u}_i \text{ on } \partial B_1 \times [0,t_0),\; t_i = \bar{t}_i \text{ on } \partial B_1^c \times [0,t_0), \\ &\varphi_i = \overline{\varphi}_i \text{ on } \partial B_2 \times [0,t_0),\; m_i = \overline{m}_i \text{ on } \partial B_2^c \times [0,t_0), \\ &\omega = \overline{\omega} \text{ on } \partial B_3 \times [0,t_0),\; p = \overline{p} \text{ on } \partial B_3^c \times [0,t_0), \\ &\theta = \overline{\theta} \text{ on } \partial B_4 \times [0,t_0),\; q = \overline{q} \text{ on } \partial B_4^c \times [0,t_0), \end{aligned} \tag{9}$$

where t_0 is some instant that may be infinite. Also, $\partial B_1, \partial B_2$, ∂B_3 and ∂B_4 with respective complements $\partial B_1^c, \partial B_2^c$, ∂B_3^c and ∂B_4^c are subsets of the surface ∂B such that

$$\begin{aligned} &\partial B_1 \cap \partial B_1^c = \partial B_2 \cap \partial B_2^c = \partial B_3 \cap \partial B_3^c = \partial B_4 \cap \partial B_4^c = \varnothing \\ &\partial B_1 \cup \partial B_1^c = \partial B_2 \cup \partial B_2^c = \partial B_3 \cup \partial B_3^c = \partial B_4 \cup \partial B_4^c = \partial B. \end{aligned}$$

Also, $u_i^0, u_i^1, \varphi_i^0, \varphi_i^1, \omega^0, \omega^1, \theta^0, \overline{u}_i, \bar{t}_i, \overline{\varphi}_i, \overline{m}_i, \overline{\omega}$, $\overline{p}$, $\overline{\theta}$ and $\overline{q}$ are prescribed functions in their domains.

By a solution of the mixed initial boundary value problem of the theory of thermoelasticity of microstretch bodies with voids in the cylinder $\Omega_0 = B \times [0,t_0)$ we mean an ordered array $(u_i, \varphi_i, \omega, \theta)$ which satisfies the equations (1), (2) and (3) for all $(x,t) \in \Omega_0$, the boundary conditions (9) and the initial conditions (8).

3. Main results

Let u and v be functions defined on $\overline{B} \times [0,\infty)$ and continuous on $[0,\infty)$ with respect to the time variable t for each spatial variable $x \in \overline{B}$. We denote by $u * v$ the convolution of u and v, that is

$$(u * v)(x,t) = \int_0^t u(x,t-\tau)v(x,\tau)d\tau. \tag{10}$$

Let us introduce the notations

$$g(t)=t,\ h(t)=1,\ f_i=\rho g*F_i+\rho\left[tu_i^1(x)+u_i^0(x)\right],$$
$$g_i=\rho g*G_i+I_{ij}\left[t\varphi_j^1(x)+\varphi_j^0(x)\right],\ l=\rho g*L+J\left[t\omega^1(x)+\omega^0(x)\right],\ w=\rho h*r+\rho T_0\eta_0 \tag{11}$$

Following the same procedure used by Iesan in [5], it is easy to prove the following result, which enables us to give an alternative formulation of the initial boundary value problem in which the initial data are incorporated into the field of equations.

Theorem 1. The functions $u_i, \varphi_{jk},\ \sigma, \theta,\ \ \tau_{ij},\ \eta_{ij}, \mu_{ijk}$ and q_i satisfy the equations (1), (2), (3) and the initial conditions (8) if and only if they satisfy the following system of equations

$$\begin{aligned}
&g*t_{ji,j}+f_i=\rho u_i\\
&g*\left(m_{ji,j}+\varepsilon_{ijk}t_{jk}\right)+g_i=I_{ij}\varphi_j\\
&g*\left(\lambda_{i,i}-s\right)+l=J\omega\\
&h*q_{i,i}+w=\rho T_0\eta
\end{aligned} \tag{12}$$

In our following estimations, we will use the formulation (12) of the mixed problem. We wish to find the behavior of the considered medium when embedded in B there is a discontinuity surface Σ for the displacements, the microrotation vector, the microstretch function and the temperature. The sides of Σ are denoted by Σ^- and Σ^+.

