Annals of the Academy of Romanian Scientists
Series on Engineering Sciences
ISSN 2066 – 8570 Volume **5**, Number **2**/**2013**

STUDIES AND APPLICATIONS OF ABSOLUTE STABILITY OF THE NONLINEAR DYNAMICAL SYSTEMS

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Rezumat. *În această lucrare sunt prezentate metode de studiu pentru reglarea automată a stabilităţii absolute în cazul sistemelor dinamice neliniare. Sunt menţionate două metode pentru stabilitatea absolută cu criterii şi mod de aplicare: a) metoda Lurie cu determinarea efectivă a funcţiei Liapunov; b) metoda frecvenţială a cercetătorului român V.M. Popov utilizând funcţia de transfer în cazurile critice. Sistemele dinamice neliniare care sunt raportate la clase speciale includ blocuri liniare şi neliniare. Datorită perturbaţiilor compuse cu acţiune inversă de răspuns a componentelor regulatoarelor automate,acestea conduc la obţinerea unui regim absolut stabil. Modelarea matematică este analizată numeric, iar aplicaţia realizată cu aceste două metode este utilizată din industria tăierii metalelor, în stabilitatea absolută a aeroplanelor echipate cu pilot automat sau în oscilaţiile de ruliu pentru navele maritime*.

Abstract. *In this paper there are presented methods of study for the automatic regulation of the absolute stability in case of the nonlinear dynamical systems. There are specified two methods for the absolute stability with criteria and mode of application: a) the Lurie method with the effective determination of the Liapunov function; b) the frequencies method of the Romanian researcher V. M. Popov using the transfer function for the critical cases. The nonlinear dynamical systems which include nonlinear and linear blocks are reported to the special classes. Due to the coposed perturbations with inverse response action of the automatic regulator components these will lead to obtaining an absolute stable regime. The mathematical modelling is numerically analysed, and the realized application by these two methods is used in the metal cutting tools machine, in the absolute stability of the rate of aircrafts equipped with autopilot or in the case of the rolling oscillations for the ships*.

Keywords: Nonlinear systems; automatic stabilization; frequencies method; Liapunov method

1. Introduction

The automatic regulation for the stability of dynamical systems holds a fundamental position in science and technique, following the optimization of the technological process of the cutting tools, of the robots, of the movement vehicles regime, or of some machines components, of energetic radioactive regimes, of chemical, electromagnetic, thermal, hydro-aerodynamic regimes, etc.

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Both studies and technical achievements are completed with mathematical models for closed circuits with input - output, that enable automatic regulation to integrate some mechanisms and devices with inverse reaction of response for controlling and rapidly and efficiently eliminating the perturbations which can appears along these processes or dynamical regimes. Generally these dynamical regimes are nonlinear, thus there were necessary some contributions and special achievements for automatic regulation, in order to generate automatic regulation of absolute stability (a.r.a.s.) for these classes of nonlinearities.

We highlight two special methods (a.r.a.s.): • Liapunov's function method discovered by A. I. Lurie [13,15,20] and developed into a series of studies by M. A Aizerman, V. A. Iakubovici, F. R. Gantmaher, R. E. Kalman, D. R. Merkin [14] and others [1,17].

• The frequency method developed by the Romanian researcher V. M. Popov [18] generalizing the criterion of Nyquist, then developed in many studies [1, 2, 15].

We note the well-known contributions of Romanian researchers on the stability and optimal control theory: C. Corduneanu, A. Halanay, V. Barbu, Th. Morozan, G. Dinca, M. Megan, Vl. Rasvan, V. Ionescu, M. E. Popescu, S. Chiriacescu, A. Georgescu and also directly on (a.r.a.s.): I. Dumitrache [4] D. Popescu [16], C. Belea [2], V. Rasvan [19], S. Chiriacescu [3] and other recent works [6-12].

The research has shown that both methods are equivalent, and studies can be qualitatively or numerically. In this paper we will present the actual working methods in the cases of singularity studies through applications.

2. (A.R.A.S.) Using the Liapunov's function method

In this part we'll present Lurie's ideas and the effective method founding Liapunov's function [2, 13, 14, 19]. Generally, systems of automatic regulation are composed of the controlled processor system, the sensory elements of measurement, the acquisition board, and the feedback controller mechanism. The regulator represents all the sensors and the acquisition board, but the controller is including feedback mechanism. Parameters characterizing the object of the control system that controls woring mode are measured by sensors, and their records on the sensor response mechanism ζ are transmitted to the acquisition board. This processes of command σ, are mechanically transmitted to the controller which, on its turn, distributes the object state, and interact simultaneously adjusting the response mechanism. We highlight the dynamic system equations. We note by $x_1, x_2, \ldots, x_n$ the state parameters of the regime's subject which must be controlled, the coordinates and the sensorial speeds. We rename in variation of these parameters if the open circuit (excluding the controller) system is described by linear differential equations with constant coefficients: $\dot{x}_k = \sum_{j=1}^{n} a_{kj} x_j, k = 1,\dots,n$. If the system is with closed loop then the variables $x_1, x_2, \dots, x_n$ will influence the regulation body, and we note by ξ its state. In this case for the autonomous closed system we have the equations:

$$\dot{x}_k = \sum_{j=1}^{n} a_{kj} x_j + b_k \xi, k = 1,\dots,n \tag{1}$$

We'll consider that the mechanism or inverse reaction is determined on the output ζ with the rigidity connection on the input ξ :

$$\zeta = k\xi \tag{2}$$

The acquisition board collects the signals and transmits the input sensors in order to obtain the embedded system:

$$\sigma = \sum_{j=1}^{n} c_j x_j - r\xi \tag{3}$$

where c_j, r are transfer numbers, r is the transfer coefficient of the inverse rigid connection, $r > 0$ (the regulator characteristics) [13,14,15]. The connection between the output function σ (linear) of the controller and the nonlinear input φ in the case of automatic regulation is express by the relation:

$$\dot{\xi} = \varphi(\sigma) \tag{4}$$

The characteristic function of the controller $\varphi(\sigma)$, $\sigma \in (-\infty, +\infty)$ is continuous and verifies the conditions [14,6,7]:

$$\begin{aligned} &a) \quad \varphi(0) = 0 \\ &b) \quad \sigma \cdot \varphi(\sigma) > 0, \quad \forall \sigma \neq 0 \\ &c) \quad \int_0^{\pm\infty} \varphi(\sigma) d\sigma = \infty \end{aligned} \tag{5}$$

Observe that $\varphi = \varphi(\sigma)$ is graphically ascending in the quarters I, III. The functions $\varphi(\sigma)$ are held admissible, and is verified the sector condition:

$$0 < \frac{\varphi(\sigma)}{\sigma} < k \tag{6}$$

where k is the amplification coefficient.

