Annals of the Academy of Romanian Scientists
Series on Engineering Sciences
ISSN 2066 – 8570 Volume **5**, Number **2**/**2013**

MODELING AND CONTROL OF URBAN TRAFFIC, IN THE MACROSCOPIC REPRESENTATION

Catalin DIMON[1],
Dumitru POPESCU[2],
Geneviève DAUPHIN-TANGUY[3]

Rezumat. *Lucrarea propune o metodologie pentru modelarea traficului rutier, organizată pe două direcţii: o abordare descentralizată, în care structura de trafic este descompusă în elemente componente de bază, modelate cu ajutorul teoriei mecanicii fluidelor şi o abordare centralizată, în care reţeaua rutieră este văzută ca o reţea compartimentală, modelată pe baza teoriei sistemelor pozitive. În primul caz, controlul este asigurat prin algoritmi polinomiali numerici de tip RST, calculaţi pentru a asigura performanţele dorite pentru sistemul în buclă închisă. În cel de-al doilea caz, regulatorul este el însuşi o reţea compartimentală, care are rolul de a pondera fluxurile de intrare pentru a evita congestionarea reţelei.*

Abstract. *A methodology for modeling road traffic is proposed, organized in two directions: a decentralized approach, in which the traffic structure is decomposed into basic elements, modeled using the theory of fluid mechanics, and a centralized approach, where the road network is seen as compartmental network, modeled on the theory of positive systems. In the first case the control is ensured by numerical algorithms of RST type, calculated to provide the desired performance for the closed loop system. In the second case, the controller is itself a compartmental network that has the role of weighting inflows to avoid network congestion.*

Keywords: road traffic, macroscopic model, modeling and simulation, numerical control

1. Introduction

In the last decades different concerns related to road transport have been challenging specialists. The main topics are focused on resolving the shortcomings related to increased comfort and safety in traffic, environmental pollution and fuel quality and consumption.

Effective management and modernization of traffic relies on modeling and controlling the flow of vehicles in circulation, the increasing interest in equipping vehicles and transport infrastructure with electronic devices and computer components that make possible the prospect of intelligent traffic circulation.

[1]S.l., PhD, Faculty of Automatic Control and Computers, Department of Automatic Control and Systems Engineering, Bucharest, Romania (e-mail: catalin.dimon@acse.pub.ro).
[2]Prof., PhD, Faculty of Automatic Control and Computers, Department of Automatic Control and Systems Engineering, Bucharest, Romania.
[3]Prof., PhD, Ecole Centrale de Lille, LAGIS, Villeneuve d'Ascq, Lille, France.

In this context, the topic of this paper is to propose appropriate solutions to the new conditions and developing requirements of traffic by modeling, controlling and optimizing network traffic.

The research is focused on two directions of study:

a) *decentralized approach*, considering the decomposition of traffic configurations into simple components, easily accessible for mathematical modeling and automatic control. The urban network, considered as part of a city's road infrastructure, is split into multiple objects representing road sections (circulated streets). At the last level of complexity there are road segments which represent parts of a road section.

b) *centralized approach*, where the road configuration is represented by a compartmental network. In a compartment network, seen as a smaller component of the metropolitan traffic, the nodes consist of material storage elements (mass, energy, items) and the arcs represent the transfer flow from one compartment to another. The network is connected to the outside by additional arcs; the inflows inject material in specific compartments and the outflows extract material from in specific compartments outward.

Often encountered in the traffic literature are three types of models used for describing road traffic [1]: microscopic models, mesoscopic models and macroscopic models.

Microscopic models describe both the spatial and temporal behavior of the system components (vehicles and drivers) and their interactions at a high level of detail. As an example, for each vehicle, the changing of lanes is described as a chain of driver decisions.