Let ν_i be the components of the unit normal vector of Σ, directed from the side $(-)$ to the side $(+)$. Then on surface Σ we consider the conditions

$$\begin{aligned}
&u_i^+-u_i^-=U_i,\ t_{ji}^+\nu_j=t_{ji}^-\nu_j\\
&\varphi_i^+-\varphi_i^-=\Phi_i,\ m_{ij}^+\nu_j=m_{ij}^-\nu_j\\
&\omega^+-\omega^-=\Psi,\ \lambda_j^+\nu_j=\lambda_j^-\nu_j\\
&\theta^+-\theta^-=\Theta,\ q_j^+\nu_j=q_j^-\nu_j
\end{aligned} \tag{13}$$

where f^+ and f^- are the limits of the function $f(x)$ as x approaches a point on the side $(+)$ or $(-)$ of the surface Σ, respectively, and U_i, Φ_i, Ψ and Θ are prescribed functions. In this way we can consider the equations (12) in the domain $B\setminus\Sigma$.

Let us consider two different systems of external data for the body

$$G^{(\alpha)}=\left\{F_i^{(\alpha)}, G_i^{(\alpha)}, L^{(\alpha)}, r^{(\alpha)}, \bar{u}_i^{(\alpha)}, \bar{\varphi}_i^{(\alpha)}, \bar{\omega}_i^{(\alpha)}, \bar{\theta}^{(\alpha)}, \bar{t}_i^{(\alpha)}, \bar{m}_i^{(\alpha)}, \bar{h}^{(\alpha)}, \bar{q}^{(\alpha)}, U_i, \Phi_i, \Psi, \Theta\right\}, \alpha=1,2$$

and two corresponding solutions

$$S^{(\alpha)}=\left\{u_i^{(\alpha)}, \varphi_i^{(\alpha)}, \omega^{(\alpha)}, \theta^{(\alpha)}, \varepsilon_{ij}^{(\alpha)}, \mu_{ij}^{(\alpha)}, t_{ij}^{(\alpha)}, m_{ij}^{(\alpha)}, \lambda_i^{(\alpha)}, s^{(\alpha)}, q_i^{(\alpha)}\right\}, \alpha=1,2,$$

For the sake of simplicity, we now introduce the notations

$$\begin{aligned} & t_i = t_{ij} n_j ,\, T_i = t_{ij}^{+} \nu_j \\ & m_i = m_{ij} n_j ,\, M_i = m_{ij}^{+} \nu_j \\ & \lambda = \lambda_i n_i ,\, \Lambda = \lambda_i^{+} \nu_i \\ & q = q_i n_i ,\, Q = q_i^{+} \nu_i \end{aligned} \tag{14}$$

In the following theorem, we prove a reciprocity relation of Betti type.

Theorem 2. *If a microstretch thermoelastic body is subjected to two systems of loadings* $G^{(\alpha)}$ *then between the corresponding solutions* $S^{(\alpha)}$ *there is the following reciprocity relation*