Example 1.

- $\varphi(\sigma) = sgn(\sigma) \cdot \ln(\sigma^2 + 1), k > 1$;
- $\varphi(\sigma) = a(e^{\sigma} - 1), k \le a$.

The equations (1), (3), (4) model the perturbed system with zeros $x(0,0,\ldots,0)$, $\xi = 0$. Using the nonsingular square matrix $A = \|a_{kj}\|$ of degree $n > 1$, $B = \begin{pmatrix} b_1 \\ \cdots \\ b_n \end{pmatrix}, C = (c_1 \quad \ldots \quad c_n)$, C' the transpose matrix of C, this system can be:

$$\dot{X} = AX + B\xi, \quad \dot{\xi} = \varphi(\sigma), \qquad \sigma = C'X - r\xi, \quad X = \begin{pmatrix} x_1 \\ \cdots \\ x_n \end{pmatrix} \tag{7}$$

Observation 1. It is known that for the linear system $\dot{X} = AX$, the second method of Liapunov for the null solution stability consists in ascertaining a Liapunov function $V = V(x)$ fulfilling the regularity conditions associated with this system [1,20]. A simple technique is to search V like square form positive defined $V = X'PX$ and $\dot{V} = X'(A'P + PA)X$ associated with the autonomous system where $V(0) = 0, \dot{V}(0) = 0$. For the simple or asymptotic stability in the proximity of the null solution must have negative sign (or be negatively defined). It must:

$$A'P + PA = -Q \tag{*}$$

Where the matrix $P, Q \in \mathbf{R}_{n \times n}$ Q are symmetrically and positives. So, practically it is chosen Q randomize fixed and is determined the matrix P from the equation (*) with A nonsingular.

Bringing the system (7) to the canonical form and determining the Liapunov function:

Suppose that A with $\det A = \Delta_0 \neq 0$ is Hurwitz, which means the characteristic polynomial $P(\lambda)$ has simple roots with $Re(\lambda_k) < 0, k = 1, \ldots, n$

$$P(\lambda) = (-1)^n \det(A - \lambda E) = 0 \tag{8}$$

The system (7) is brought to the canonical form if the matrix A is brought to the Jordan form $J = diagA = \begin{pmatrix} \lambda_1 & & 0 \\ & \ddots & \\ 0 & & \lambda_n \end{pmatrix}$. It is determined a non-degenerate matrix $T = (t_{kj})$ for the diagonalization of matrix A with the relation:

$$T^{-1}AT = J, \quad AT = TJ, \quad \det T \neq 0 \tag{9}$$

We make the linear transformation:

$$X = TY, Y = \begin{pmatrix} y_1 \\ \cdots \\ y_n \end{pmatrix} \tag{10}$$

Obtaining from (7):

$T\dot{Y} = ATY + B\xi, \quad \dot{\xi} = \varphi(\sigma), \sigma = C'TY - r\xi$ which means:

$$\begin{aligned} &\dot{Y} = JY + B_1\xi, \quad \dot{\xi} = \varphi(\sigma), \\ &\sigma = C_1^{'}Y - r\xi, B_1 = T^{-1}B, C_1' = C'T \end{aligned} \tag{11}$$

Reducing the system (1) with the linear transformation:

$$Z = JY + B_1\xi, \sigma = C_1'Y - r\xi, Z = \begin{pmatrix} z_1 \\ \cdots \\ z_n \end{pmatrix} \tag{12}$$

$$\begin{cases} \dot{Z} = JZ + B_1\varphi(\sigma) \\ \dot{\sigma} = C_1'Z - r\varphi(\sigma) \end{cases} \tag{13}$$

The disturbed system (13) with the equilibrium solution ($z_k = 0, \sigma = 0)$ will be equivalent with the system (7) with the equilibrium solution $(x_k = 0, \xi = 0)$ and the transformation (12) will be non-degenerate if the determinant of the system (13) is non-null.

$$\Delta = \begin{vmatrix} J & B_1 \\ C_1' & -r \end{vmatrix} \neq 0, r + C_1'J^{-1}B_1 \neq 0 \tag{14}$$

Retuning to $J^{-1} = T^{-1}AB, B_1 = T^{-1}B, C_1' = C'T$ transforms we obtain from (14) the final condition:

$$r + C'A^{-1}B \neq 0 \tag{15}$$

Lurie's problem consists in calculating the asymptotic stability conditions of the (7) equivalent with (13) with the null solution respectively $(x_k = 0, \xi = 0)$, $(z_k = 0, \sigma = 0)$ for the initial perturbations and for any admissible functions $\varphi(\sigma)$ defined in (5), (6). This type of stability where the systems (7), (13) have a linear part which is the A and a nonlinear part which is $\varphi(\sigma)$, is named absolute stability (a.s), [1, 16] It is observed that if $\varphi(\sigma)$ is linear, than the systems are linearized being asymptotically stable. The simplicity of system (13) entails immediate techniques for determining the Liapunov function $V = V(z_1, \ldots, z_n, \sigma)$ attached to the system (13). The function $V(z,\sigma)$ of class C^1 is Liapunov function from the system (13) if $V(z = 0, \sigma = 0) = 0$ and if it is positively defined $V(z,\sigma) > 0$ and radial unlimited to ∞ , with the absolute derivative $\dot{V} = \dfrac{dV}{dt}$ $\dot{V}(0,0) = 0$ and $\dot{V}$ negative defined $\dfrac{dV}{dt} < 0$ for $(z \neq 0, \sigma \neq 0)$ in the proximity of the equilibrium point, therefore we obtain absolute stability. Here, for the case of automatic regulation we choose $V, \dot{V}$ having the special form that verifies these conditions.