Mesoscopic models do not differentiate vehicles as individual entities, but describe their behavior in probabilistic terms. Thus, traffic is represented by small groups of vehicles for which the activities and the interactions are described in low level details. For example, a lane change maneuver for a vehicle can be represented as an instantaneous event, the decision to make the lane change being based on relative densities of lanes and speed differences. Mesoscopic models are obtained by analogy with the kinetic theory of gases used to describe the dynamics of speed distributions.

Macroscopic models describe traffic at a high level of aggregation, as a steady flow of vehicles, without distinguishing between components. These models are typically used for planning and control operations, in major networks and over long periods of time. The traffic is represented in a compact way, using a number of interdependent variables such as flow rate, density and speed. Vehicle specific maneuvers, such as changing lanes, will not be represented.

The present research is focused on the macroscopic models; the manner of modeling the flow of vehicles in approach a) uses the laws of fluid mechanics, while the compartmental modeling of traffic networks in approach b) is developed through the theory of positive systems.

Furthermore, traffic control algorithms are proposed to ensure performance in tracking and control towards adaptive robust and optimal implementations. Acquisition and control equipment are associated with the vehicle and integrated in embedded systems configurations. In the case of complex networks state-space MIMO configurations are proposed, controlled by compartmental systems.

2. Traffic modeling and control, decentralized approach

An urban network is considered as being composed of several traffic objects, which make up the road infrastructure: road sections (streets) and road segments (street segments). The objective is to control traffic at section level thereby it guarantees free-flow traffic without congestion. The section dynamic is highlighted on models of elementary segments of the road, assembled together in accordance with that section's configuration. The typology of a section is represented by capacitive segments with constant density of vehicles, by segments with constant flow rate (speed) traffic and by reservoir-type vehicles storage segments. For each of the mentioned types we will present examples of calculating the model for the segments composing the section.

2.1. Elementary road segment

We consider a road segment with pressures $p_1(t)$ and $p_2(t)$ at the two ends; the segment has a length L and a capacity C. The number of vehicles within the segment is given by the difference between the incoming and outgoing flows of vehicles. A representation of this segment is given in Figure 1.

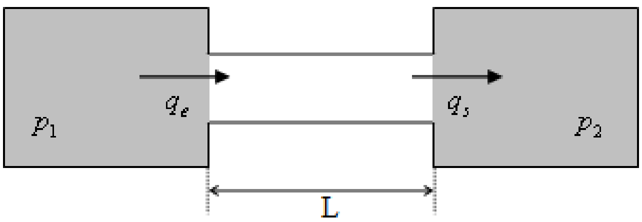

Fig. 1. Model of an elementary road segment.

From the vehicles flow equation, through linearization and normalization operations, it is obtained the dynamic model of the segment, as a first-order element:

$$\tau_{P_1} \frac{dy(t)}{dt} + y(t) = k_{P_1} u(t) \quad (1)$$

with the parameters:

$$\tau_P = \frac{2N_0}{q_0};\ k_P = \frac{2N_0}{q_0} \tag{2}$$

where τ_P is the time constant and k_P is the gain; N_0 represents the number of vehicles within the segment in steady state and q_0 represents the steady state flow.

2.2. Road segment with vehicles queue

If the road segment in Figure 1 is complicated by considering a queue of vehicles, we obtain the model from Figure 2, where l_q is the length of the queue.

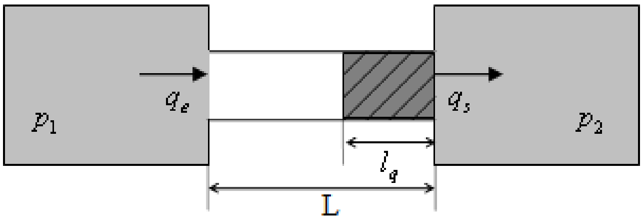

Fig. 2. Model of a road segment with vehicles queue.