$$\begin{aligned} & \int_B \left(f_i^{(1)} * u_i^{(2)} + g_i^{(1)} * \varphi_i^{(2)} + l^{(1)} * \omega^{(2)} - \frac{1}{T_0} g * \omega^{(1)} * \theta^{(2)} \right) dV + \\ & + \int_{\Sigma} g * \left(T_i^{(1)} * U_i^{(2)} + M_i^{(1)} * \Phi_i^{(2)} + \Lambda^{(1)} * \Psi^{(2)} - \frac{1}{T_0} h * Q^{(1)} * \Theta^{(2)} \right) dA = \\ & = \int_B \left(f_i^{(2)} * u_i^{(1)} + g_i^{(2)} * \varphi_i^{(1)} + l^{(2)} * \omega^{(1)} - \frac{1}{T_0} g * \omega^{(2)} * \theta^{(1)} \right) dV + \\ & + \int_{\partial B} g * \left(t_i^{(2)} * u_i^{(1)} + m_i^{(2)} * \varphi_i^{(1)} + \lambda^{(2)} * \omega^{(1)} - \frac{1}{T_0} h * q^{(2)} * \theta^{(1)} \right) dA - \\ & + \int_{\Sigma} g * \left(T_i^{(2)} * U_i^{(1)} + M_i^{(2)} * \Phi_i^{(1)} + \Lambda^{(2)} * \Psi^{(1)} - \frac{1}{T_0} h * Q^{(2)} * \Theta^{(1)} \right) dA \end{aligned} \tag{15}$$

Proof. In view of symmetry relations (6) and with the aid of the constitutive relations (4), by direct calculations it is easy to obtain

$$\begin{aligned} & t_{ij}^{(1)} * \varepsilon_{ij}^{(2)} + m_{ij}^{(1)} * \mu_{ij}^{(2)} + \lambda_i^{(1)} * \gamma_i^{(2)} + s^{(1)} * \omega^{(2)} - \rho \theta^{(1)} * \eta^{(2)} = \\ & = t_{ij}^{(2)} * \varepsilon_{ij}^{(1)} + m_{ij}^{(2)} * \mu_{ij}^{(1)} + \lambda_i^{(2)} * \gamma_i^{(1)} + s^{(2)} * \omega^{(1)} - \rho \theta^{(2)} * \eta^{(1)} \end{aligned} \tag{16}$$

Let us introduce the notation

$$I_{\alpha\beta} = \int_B g * \left[t_{ij}^{(\alpha)} * \varepsilon_{ij}^{(\beta)} + m_{ij}^{(\alpha)} * \mu_{ij}^{(\beta)} + \lambda_i^{(\alpha)} * \gamma_i^{(\beta)} + s^{(\alpha)} * \omega^{(\beta)} - \rho \theta^{(\alpha)} * \eta^{(\beta)} \right] dV \tag{17}$$

Based on the identity (16) and the notation (17), it is easy to see that

$$I_{\alpha\beta} = I_{\beta\alpha} \tag{18}$$

With the aid of the equations of motion and the equations (12), we can write

$$\begin{aligned} & g * \left[t_{ij}^{(\alpha)} * \varepsilon_{ij}^{(\beta)} + m_{ij}^{(\alpha)} * \mu_{ij}^{(\beta)} + \lambda_i^{(\alpha)} * \gamma_i^{(\beta)} + s^{(\alpha)} * \omega^{(\beta)} - \rho \theta^{(\alpha)} * \eta^{(\beta)} \right] = \\ & = g * \left[t_{ji}^{(\alpha)} * u_j^{(\beta)} + m_{ji}^{(\alpha)} * \varphi_j^{(\beta)} + \lambda_i^{(\alpha)} * \omega^{(\beta)} - \frac{1}{T_0} h * q_i^{(\alpha)} * \theta^{(\beta)} \right]_{,i} + \\ & + f_i^{(\alpha)} * u_i^{(\beta)} + g_i^{(\alpha)} * \varphi_i^{(\beta)} + l^{(\alpha)} * \omega^{(\beta)} - \frac{1}{T_0} g * w^{(\alpha)} * \theta^{(\beta)} - \\ & - \left[\rho u_i^{(\alpha)} * u_i^{(\beta)} + I_{ij} \varphi_i^{(\alpha)} * \varphi_j^{(\beta)} + J \omega^{(\alpha)} * \omega^{(\beta)} \right] + \frac{1}{T_0} g * h * k_{ij} \theta_{,i}^{(\alpha)} * \theta_{,i}^{(\beta)} . \end{aligned} \tag{19}$$