So we search the function $V = V(z,\sigma)$ composed by a square form z_k corresponding to the linear block A and an integral term corresponding to the nonlinear part.

$$V(z,\sigma) = Z'PZ + \int_0^\sigma \varphi(\sigma)d\sigma = V_1(z,\sigma) + \int_0^\sigma \varphi(\sigma)d\sigma \tag{16}$$

From theory [1,4] $Z'PZ$ is the square form defined strictly positive if the matrix P is symmetric $(P = P')$ and we have $A'P + PA = -Q$ where Q is symmetric and positive (with the eigenvalues positive). The integral term from (16) is strictly positive from the conditions (5) with $\sigma \neq 0$ and $V(z = 0, \sigma = 0) = 0$. Next are verified the regularity conditions with $\dot{V}$ attach to (13), and with (15) are met the conditions for the parameters c_k, r in order to obtain (a.r.a.s.). From (16) using (13) and: $Q = Q', P = P', B_1'PZ + Z'PB_1 = B_1'PZ + (PB_1)'Z = 2(PB_1)'Z$, for:

$$\frac{dV(z,\sigma)}{dt} = Z'(J'P + PJ)Z - r\varphi^2(\sigma) + \varphi(\sigma)(B_1'PZ + Z'PB_1) + \varphi(\sigma)C_1$$

we obtain:

$$\frac{dV}{dt} = -Z'QZ - r\varphi^2(\sigma) + 2\varphi(\sigma)\left(PB_1 + \frac{1}{2}C_1\right)Z; \dot{V}(z = 0, \sigma = 0) = 0 \tag{17}$$

The connection between the steps is obvious: from the matrix components $P(p_{ij}), Q(q_{ij})$ to $\lambda_i + \lambda_j \neq 0, i, j = 1,\ldots,n, P = P', J = \mathrm{diag}A$ than from $Q = Q'$ to $q_{ij} = -(\lambda_i p_{ij} + \lambda_j p_{ij})$ which means:

$$p_{ij} = -\frac{q_{ij}}{\lambda_i + \lambda_j} \tag{18}$$

Observation 2. The matrix A is stable with $\lambda_i + \lambda_j \neq 0$ if Q is a square form positively defined.

Example 2. If choose $Q = E$ the unit matrix and P obtained from (18) than the observation below is valid. Because $\dot{V} < 0$ we prove that $(-\dot{V})$ is positively defined. We apply in (17) the Sylvester criterion demanding that all diagonal minors of (17) to be positive. Because Q is positive like square form, than the first n inequalities are verified; it rest the last inequality from (17) after the square form in z and which is:

$$r > \left(PB_1 + \frac{1}{2}C_1\right)' Q^{-1}\left(PB_1 + \frac{1}{2}C_1\right);\ Q = E, \sqrt{r} > \left\|PB_1 + \frac{1}{2}C_1\right\| \tag{19}$$

If the regulator parameters verify the conditions (15), (19) there are sufficient conditions for the asymptotic stability of the system (1), (3), (4) for the solution $(x=0,\xi=0)$ [13, 19, 11].

Remark1. A technique choice of the square form $V_1(z)$ for p_{ij} according to Lurie is:

$$V_1(z)=\varepsilon\sum_{k=1}^{s}z_{2k-1}z_{2k}+\frac{\varepsilon}{2}\sum_{k=1}^{n-2s}z_{2s+k}^2-\sum_{k=1}^{n}\sum_{j=1}^{n}\frac{a_k z_k a_j z_j}{\lambda_k+\lambda_j},\ \varepsilon>0$$

where $a_1,a_2,\ldots,a_{2s}$ are intricately conjugated, $a_{2s+1},\ldots,a_n$ are real, corresponding to roots λ_k determining the coefficients a_k .

Remark2. The two transformations for the diagonal system (1), (3), (4) to obtain (13) can be directly replaced with the transformation [15]:

$$x_k=-\sum_{i=1}^{n}\frac{N_k(\lambda_i)}{D'(\lambda_i)}z_i \tag{20}$$

where from (7) it is obtained $P(\lambda)=(-1)^n D(\lambda), N_k(\lambda)=\sum_{i=1}^{n}b_i D_{ik}(\lambda)$, D_{ik} are the corresponding algebraic complements of (i,k) from $D(\lambda)=A-\lambda E$. In this case the simplified system analogous (13):

$$\dot z_k=\lambda_k z_k+\varphi(\sigma),\dot\sigma=\sum_{i=1}^{n}f_i z_i-r\varphi(\sigma), k=1,\ldots,n \tag{21}$$

for which we will build easier $V(z,\varphi)$.

Determining of $V(z,\varphi)$ with a new efficient method for (13) or (21)

Following the form of $V_1(z)$ we choose the function $V(z,\sigma)$ for (21).

$$V(z,\sigma)=\frac{1}{2}\sum_{j=1}^{n}A_j z_j^2+F(\alpha_1 z_1,\alpha_2 z_2,\ldots,\alpha_n z_n)+\int_0^{\sigma}\varphi(\sigma)d\sigma \tag{22}$$

$$F(z_1,z_2,\ldots,z_n)=-\sum_{j.k=1}^{n}\frac{1}{\lambda_j+\lambda_k}z_j z_{k,}\lambda_{k<0} \tag{23}$$

where, $A_j>0,\alpha_j\in\mathbf{R}$ will be determined. From:

$$-\frac{1}{\lambda_j+\lambda_k}=\int_0^{\infty}e^{(\lambda_j+\lambda_k)s}ds>0 \qquad F(z_1,z_2,\ldots,z_n)=\int_0^{\infty}\sum_{j,k}z_j z_k e^{(\lambda_j+\lambda_k)s}ds=\int_0^{\infty}\left(\sum_{j=1}^{\infty}z_j e^{\lambda_j s}\right)^2 ds\ge 0$$

results that F is nullify just for $F(z_1=0, z_2=0,\ldots,z_n=0)=0$ and $\int_0^\sigma \varphi(\sigma)d\sigma > 0$.

So, $V(z,\sigma)$ has the positive sign defined and $V(z=0,\sigma=0)=0$.

Compute $\frac{dV}{dt}$ associate to the system (21) and it must be $(-\dot{V})$ of positive sign defined.

$$-\frac{dV}{dt} = -\sum_{j=1}^{n} A_j\lambda_j z_j^2 - 2\sum_{j,k=1}^{n}\frac{\lambda_j\alpha_j\alpha_k}{\lambda_j+\lambda_k}z_j z_k + r\varphi^2(\sigma) + \sum_{j=1}^{n} z_j\left[A_j + f_j - 2\alpha_j\sum_{k=1}^{n}\frac{\alpha_k}{\lambda_j+\lambda_k}\right]\varphi$$

From $2\sum_{j,k=1}^{n}\frac{\lambda_j\alpha_j\alpha_k}{\lambda_j+\lambda_k}z_j z_k = \left(\sum_{k=1}^{n}\alpha_k z_k\right)^2, r>0, \lambda_j>0$

we obtain the first three terms positives and we must nullify the coefficient of φ:

$$A_j + f_j - 2\alpha_j\sum_{k=1}^{n}\frac{\alpha_k}{\lambda_j+\lambda_k} = 0,\ j=1..n \tag{24}$$

In this quadratic algebraic system (24) we can consider $A_j = -\frac{1}{\lambda_j}$, and f_j,λ_j as known, we determine the coefficients $\alpha_j, j=1..n$ and the other conditions from (19). If in (24) divided with λ_j and summing we obtain

$$\left(\sum_{j=1}^{n}\frac{\alpha_j}{\lambda_j}\right)^2 = -\sum_{j=1}^{n}\frac{A_j+f_j}{\lambda_j} \equiv \Gamma^2, \sum_{j=1}^{n}\frac{\alpha_j}{\lambda_j} = \pm\Gamma \tag{25}$$

So, we must have $\sum_{j=1}^{n}\frac{A_j+f_j}{\lambda_j} < 0$, and the solution of the system (24) $(\alpha_1,\alpha_2,\ldots,\alpha_n)$ is in this hyper-plane (25).