From the mass balance equation it is deduced the linearized dynamic model, dimensionless, also represented by a first order element described by the equation:

$$\tau_P \frac{dy(t)}{dt} + y(t) = k_P u(t) \tag{3}$$

with the time constant and the gain:

$$\tau_P = \frac{2l_{q0}}{l_{veh} q_0};\ k_P = \frac{2l_{q0}}{q_0} \tag{4}$$

where l_{q0} is the length of the vehicles queue in steady state and l_{veh} is the estimated average length of a vehicle.

2.3. Control of a road section

We considered a road section (circulated street) as a combination of cascade-connected segments, represented by a SISO model given by:

$$P(s) = \prod_i P_i(s) = \prod_i \frac{k_i}{\tau_i s + 1} = \frac{B(s)}{A(s)} \tag{5}$$

or, in discrete representation:

$$P(z^{-1}) = \prod_i \frac{b_{1i} z^{-1}}{1 + a_{1i} z^{-1}} = \frac{B(z^{-1})}{A(z^{-1})} \tag{6}$$

The proposed control algorithm is a polynomial one, RST type, and ensures individual performances both in tracking and regulation.

The regulation performances are achieved by pole placement method. The closed loop system structure is represented in Figure 3.

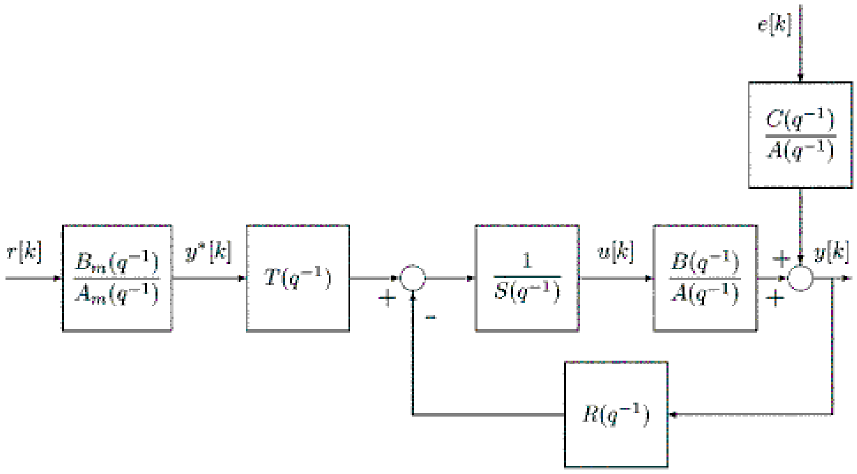

Fig. 3. Numerical RST controller with trajectory generator.

The discretized model of the controlled section is given by $A(z^{-1})$ and $B(z^{-1})$ polynomials:

$$\begin{aligned} A(z^{-1}) &= 1 + a_1 z^{-1} + a_2 z^{-2} + \ldots + a_{n_A} z^{-n_A} \\ B(z^{-1}) &= b_1 z^{-1} + b_2 z^{-2} + \ldots + b_{n_B} z^{-n_B} \end{aligned} \quad (7)$$

The trajectory model, given by $A_m(z^{-1})$ and $B_m(z^{-1})$ polynomials:

$$\begin{aligned} A_m(z^{-1}) &= 1 + a_{m1} z^{-1} + a_{m2} z^{-2} + \ldots + a_{n_{Am}} z^{-n_{Am}} \\ B_m(z^{-1}) &= b_{m0} + b_{m1} z^{-1} + b_{m2} z^{-2} + \ldots + b_{n_{Bm}} z^{-n_{Bm}} \end{aligned} \quad (8)$$

generates the desired trajectory $y^*(k)$, which must be reproduced at the system output, $y(k)$.

The reference $r(k)$ contributes to the calculation of the output value through the trajectory generator.

The closed loop poles are assigned through the characteristic polynomial $P(z^{-1})$.