By integrating in (19) and using the divergence theorem, we are lead to

$$\begin{aligned} I_{\alpha\beta} &= \int_B \left(f_i^{(\alpha)} * u_i^{(\beta)} + g_i^{(\alpha)} * \varphi_i^{(\beta)} + l^{(\alpha)} * \omega^{(\beta)} - \frac{1}{T_0} g * w^{(\alpha)} * \theta^{(\beta)} \right) dV + \\ &+ \int_{\partial B} g * \left(t_i^{(\alpha)} * u_i^{(\beta)} + m_i^{(\alpha)} * \varphi_i^{(\beta)} + \lambda^{(\alpha)} * \omega^{(\beta)} - \frac{1}{T_0} h * q^{(\alpha)} * \theta^{(\beta)} \right) dA - \\ &- \int_\Sigma g * \left(T_i^{(\alpha)} * U_i^{(\beta)} + M_i^{(\alpha)} * \Phi_i^{(\beta)} + \Lambda^{(\alpha)} * \Psi^{(\beta)} - \frac{1}{T_0} h * Q^{(\alpha)} * \Theta^{(\beta)} \right) dA \end{aligned} \tag{20}$$

Finally, introducing (20) into (17), we arrive at the desired result (15).

It is easy to see that in the absence of discontinuities we obtain the generalization, in the context of the thermoelasticity of thermoelastic microstretch bodies, of the previous results established in the classical thermoelastodynamics.

Based on the relation (15), we now calculate the thermomechanical body loadings equivalent to a given dislocation. To this aim, we assume that $u_i^{(2)}, \varphi_i^{(2)}$, $\omega^{(2)}$ and $\theta^{(2)}$, as functions of (t,x), where $x=(x_i)$, are of class $C^\infty(B\times[0,\infty))$. Of course, if the functions $u_i^{(2)}$, $\varphi_i^{(2)}$, $\omega^{(2)}$ and $\theta^{(2)}$ are given, then by means of equations (12), we can determine the functions $F_i^{(2)}$, $G_i^{(2)}$, $L^{(2)}$ and $r^{(2)}$.

Now, we restrict our considerations only to the case when $U_i^{(2)} = \Phi_i^{(2)} = 0, i = 1,2,3,$ $\Psi^{(2)} = \Theta^{(2)} = 0$ and $\eta^{(2)}$ correspond to the faulted medium. Then, in the case of identical boundary conditions, we obtain;

$$\begin{aligned} &\int_B \rho \left(F_i^{(1)} * u_i^{(2)} + G_i^{(1)} * \varphi_i^{(2)} + L^{(1)} * \omega^{(2)} - \frac{1}{T_0} h * r^{(1)} * \theta^{(2)} \right) dV = \\ &= \int_B \rho \left(F_i^{(2)} * u_i^{(1)} + G_i^{(2)} * \varphi_i^{(1)} + L^{(2)} * \omega^{(1)} - \frac{1}{T_0} h * r^{(2)} * \theta^{(1)} \right) dV - \\ &- \int_\Sigma g * \left(T_i^{(2)} * U_i^{(1)} + M_i^{(2)} * \Phi_i^{(1)} + \Lambda^{(2)} * \Psi^{(1)} - \frac{1}{T_0} h * Q^{(2)} * \Theta^{(1)} \right) dA \end{aligned} \tag{21}$$

In view of (14), we have

$$\begin{aligned} T_i^{(2)} &= \left[A_{ijmn}\varepsilon_{mn} + B_{ijmn}\mu_{mn} + D_{ijm}\gamma_m + a_{ij}\,\omega - E_{ij}\,\theta \right]\nu_j \\ M_i^{(2)} &= \left[B_{ijmn}\varepsilon_{mn} + C_{ijmn}\mu_{mn} + E_{ijm}\gamma_m + b_{ij}\,\omega - D_{ij}\,\theta \right]\nu_j \\ \Lambda^{(2)} &= \left[D_{mni}\varepsilon_{mn} + E_{mni}\mu_{mn} + C_{ij}\gamma_j + d_i\,\omega - L_i\,\theta \right]\nu_i \\ \Theta^{(2)} &= k_{ij}\,\theta_{,i}\nu_j \end{aligned}$$