For the case when a root is null $P(0)=0$ and the others have $Re(\lambda_k)<0, k=1,\ldots,n-1$ the system (13) with $Z = \begin{pmatrix}\tilde{z}\\ z_n\end{pmatrix}$ becomes:

$$\dot{\tilde{z}} = \tilde{J}\tilde{Z} + \tilde{B}_1\varphi, \dot{z}_n = b_0\varphi, \dot{\varepsilon} = \tilde{C}_1'\tilde{Z} + C_0 z_n - r\varphi \tag{26}$$

where for $\tilde{z}$ we have the matrix $\tilde{Z}$ and $\tilde{J}$ of degree $(n-1)$, $\tilde{B}_1,\tilde{C}_1$ row, column matrix $(n-1,1),(1,n-1)$. In this case, the Liapunov function receives the form:

$$V(\tilde{z}, z_1, \sigma) = az_1^2 + \left\{ \tilde{z}' P \tilde{z} + \int_0^{\sigma} \varphi(\sigma) d\sigma \right\} \tag{27}$$

For further evidences and recent applications we recommend the bibliography [2, 15, 14, 11, 12].

2.1. The frequency method for (a.r.a.s.)

This method obtained by V. M. Popov [18] is applied to the dynamical system with continuous nonlinearity. We will present in this section the method with the criteria given by Aizerman, Kalman, Jakubovici [19,14]. Let be the dynamical, autonomous, non homogeneous system:

$$\dot{x}_i = \sum_{l=1}^{n} a_{il} x_l + b_i u, i = 1, \ldots, n; \dot{x} = \frac{dx}{dt}; \sigma = \sum_{l=1}^{n} c_l x_l, u = -\varphi(\sigma) \tag{28}$$

where a_{il}, b_i, c_l are real constants, u is the arbitrary function of input , continuous, nonlinear with $\varphi(\sigma)$ and σ is the output function. Using the Laplace transformation, replacing the operator $\frac{d}{dt}$ with s we obtain from (2):

$$sx_i = \sum_{l=1}^{n} a_{il} x_l + b_i u, \sigma = \sum_{l=1}^{n} c_l x_l, i = 1, \ldots, n \tag{29}$$

Eliminating from (21) the characteristic parameters of the regulator is obtained:

$$\sigma = W(s)u, \sigma = W(s)(-\varphi) \tag{30}$$

where $W(s) = \frac{Q_m(s)}{Q_n(s)}$ is the transfer function and $Q(s)$ are polynomials $m < n$. [4,6,16] The transfer function connect σ and φ ; the function φ verifies the conditions (5) and the sector condition (6) $0 < \frac{\varphi(\sigma)}{\sigma} < k \leq \infty$ - the plot $\varphi = \varphi(\sigma)$ in the plane (σ, φ) will be the sector $0 \leq \varphi(\sigma) \leq k\sigma$. The sector condition and the nonlinearity of φ determine the system (σ, φ) with closed loop through the impulse function φ. We study the absolute stability of the perturbed system (29) from the null solution $(x = 0, u = 0)$. Because the system is closed and nonlinear we can't applied directly the Nyquist criterion, [4, 6, 18]. If $\varphi \equiv k\sigma$ then the system is linear and it can be applied this criterion. It can be observed that the block $\sum a_{il} x_l$ is linear and $b_i u$ is nonlinear, thus it results that the roots of characteristic polynomial $P(\lambda) = (-1)(A - \lambda E) = 0, P(\lambda_i) = 0,$ the poles of $W(s)$ and k will influence the determination of the absolute stability criteria. From $W(s = j\omega) = U(\omega) + jV(\omega), j = \sqrt{-1}$ we have the hodograph for the axis (U, V) [2, 4, 6, 7, 15]:

$$U = U(\omega), V = V(\omega), 0 \le \omega \le \infty \tag{31}$$

If all poles of $W(s)$ have $Re(s_i) < 0$ then the system is uncritical; if through the poles of $W(s)$ is a null part or on the imaginary axis and the rest we have $Re(s_i) < 0$ then the system is in the critical case. We enunciate the criteria for absolute stability of automatic control (a.r.a.s.) with respect to the frequency method.

Criterion1. *(the uncritical case). Let there be the conditions:*

a) The function $\varphi(\sigma)$ *verifies (5), (6)*

b) All poles of $W(s)$ *have* $Re(s_i) < 0$

c) If there exists a real number $q \in R$ *that* $\forall \omega \ge 0$ *is satisfied the condition:*

$$\frac{1}{k} + Re[(1 + j\omega q)W(j\omega)] \ge 0 \tag{32}$$

Then the system (20) is automatic regulated and absolute stable for the null solution $(x = 0, u = 0)$.

From (32) is obtained:

$$\frac{1}{k} + U(\omega) - q\omega V(\omega) \ge 0 \tag{33}$$

The criterion (32) geometrically shows that in the plane geometric $U_1 = U, V_1 = \omega V$ exists the line (33) passing through $\left(-\frac{1}{k}, 0\right)$ and the plot of the hodograph is under this line for $\omega \ge 0, k > 0$.

Criterion2. *(the critical case when there is a simple null pole* $s_0 = 0$ *). Let there be satisfied the conditions:*

a) The function φ *verifies (5), (6).*

b) $W(s)$ *has a simple null pole, and the others poles* s_i *have* $Re(s_i) < 0$.

c) We have $\rho = \lim_{s \to 0} sW(s) > 0$ *and exists* $q \in R$ *for* $\forall \omega \ge 0$ *verifying the condition (33). Then for the system (28) for the null solution we have (a.r.a.s.).*

Criterion3. *(the critical case when* $s = 0$ *is a double pole). Let there be the conditions:*

a) The function $\varphi(\sigma)$ *verifies (5), (6) and the sector condition for* $k = \infty$ *in the quarters I, III.*

b) $W(s)$ *has a double pole in s=0 and the others poles have* $Re(s_i) < 0$.

c) It is verifying $\rho = \lim_{s\to 0} s^2 W(s) > 0$, $\mu = \lim_{s\to 0} \frac{d}{ds}\left[s^2 W(s)\right] > 0$, $\pi(\omega) = \omega Im W(j\omega) < 0$ *for* $\forall\omega \geq 0$ *then for the system (28) we have (a.r.a.s.) for the null solution*.