The RST polynomial algorithm is given by:

$$u[k] = \frac{T(z^{-1})}{S(z^{-1})} r[k] - \frac{R(z^{-1})}{S(z^{-1})} y[k] \quad (9)$$

with:

$$\begin{aligned} R(z^{-1}) &= r_0 + r_1 z^{-1} + r_2 z^{-2} + ... + r_{n_R} z^{-n_R} \\ S(z^{-1}) &= 1 + s_1 z^{-1} + s_2 z^{-2} + ... + s_{n_S} z^{-n_S} \\ T(z^{-1}) &= t_0 + t_1 z^{-1} + t_2 z^{-2} + ... + t_{n_T} z^{-n_T} \end{aligned} \tag{10}$$

where $r[k]$ is the reference signal and $y[k]$ is the system output.

The calculation of $R(z^{-1})$, $S(z^{-1})$ and $T(z^{-1})$ polynomials is done in two stages. The first step is to calculate the $R(z^{-1})$ and $S(z^{-1})$ polynomials, by pole placement with $P(z^{-1})$ (regulation performances).

In the second step, we calculate the pre-compensator $T(z^{-1})$, by anticipating and reproducing the of desired trajectory $y^*(k)$ at the closed loop system output (tracking performances).

From the diagram in Figure 3, the transfer function of the closed loop system is expressed as:

$$H_{RS}\left(q^{-1}\right) = \frac{B\left(q^{-1}\right)}{A\left(q^{-1}\right)S\left(q^{-1}\right) + B\left(q^{-1}\right)R\left(q^{-1}\right)} \tag{11}$$

By imposing the condition:

$$H_{RS}(q^{-1}) = \frac{1}{P(q^{-1})} \tag{12}$$

from the polynomial equation:

$$A\left(q^{-1}\right)S\left(q^{-1}\right) + B\left(q^{-1}\right)R\left(q^{-1}\right) = P\left(q^{-1}\right) \tag{13}$$

we can compute $R(z^{-1})$ and $S(z^{-1})$.

This equation has a unique solution for:

$$\begin{cases} \text{grd}\, P\left(q^{-1}\right) = np \le na + nb + 1 \\ \text{grd}\, S\left(q^{-1}\right) = nb + 1 \\ \text{grd}\, R\left(q^{-1}\right) = na - 1 \end{cases} . \tag{14}$$

The equation (13) can be put in matrix form:

$$M\, x = p \tag{15}$$

where M is a Sylvester matrix. Unknown parameter vector x is obtained by inverting the matrix M :

$$x = M^{-1} p \tag{16}$$

Tracking performances are ensured by calculating pre-compensator $T(z^{-1})$, imposing the condition that the system in Figure 3 (obtained by attaching the trajectory generator and the pre-compensator) acts as a tracking model:

$$H^{*}_{RST}\left(q^{-1}\right) \stackrel{def}{=} \frac{B_m\left(q^{-1}\right)}{A_m\left(q^{-1}\right)} T\left(q^{-1}\right) H_{RS}\left(q^{-1}\right) = \frac{B_m\left(q^{-1}\right)}{A_m\left(q^{-1}\right)} \tag{17}$$

The closed loop configuration verifies equation (12) and by imposing the ideal (unitary) transfer condition from reference $r(k)$ to output $y(k)$, it results the identity:

$$T(z^{-1}) = P(z^{-1}) \tag{18}$$

A robust approach will provide tolerance to system nonlinearities and to model structural modifications caused by changes in the road section's configuration. Robust correction is achieved by choosing pre-specified polynomials, which ensure a certain gauge for the system's disturbance-output sensitivity function. Thus, it can be achieved a reasonable robustness margin for the designed system. Acquisition and control equipment necessary for the implementation of the control systems are associated with the vehicle and the road, integrated in the concept of intelligent traffic system.

3. Traffic modeling and control, centralized approach (compartmental networks)

We considered a part of a metropolitan road network, with the structure represented in Figure 4. The nodes represent mass storage units, defined by the states $x_i\,(i=1,2,...,n)$; the arcs have associated transfer flows f_{ij} (from compartment *i* to compartment *j*) which depends on the network states. The network is connected to the outside by additional arcs. Inflows e_i inject matter into compartments and outflows s_i extract matter from certain compartment. Such a configuration is associated with a conservative compartmental system – compartmental due to storage capacity in nodes and conservative due to capacity to preserve the mass or the energy circulated within the network.