Taking into account the definition of the Dirac translated measure, δ, we can prove the relation of the following type

$$\psi_i(\xi,t) = \int_B \psi_i(x,t)\,\delta(x-\xi)dV, \psi_{i,j}(\xi,t) = \int_B \psi_i(x,t)\,\delta_{,j}(x-\xi)dV \tag{22}$$

and then the relation (21) can be rewritten as follows:

$$\int_B \rho \Big[\big(F_i^{(1)} + F_i\big) * u_i^{(2)} + \big(G_i^{(1)} + G_i\big) * \varphi_i^{(2)} + \big(L^{(1)} + L\big) * \omega^{(2)} - \frac{1}{T_0} * \big(r^{(1)} + R\big) * \theta^{(2)} \Big] dV = \\ = \int_B \rho \left(F_i^{(2)} * u_i^{(1)} + G_i^{(2)} * \varphi_i^{(1)} + L^{(2)} * \omega^{(1)} \frac{1}{T_0} h * r^{(2)} * \theta^{(1)} \right) dV. \tag{23}$$

In the above relations we have used the notations

$$F_k = -\frac{1}{\rho} \int_\Sigma \Big[A_{jirk} U_i^{(1)} + B_{jirk} \Phi_i^{(1)} + D_{jkr} \Psi^{(1)} \Big] \delta_{,r}(x-\xi) \upsilon_j dA_\xi$$

$$G_k = -\frac{1}{\rho} \int_\Sigma \Big[\big(B_{jirk} U_i^{(1)} + C_{jirk} \Phi_i^{(1)} + E_{jkr} \Psi^{(1)} \big) \delta_{,r}(x-\xi) + \\ \varepsilon_{mnk} \big(A_{jimn} U_i^{(1)} + B_{jimn} \Phi_i^{(1)} + D_{jmn} \Psi^{(1)} \big) \delta(x-\xi) \Big] \upsilon_j dA_\xi$$

$$L = -\frac{1}{\rho} \int_\Sigma \Big[\big(D_{jim} U_i^{(1)} + E_{jim} \Phi_i^{(1)} + C_{jm} \Psi^{(1)} \big) \delta_{,m}(x-\xi) + \\ \big(a_{ji} U_i^{(1)} + b_{ji} \Phi_i^{(1)} + d_j \Psi^{(1)} \big) \delta(x-\xi) \Big] \upsilon_j dA_\xi$$

$$R = \frac{1}{\rho} \int_\Sigma T_0 \Big[\big(E_{ji} U_i^{(1)} + D_{ji} \Phi_i^{(1)} + L_j \Psi^{(1)} \big) \delta(x-\xi) - \\ k_i \delta_{,i}(x-\xi) \Big] \upsilon_j dA_\xi$$

4. Conclusions

In the absence of discontinuities we obtain the generalization, in the context of the thermoelasticity of thermoelastic microstretch bodies, of the previous results established in the classical thermoelastodynamics.

In the classical elasticity in the system of equations of motion occur only three unknown functions, while in our case the number of unknowns is seven.

Also, while in the classical elasticity the equations of motion are parabolic, hyperbolic or elliptic only, in our case we have a mixed system of equations.

Taking into account the relation (23) we deduce that the effect of discontinuities across the surface Σ can be represented by extra external body loads and heat supply.

Although these are supposed to act in an unfaulted medium and cannot in any sense represent real forces acting in the real medium, they may nevertheless provide, as pointed in the papers [2], [3] and [8], a useful theoretical tool and this because if two dislocations have the same equivalent force, they also emit the same radiation.

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