Observation3. The shape of these criteria (I, II, III) has an analytical character and their verification is required for constructing hodograph values of the coefficients by numbers. For special cases the recommended monographs are [2, 4, 15, 19].

3. The study of the absolute stability of aircraft course with automatic pilot

We'll consider the airplane fly in the vertical plane xOy, the longitudinal axis of the aircraft is parallel with the horizontal axis Ox and the vertical plane constitutes the symmetry plane for the aircraft. In the longitudinal fly course (horizontal) can appear some perturbations with angular variations for: the pitch angle ψ, between the longitudinal axis and Ox, the speed angle on the trajectory of fly θ, with the axis Ox compared with the considered system $\psi - \theta = \alpha$, represents the attack angle [17].

Considering these 3 angles without yaw and roll, it is written the system of disturbed differential equations compared with the mass centre, corresponding to ψ, θ, α, the coefficients being linearized, depending of the gyroscopic momentum created by the stability gyroscopes and the automatic regulation mechanisms for the pitch stability [5,17]. Eliminating θ, α from the system we'll study the equation for ψ in concordance with the regulator characteristics. The object of automatic regulation is the horizontal course of the plane. The important elements of the measurement, control, sensors and with response with inverse reaction to the perturbations that compose the regulator are considered: a gyroscope that measures the pitch speed ψ and a gyrotachometer that measures the angular speed $\dot{\psi}$, [5,17]. With sensors and potentiometers help these values are transmitted on the collector plate, while transducers and amplifiers are turned into electrical signals, they are transmitted through the input function φ for the output command function to the server $\sigma = -C_1\psi - C_2\dot{\psi} - r\xi$. By mechanical, electromagnetic, hydroelectric and gyroscopic effects, with the reaction parameter ξ determined, in accordance with the conditions from §2, it is obtained the stability for the null solution.

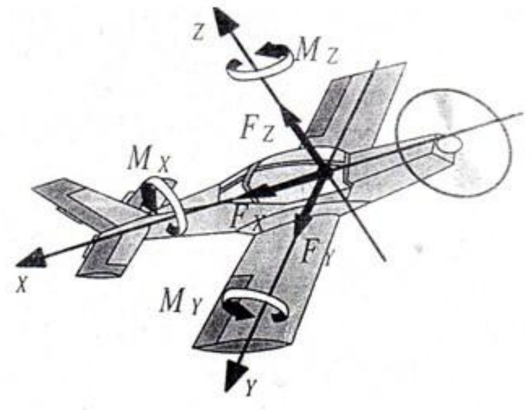

Fig. 1 The momentums in the case of the airplane dynamics.

The mechanical reactions of replay to the control will be transmitted by the commanded stabilizer to the ailerons, shutters (solid or jet type), horizontal empennage, horizontal rudder, to the pitch momentum around the Oy axis to converge to zero, considering that the perturbations moments by rolling or yaw are very small; in this way it is obtained the absolute stability of the horizontal course, (fig. 1).

3.1. The method of the Liapunov solution for (a.r.a.s.).

We'll write the reduce system of equations dimensionless [17], corresponding to the pitch perturbation $\psi = x$ in concordance with the functions and characteristics of the regulator connections.

$$\dddot{x} + a_1\ddot{x} + a_2\dot{x} = l\dot{y} + lmy; \sigma = -c_1x - c_2\dot{x} - r\xi; \psi = x; \dot{\psi} = \frac{dx}{dt} \tag{34}$$

Here, in the constants that appear have been included mass moments, moments of inertia, gyroscopic moments $a_1, a_2, l, m > 0, a_1^2 > 4a_2$ and the characteristic parameters of the regulator $c_1, c_2, r > 0, b_2 = l, b_3 = l(m - a_1)$. The right side of the equation is actually the expression of the server represented by the nonlinear function $\varphi(\sigma)$. We will write the system (34) with (1)-(4) using the next notations: $x_1 = x = \psi$, $x_2 = \dot{x} = \dot{x}_1 = \dot{\psi}$, $x_3 = \dot{x}_2 - ly$, $y = \xi$, $\dot{y} = \dot{\xi} = \varphi(\sigma)$.

$$\dot{x} = Ax + By, \dot{y} = \dot{\xi} = \varphi(\sigma), \sigma = c'x - r\xi \tag{35}$$

The matrix from (35) are:

$$x = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}, A = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & -a_2 & -a_1 \end{pmatrix}, B = \begin{pmatrix} 0 \\ b_2 \\ b_3 \end{pmatrix}, C = \begin{pmatrix} -c_1 \\ -c_2 \\ 0 \end{pmatrix} \tag{35'}$$

Using the linear transformation:

$$u = AX + B\xi, \dot{\sigma} = C'x - r\xi, u = \begin{pmatrix} u_1 \\ u_2 \\ u_3 \end{pmatrix} \tag{36}$$

We obtain the simplified system, by derivation:

$$\dot{U} = AU + B\varphi(\sigma), \dot{\sigma} = C'U - r\varphi(\sigma) \tag{37}$$

The system (35) has the unique solution $(x = 0, \xi = 0)$ and (37) $(U = 0, \sigma = 0)$. The absolute stability will be achieved relatively to these null solutions. The characteristic polynomial $P(\lambda) = \det(A - \lambda E) = 0$, $\lambda(\lambda^2 + a_1\lambda + a_2) = 0$ with the notations: $a_1 = 2p, a_2 = q$ has the roots:

$$\lambda_1 = -p + \sqrt{p^2 - q}, \lambda_2 = -p - \sqrt{p^2 - q}; \lambda_1 < 0, \lambda_2 < 0, \lambda_3 = 0 \tag{38}$$

After the diagonalization method (9) – (13), the system (37) will be transformed with $U = Tz$, $T(t_{ij}), i, j = 1,2,3$, determining the matrix T with (9) $AT = TJ, J = diagA$, and thus obtaining :

$$T = \begin{pmatrix} \dfrac{1}{\lambda_1(\lambda_1 - \lambda_2)} & -\dfrac{1}{\lambda_2(\lambda_1 - \lambda_2)} & \dfrac{1}{\lambda_1\lambda_2} \\ \dfrac{1}{\lambda_1 - \lambda_2} & -\dfrac{1}{\lambda_1 - \lambda_2} & 0 \\ \dfrac{\lambda_1}{\lambda_1 - \lambda_2} & -\dfrac{\lambda_2}{\lambda_1 - \lambda_2} & 0 \end{pmatrix} \quad T^{-1} = \begin{pmatrix} 0 & -\lambda_2 & 1 \\ 0 & -\lambda_1 & 1 \\ \lambda_1\lambda_2 & -(\lambda_1 + \lambda_2) & 1 \end{pmatrix}, z = \begin{pmatrix} z_1 \\ z_2 \\ z_3 \end{pmatrix} \tag{39}$$

$$\dot{z} = Jz + T^{-1}B\varphi(\sigma), \qquad \dot{\sigma} = C'Tz - r\varphi(\sigma) \tag{40}$$