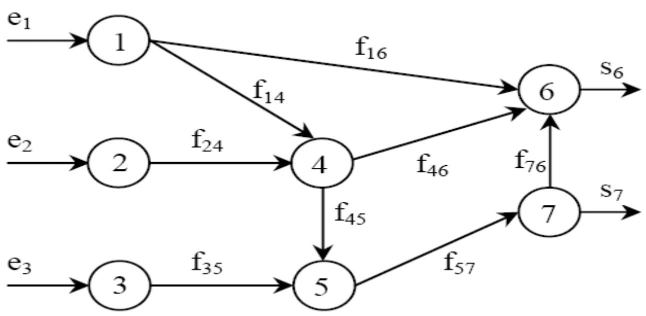

Fig. 4. Compartmental network.

The balance of dynamic transfer between compartments is described by the equations:

$$\dot{x}_j = e_j + \sum_{i \neq j} f_{ij}(x) - \sum_{k \neq j} f_{jk}(x) - s_j, \quad j = 1, ..., n \tag{19}$$

This model has the remarkable property that all state variables remain non-negative on the entire dynamics of the system. The functions f_{ij} and s_i have a non-negative admissible domain and give non-negative values $f_{ij}, s_i : \mathbb{R}_+ \to \mathbb{R}_+$, while the functions e_i are considered non-negative. If f_{ij} and s_i are differentiable, we can write:

$$\begin{aligned} f_{ij}(x) &= r_{ij}(x)x_i \\ s_i(x) &= q_i(x)x_i \end{aligned} \tag{20}$$

The state model of the traffic network becomes:

$$\dot{x}_i = \sum_{j \neq i} r_{ji}(x)x_j - \sum_{i \neq k} r_{ik}(x)x_i - q_i(x)x_i + e_i, \quad i = 1, ..., n \tag{21}$$

In matrix-vector representation, the previous set of equations can be written:

$$\begin{aligned} \dot{x} &= A(x)x + B(d)u \\ y &= C(x)x \end{aligned} \tag{22}$$

with the matrices having the following form:

$$A(x) = \begin{bmatrix} -\left(q_1(x) + \sum_{i \neq 1} r_{1i}(x)\right) & r_{21}(x) & \cdots & r_{n1}(x) \\ r_{12}(x) & -\left(q_2(x) + \sum_{i \neq 2} r_{2i}(x)\right) & \cdots & r_{n2}(x) \\ \vdots & \vdots & \vdots & \vdots \\ r_{1n}(x) & r_{2n}(x) & \cdots & -\left(q_n(x) + \sum_{i \neq n} r_{ni}(x)\right) \end{bmatrix}$$

$$B(d) = \begin{bmatrix} d_{11} & 0 & \cdots & 0 \\ 0 & d_{22} & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \cdots & d_{nm} \end{bmatrix} \tag{23}$$

$$C(x) = \begin{bmatrix} q_1(x) & 0 & \cdots & 0 \\ 0 & q_2(x) & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \cdots & q_n(x) \end{bmatrix}$$

The matrix $A(x)$ is called compartmental matrix and it is a Metzler matrix – diagonal dominant with non-negative elements outside the main diagonal elements. The non-singularity and stability of this compartmental matrix are closely related to the notion of outflow connection. A compartment *i* is said to be "outflow connected" if there is a path $i \to j \to ... \to k$ from compartment *i* to compartment *k*, from which we have an output flow q_k. The network is called fully outflow connected (FOC) if all compartments are connected to output flows. A compartment *k* is said to be "inflow connected", if there is a path $i \to j \to ... \to k$ from compartment *i* to compartment *k*, where compartment *i* has an input flow b_i. The network is called fully inflow connected (FIC) if all compartments are connected to input flows.