The system (40) is equivalent with (35) (36), and has the unique solution $(z = 0, \sigma = 0)$ and for this solution we study (a.r.a.s), determining the Liapunov function. To build the Liapunov function corresponding to the transformed system (40) $V = V(z, \varphi(\sigma))$, we apply the calculus technique presented in (22) – (25) for the special case $\mathrm{Re}(\lambda_{1,2}) < 0, \lambda_3 = 0$ at (26), (27). The system (40) became:

$$\dot{z}_1 = \lambda_1 z_1 + b_1'\varphi(\sigma); \dot{z}_2 = \lambda_2 z_2 + b_1'\varphi(\sigma), \dot{z}_3 = b_3'\varphi(\sigma); \dot{\sigma} = f_1 z_1 + f_2 z_2 + f_3 z_3 - r\varphi(\sigma) \tag{41}$$

$$b_1' = b_3 - \lambda_2 b_2, b_2' = b_3 - \lambda_1 b_2, b_3' = b_3 - (\lambda_1 + \lambda_2)b_2; \quad f_1 = -\frac{c_1 + \lambda_1 c_2}{\lambda_1(\lambda_1 - \lambda_2)}, f_2 = \frac{c_1 + \lambda_2 c_2}{\lambda_2(\lambda_1 - \lambda_2)}, f_3 = -\frac{c_1}{\lambda_1\lambda_2}.$$

In this case, we choose the Liapunov function in accordance with (22), (27)

$$V(z, \sigma) = \frac{1}{2}A_1 z_1^2 + \frac{1}{2}A_2 z_2^2 + \frac{1}{2}A\, z_3^2 + \int_0^\sigma \varphi(\sigma)d\sigma \tag{42}$$

where $A_1, A_2, A > 0$ are fixed, $V(z=0,\sigma=0)=0$ and $V(z,\sigma)$ is positively defined. Compute the derivative $\dot{V}$ associated to the system (41)

$$\dot{V} = \sum_{j=1}^{2} A_j \lambda_j z_j^2 - r\varphi^2 + \sum_{j=1}^{2} (A_j \lambda_j b_j' + f_j) z_j \varphi + (Ab_3' + f_3) z_3 \varphi(\sigma) \tag{43}$$

We observe that taking $A_j = -\dfrac{1}{\lambda_j} > 0$ the negativity of this form is ensured from the first terms, forcing the cancellation of the last term: $Ab_3' + f_3 = 0$, that means:

$A = -\dfrac{f_3}{b_3'} = \dfrac{c_1}{a_1(b_3 + a_1 b_2)} = \dfrac{c_1}{a_1 lm} > 0$. From

$$\dot{V} = -(z_1^2 + z_2^2) - r\varphi^2 + \sum_{j=1}^{2} \varphi z_j \left(\frac{b_j'}{\lambda_j} - f_j \right) \tag{44}$$

The quadratic form is positively defined for $(-\dot{V})$ in relation with z_1, z_2, φ, with the system (41) or (9). From the Sylvester determinant is obtained the necessary and sufficient condition (41) for the rigidity coefficient.

$$r > \left(\frac{b_1'}{\lambda_1} - f_1 \right)^2 + \left(\frac{b_2'}{\lambda_2} - f_2 \right)^2 \tag{45}$$

In this way, the characteristic parameters of the regulator r, c_1, c_2 verify the condition (45), and ensure the absolute stability of the horizontal fly course of the aircraft. It is observed that whenever the function φ does not appear, the nonlinear control function can be choose arbitrarily from the admissible class (5), (6).

3.2. The frequency method for (a.r.a.s.).

For this study we have applied the frequency method used in §3. Since the system (35) is equivalent with (37) and (41), the function $u = -\varphi(\sigma)$ verifies the sector condition. By replacing the operator $\dfrac{d}{dt}$ with the factors is found the transfer function $W(s)$. For simplicity we choose the system (37) with (35), we deduce the transfer function $W(s)$ that is the same for (35) and (41). Applying the Laplace operator in (37) we have:

$$U_1 s = U_2, U_2 s = U_3 + b_2\varphi, U_3 s = -a_2 U_2 - a_1 U_3 + b_3 \varphi; \quad \sigma s = -c_1 U_1 - c_2 U_2 - r\varphi \tag{46}$$

Eliminating from these relations U_1, U_2, U_3 it is found the connection $\sigma = W(s)(-\varphi)$:

$$W(s) = \frac{1}{s^2} \left(rs + \frac{[b_2(s + a_1) + b_3](c_2 s + c_1)}{s^2 + a_1 s + a_2} \right) \tag{47}$$

We observe that $W(s)$ has a double pole in $s_0 = 0$ and $s_1 = \lambda_1 < 0, s_2 = \lambda_2 < 0$, meeting the special case of the frequency method, Criterion3 (a.r.a.s) from §3. Then, we verify the conditions from Criterion3.

$$\rho = \lim_{s \to 0} s^2 W(s) = \frac{lmc_1}{a_2} > 0, b_2 = l > 0, b_3 = l(m - a_1) > 0, a_1 > 0, a_2 > 0, c_1 > 0 \tag{48}$$

$$\mu = \lim_{s \to 0} \frac{d}{ds}(s^2 W(s)) = r + \frac{l}{a_2^2}\left[c_1(a_1^2 + a_2) - m(a_1 c_1 - a_2 c_2)\right] > 0 \tag{49}$$

From (49) we obtain the conditions for r, m, c_2

$$r > \frac{l}{a_2^2}\left[m(a_1 c_1 - a_2 c_2) - c_1(a_1^2 + a_2)\right] > 0; \quad m > \frac{c_1(a_1^2 + a_2)}{a_1 c_1 - a_2 c_2} > 0, \frac{a_1 c_1}{a_2} > c_2 > 0 \tag{50}$$

$$\pi(\omega) = \omega \operatorname{Im} W(j\omega) = -r - l\frac{\omega^2[a_1 c_2 - (c_1 + mc_2)] + [a_2(c_1 + mc_2) - a_1 c_1(m - a_1)]}{(a_2 - \omega^2)^2 + a_1^2 \omega^2} = -r + g(\omega) \tag{51}$$

$$\lim_{\omega \to \infty} \pi(\omega) = -r < 0, \lim_{\omega \to 0} \pi(\omega) = -r + g(0) < 0 \tag{52}$$

From (52) we observe that $r > g(0)$ is from (50) condition. For the rigidity coefficient r we obtain the equivalence with (45). It is observed that by this qualitative criterion are also necessary numerical data in the space of the parameters for the regulator. The condition $\pi(\omega) = -r + g(\omega) < 0, \forall \omega \geq 0$ as $g(0) > 0$ it is the right member from (50), $g(\omega)$ is derivable, $g'(\omega) < 0$, $\lim_{\omega \to \infty} g(\omega) = 0$ ($g = g(\omega)$ is an even function on $(-\infty, \infty)$ with $g(0)$ maximal. The automatic regulation for the absolute stability of the horizontal course of flying is presented in the figures 2 and 3.

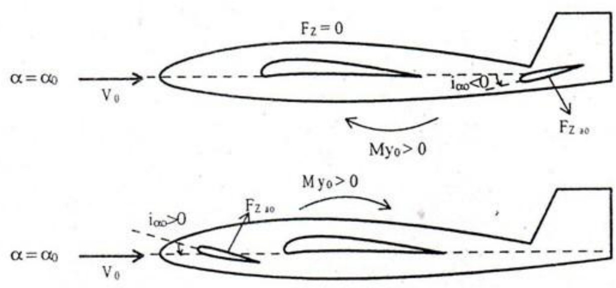

Fig. 2 The scheme of the arrangement for the horizontal empennage compare with the wing.

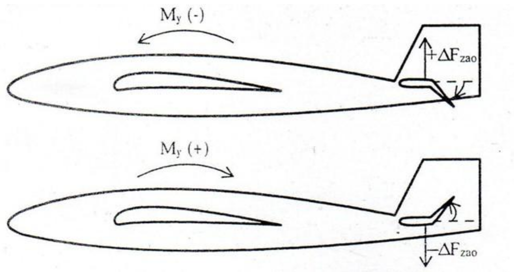

Fig. 3 The command of the pitch momentum.

4. The absolute stability in the automatic regulation of the wood cutting

The high precision of wood cutting with tools machine implies an automatic regulation of these processes. Our approach aims at studying and modelling the nonlinear dynamics of the cutting processes (CP) with tools inside wood blocks, composite materials blocks, or hardwood. [3]

These (CP) are: CP of drilling, CP of milling, CP of grinding, screw machine, spindle bearing. Machine tool bar is provided with an inner elastic hard wood cutting, cutting inside to run the required geometric rotation and advancing in slow step. Because of the variation in hardness, density, coefficient of elasticity, material composition manufactured by the process disturbances will occur in the working mode: transverse vibration due to shaft rotation or longitudinal vibrations due to advance. Automatic controller is equipped with sensors, micrometers, tensiometers, rigid response mechanisms of signals, output power amplifiers, and accelerators. Their purpose is to adjust the characteristics in order to obtain asymptotic stability of the system work, resulting in high precision components. We will study the two methods described above in §2, §3.

4.1. The (a.r.a.s.) method by Liapunov function

Consider the dynamic system mathematically modelled, brought to a canonical form of Cauchy, autonomous, with features automatically adjusted for absolute stability of dynamic cutting machining processes. [3] [14]

$$\begin{cases} \dot{x}_1 = a_{11}x_1 + b_1\xi \\ \dot{x}_2 = a_{23}x_3 \\ \dot{x}_3 = a_{31}x_1 + a_{32}x_2 + a_{33}x_3 \\ \dot{x}_4 = a_{44}x_4 + b_4\xi \end{cases}, \sigma = c_2x_2 + c_4x_4 - r\xi, \dot{\xi} = \varphi(\sigma) \tag{53}$$

where a_{ij}, b_i, r, ξ are constants $i, j = 1,2,3,4$.

$$\begin{aligned} &a_{11} = -m < 0, a_{31} = n > 0, a_{32} = -\varepsilon n < 0, a_{33} = -p < 0, a_{44} = -l < 0 \\ &a_{23} = 1. c_2 = 1, c_4 = c < 0, b_1 = b > 0, b_4 = d - r > 0, r > 0 \end{aligned} \tag{54}$$

These, according to Lagrange's equations of the parameters are mass produced; mass inertia, elastic constants, strain or pressure coefficients, and σ, r, ξ are the characteristics of the server. We assume that the input function φ is generally nonlinear, and checks conditions (5), (6). We observe that the linear response function σ of the server controls the elements x_2 - the rotational speed of the cutting bar, and x_4 - the speed of its advancing material.

We check the absolute stability of the system solution from zero $(x = 0, \xi = 0)$. Suppose that the block linear system (XA) is asymptotically stable as follows from relations: $\det A \neq 0, \mathrm{Re}(\lambda_i) < 0, i = 1,2,3,4$.

$$P(\lambda) = D(\lambda) = \begin{vmatrix} a_{11} - \lambda & 0 & 0 & 0 \\ 0 & -\lambda & a_{23} & 0 \\ a_{31} & a_{32} & a_{33} - \lambda & 0 \\ 0 & 0 & 0 & a_{44} - \lambda \end{vmatrix} = (a_{11} - \lambda)(a_{44} - \lambda)(\lambda^2 - \lambda a_{33} - a_{23}a_{32}) = 0 \tag{55}$$

$$\lambda_1 = a_{11} = -m < 0, \lambda_4 = a_{44} = -l < 0, \lambda_{2,3} = \frac{1}{2}\left(-p \pm \sqrt{p^2 - 4\varepsilon n}\right) < 0, \lambda_i \in \mathbf{R} \tag{56}$$

In this case, following the diagonalization method §2 with the formulas (9) - (13) or directly choosing the option remark (R2), we get the diagonal system in z_i and $\dot{\sigma}$ (12), (13):

$$\dot{z}_i = \lambda_i z_i + \varphi(\sigma); \dot{\sigma} = \sum_{i=1}^{4} f_i z_i - r\varphi(\sigma), i = 1,\ldots,4 \tag{55}$$

$$f_1 = \frac{b_1 a_{31}}{(\lambda_1 - \lambda_2)(\lambda_1 - \lambda_3)}, f_2 = \frac{-b_1 a_{31}}{(\lambda_1 - \lambda_2)(\lambda_2 - \lambda_3)}, f_1 = \frac{b_1 a_{31}}{(\lambda_1 - \lambda_3)(\lambda_2 - \lambda_3)}, f_4 = b_4 c_4 < 0 \tag{56}$$

We observe that $f_1 + f_2 + f_3 = 0$ and whatever is the choice of order quantities $\lambda_1, \lambda_2, \lambda_3$ are strictly negative, always two of the functions $f_i, i = 1,2,3$ have the same sign and the third function takes opposite sign using relation (26).