The compartmental matrix $A(x)$ is non-singular and stable if and only if the associated compartment network is fully outflow and inflow connected. Thus, the non-singularity and the stability of matrix *A* can be directly verified by analyzing the associated compartmental network.

On the other hand, the continuous system:

$$\begin{cases} \dot{x}(t) = A\,x(t) + b\,u(t) \\ y(t) = c^T x(t) \end{cases}, A \in \mathrm{R}^{N \times N},\ b, c \in \mathrm{R}^N \tag{24}$$

is a positive system if the following conditions are verified:

$$\begin{cases} b_i \geq 0 \\ c_i \geq 0 \\ a_{ij} \geq 0,\ i \neq j \\ a_{ii} + \sum\limits_{j \neq i} a_{ji} \leq 0 \end{cases},\ i, j = 1, 2, ..., N \tag{25}$$

It is easy to see that the conservative compartmental network given by (18) is a positive system and benefits from the properties of positive systems, used in controlling compartmental networks.

We consider a compartmental network, with *n* compartments, *m* input flows and *p* output flows, under the following conditions:

- the network is fully inflow connected and fully outflow connected;

- the transfer flows between network compartments are bounded;

- the capacities of network compartments are bounded;

- there is a demand for input flow, $d_i(t)$, for each input node of the network, which determines the flow that can enter the system.

Preventing network congestion requires the control of input flow $e_i(t)$, effectively injected into the system, by reducing the demand:

$$e_i(t) = u_i(t)d_i(t),\ 0 \le u_i(t) \le 1 \tag{26}$$

Thus $e_i(t)$ represents a fraction of the required flow $d_i(t)$. The proposed controller is a dynamic nonlinear controller, with the structure [4, 5]:

$$\begin{aligned} \dot{z}_i &= y_i - \Phi(z_i)\sum_{k\in Q_i}\alpha_{ki}d_k \quad (i \in I_s) \\ u_j &= \sum_{k\in P_j}\alpha_{jk}\Phi(z_k) \quad (j \in I_e) \end{aligned} \tag{27}$$

The closed loop system is represented in Figure 5. The controller has a compartmental network structure with a number of compartments equal to the number of controlled outputs y_i. Each compartment of the controller is injected (virtually) with a copy of the output flows from the controlled network. The output flows from the controller are distributed towards the control variables (controlled network's inputs), thus there is only one way through the regulator from an output node to an input node, for each possible pair.

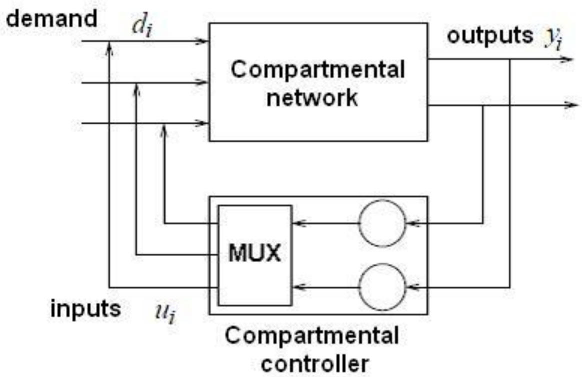

Fig. 5. Closed loop control of a compartmental system.

In matrix form, the control law is written:

$$\begin{aligned} \dot{z} &= G(d)F(z)z + y \\ u &= K(z)z \end{aligned} \tag{28}$$

with:

$$\begin{aligned} G(d) &= diag\left(\sum_{k\in Q_i} -\alpha_{ki}d_k, i \in I_s\right) \\ F(z) &= diag\left(\sum_{k\in Q_i}\Phi(z_i), i \in I_s\right) \end{aligned} \tag{29}$$

where the following notations have been used:

- I_e is the set of input nodes of the network;

- I_s is the set of output nodes of the network;