In this case the stability of the following (29) from the null solution ($z_i = 0, \sigma = 0$) and that $f_{4<0}$ we can construct such a Liapunov function:

$$V(z,\sigma) = -\frac{1}{2} f_4 z_4^2 - \frac{1}{2}\sum_{i=1}^{3} \frac{a_i^2 z_i^2}{\lambda_i} - \sum_{i=1}^{3}\sum_{j=1}^{3} \frac{a_i a_j}{\lambda_i + \lambda_j} z_i z_j + \int_0^{\sigma} \varphi(\sigma) d\sigma \tag{55}$$

where the real coefficients a_1, a_2, a_3 will be determined.

From $\lambda_i < 0, \lambda_i + \lambda_j < 0, V(0) = 0$, the summative terms after $i = 1,2,3$ determine a positive quadratic form positive definite and the integral positive term, we have $V(z,\sigma) > 0$ allowed in the vicinity.

We calculate $\dot{V}(z,\sigma)$ attached to the system (29), and we obtain:

$$\dot{V}(z,\sigma) = -f_4\lambda_4 z_4^2 - (a_1 z_1 + a_2 z_2 + a_3 z_3)^2 - \varphi\sum_{i=1}^{3} z_i\left(\frac{a_i^2}{\lambda_i} + \sum_{j\neq i}\frac{2a_i a_j}{\lambda_i + \lambda_j} - f_i\right) \tag{56}$$

Observe that $\dot{V}(z=0,\sigma=0)=0$ and to have the strict negativity, the parenthesis from the term φ must be null

$$\frac{a_i^2}{\lambda_i} + \sum_{j\neq i}\frac{2a_i a_j}{\lambda_i + \lambda_j} - f_i = 0, i, i = 1,2,3 \qquad (S^*) \tag{57}$$

The system (S*) $F_i(a_1,a_2,a_3)=0, i=1,2,3$ is implicit with three equations with three unknowns, and the existence of solutions is provided by the Jacobian system $J = \dfrac{D(F_1,F_2,F_3)}{D(a_1,a_2,a_3)} \neq 0$.

A helping calculation proves that if each equation from (33) is multiplying respectively by $\dfrac{1}{\lambda_i}$ and summing, it is obtained:

$$\Gamma^2 = \left(\sum_{i=1}^{3}\frac{a_i}{\lambda_i}\right)^2 = \sum_{i=1}^{3}\frac{f_i}{\lambda_i} = S > 0, \Gamma = \pm\sqrt{S} \tag{58}$$

A condition that indicates that the knowing sum (S) is strictly positive and in the parametric space (a_1,a_2,a_3) the symmetrical plane (π_{12}) , $\sum_{i=1}^{3}\frac{a_i}{\lambda_i} = \pm\sqrt{S}$ where exists a solution, don't admit the null solution because $f_i \neq 0$. Is obtained as:

$$J = \pm 8\frac{\sqrt{S}(a_1+a_2+a_3)[a_1(\lambda_2+\lambda_3)+a_2(\lambda_1+\lambda_3)+a_3(\lambda_1+\lambda_2)]}{\lambda_1\lambda_2\lambda_3(\lambda_1+\lambda_2)(\lambda_1+\lambda_3)(\lambda_2+\lambda_3)} \neq 0 \tag{59}$$

Because the two factors from the numerator parenthesis are planes passing through the origin, the solution is contained in the planes (π_{12}). By analysing the system (S*) after the sign of $f_i,(a_1,a_2,a_3)$ these solutions from (π_{12}) are not in the I,V octant. If all would have the same sign than $f_i > 0$. We proved that exists solutions of the system (33) and $\dot{V} < 0$; results that the Liapunov function provide the automatic regulation of the absolute stability. For this application and sufficient conditions of type (15), (16) with numerical data, are obtain.

4.2. The frequency method for (a.r.a.s.)

For this study we have applied the frequency method used in §3. Because the system (53) is equivalent with (39), the function $u = -\varphi(s)$ verifies the sector conditions. By replacing the operator $\frac{d}{dt}$ with the factor s is found the transfer function $W(s)$. So, from (29) is obtained for $\sigma = W(s)(-\varphi)$

$$sz_i = \lambda_i z_i + \varphi; s\sigma = \sum_{i=1}^{4} f_i z_i - r\varphi \tag{60}$$

Eliminating from (60) z_i we obtained the transfer function from $\sigma = W(s)(-\varphi)$

$$W(s) = \frac{1}{s}\left(r - \sum_{i=1}^{4} \frac{\lambda_i f_i}{s - \lambda_i} \right) \tag{61}$$

Because the real roots $s_i = \lambda_i$ verifies $\mathrm{Re}(\lambda_i) < 0$ the transfer function has a simple pole in $s = 0$ and the rest of real roots with $\mathrm{Re}(s_i) < 0$.

In this case we have the Criterion II of critical singularity from §3 for (a.r.a.s.). Here, the conditions (15), (19) and II a), b) are verified from the method A, and must verify the condition c).

So, $\rho = \lim\limits_{s \to 0} sW(s) = r + \sum\limits_{i=1}^{4} f_i = r + f_4 = r + b_4 c_4 > 0$ implies $r + c(d - r) > 0$ that means $r > \frac{cd}{1-c} > 0, d < 0$. From $W(s = j\omega) = U(\omega) + jV(\omega)$, we have:

$$U(\omega) = \sum_{i=1}^{4} \frac{f_i \lambda_i}{\lambda_i^2 + \omega^2}, \quad V(\omega) = -\frac{r}{\omega} + \frac{1}{\omega} \sum_{i=1}^{4} \frac{\lambda_i^2 f_i}{\lambda_i^2 + \omega^2}.$$

For given $k > 0$, from the condition $0 < \varphi(s) < k\sigma$ with $\varphi(\sigma)$ specified, it can be determined $q \in \mathbf{R}$ verifying the condition (24'). The parameters λ_i, f_i are known from (56), (60), the nonlinear function φ is chosen with σ from (53), and for specified numerical data determine k and the delimitation of q. The existence of these conditions can be performed hodographically for (a.r.a.s.) at this application.

5. Conclusions

The relevancy of this paper consists in the fact that the problem of absolute stability is systematized by the two methods. It should be noticed the fact that the application with respect to (a.r.a.s.) for the horizontal fly course with automatic pilot is studied for the critical difficult cases, when the roots of characteristic polynomial or the pole of transfer function is in its origin (on the imaginary axis). For the Liapunov function building we have applied an original method. For further and more profound details we recommend the published results of some other researchers [1,15,19,20,11].

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