- $\mathcal{R}$ is the set of pairs of nodes (j,k), with $j \in I_e$, for which there is a direct path from the input node *j* to the output node *k*;

- $P_j = \{k \mid (j,k) \in \mathcal{R}\} \subset I_s$ is the set of output nodes accessible from input node *j*;

- $Q_i = \{k \mid (k,i) \in \mathcal{R}\} \subset I_e$ is the set of input nodes that feed the output node *i*;

- $\Phi : R \to R_+$ is an increasing function, continuous and differentiable, with the properties $\Phi(0) = 1$ and $\Phi(\infty) = 1$;

- α_{jk}, $(j,k) \in \mathcal{R}$ are characteristic parameters of the network, with the properties $0 \le \alpha_{jk} \le 1$ and $\sum_{k \in P_j} \alpha_{jk} = 1$.

From equations (24) and (28), it results the set that describes the closed loop system:

$$\begin{pmatrix} \dot{x} \\ \dot{z} \end{pmatrix} = \begin{pmatrix} A(x) & B(d)K(z) \\ C(x) & G(d)F(z) \end{pmatrix} \begin{pmatrix} x \\ z \end{pmatrix} = L(x,z) \begin{pmatrix} x \\ z \end{pmatrix} \tag{30}$$

The main properties of the closed loop system are the following:

- The matrix $L(x,z)$ is a compartmental matrix; therefore the system in (29) is a positive one.

- The state variables are bounded; the upper limits are smaller than $x_{i\max}$. Thus, the congestion control objective is accomplished by ensuring that network saturation is avoided.

- The command variables $u_i(t)$, as fractions of required inflows, are functions with values in the interval (0,1).

- The controlled network is FIC and FOC and because of the controller structure, the closed-loop system is compartmental and strongly connected.

The compartmental controller has an interesting practical interpretation, namely that the designed controller is actually another traffic network that takes the saturation excess demand of the controlled network and thus avoids congestion.

Conclusions

In the current context of an expanding society and of its growing mobility needs, this paper aims to present solutions to improve and optimize road traffic by employing methods and techniques from automatic control.

For this purpose, the macroscopic representation of traffic was chosen. In this representation we have a decentralized approach to traffic in the following configuration: road network - road object - road segment. The road network is seen as the top level of the road infrastructure. The road objects form the intermediate level and are represented by road sections (streets) and road intersections. On the lower level there are road segments, which are the basic components of a traffic configuration. By interconnecting elements within a lower level elements from the upper level are obtained. Thus, multiple road segments form a road section or a road intersection and several sections and intersections create the road network. Also, a road section may be composed only of a single road segment, which is the case of a vehicle storage intersection for which the central part, that connects the intersecting sections, is a road segment.

According to hydraulic representation, models are built for road segments and their connections (sections), resulting input-output linearized mathematical models. Based on these models, numerical control algorithms of polynomial type (RST) are designed for different traffic parameters, such as the flow of vehicles, the number of vehicles or the length of vehicles queue. The designed command can be applied for simple models (segments) or for more complex models (sections).

In the case of control of road networks (complex configurations taken from a urban traffic infrastructure) we turned to a centralized approach using a compartmental representation of the network and obtained dynamic state models. Based on these compartmental models a state space feedback control was designed, taking advantage of the positive system structure.

R E F E R E N C E S

[1] S.P. Hoogendoorn, P.H.L. Bovy, Journal of Systems & Control Engineering, **215**, 283 (**2001**).

[2] G. Dauphin-Tanguy, L. Foulloy, D. Popescu, *Modélisation et commande des systèmes* (Romanian Academy Press, Bucharest, Romania, **2004**).

[3] I. Landau, G. Zica, *Digital Control Systems - Design, Identification and Implementation* (Springer Press, Berlin, **2006**).

[4] G. Bastin, V. Guffens, Systems and Control Letters, **55(8)**, 689 (**2006**).

[5] G. Bastin, Revue e-STA, **3(1)**, (**2006**).