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COST ALLOCATION OF TRANSMISSION SYSTEMS FOR REACTIVE POWER

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Rezumat. *În această lucrare, autorii propun determinarea alocării puterii reactive luând în considerare pierderilor şi costul acestora folosind metoda factorilor de distribuţie. Comparaţia se realizează folosind metoda Bialek ,bazată pe un metodă topologică pentru determinarea contribuţiilor generatoarelor (consumatorilor) la circulaţiile individuale de puteri prin elementele de reţea. Studiul de caz se referă la un sistem cu 12 noduri, având 6 surse şi 9 consumatori.*

Abstract. *In this paper, the authors propose the compute of reactive power allocation considering their losses and their cost using the distribution factors method. The comparison is performed using Bialek method, based on a topological approach for determining the contributions of generators (consumers) to individual power flow through network elements. The case study refers to the 12 buses test power system, heaving 6 P-U buses and 9 P-Q buses.*

Keywords: cost allocation, power systems, reactive power, tracing method, distribution factors

1. Introduction

Reactive power has a dominant effect on real energy transfer and an appropriate management of reactive power is very essential for supporting power system security. On the other hand, while reactive power production cost is highly dependent on active power generation, it is mainly confined to local consumption. Most researches have been focused on active power as the main good transacted in electricity markets and reactive power is studied less and superficial. As a result, to avoid market power and to maintain the secure operation of the system, a fair cost allocation method seems to be very essential.

Several methods have been developed to solve the allocation problem for reactive power costs. The Zbus method presents a solution based on Zbus matrix and considers the current injection at each bus [1], [2]. Methods based on proportional sharing principle provide efficient procedures for reactive power and reactive losses. References [3], [4], [5], [6] are example of these method.

These paper presents the distribution factors methods for reactive power allocation, considers active and reactive power losses. There are 3 categories of

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factors [7], [8]. Generation shift factors (AQ factors) refer to change of power flow through network elements due to changes of power generated distribution. Generalized generation distribution factors (DQ factors) measure the total use of transmission network facilities by generation injection. Generalized load distribution factors (CQ factors) measure the total use of transmission network facilities by loads. The distribution factors method was extended to calculate the AC power flow, heaving the object the calculus of the system cost allocation for the regime with active power losses [9]. The results are compared with Bialek method. The case study refers to the 12 buses test power system, heaving 6 P-U buses and 9 P-Q buses.

2. Method presentation

Generation Shift factors for reactive power (AQ factors) reflect the modification of reactive power through network element, corresponding to change of reactive power generated (without changing the power system overall balance). They depend on the choice of reference bus and not on the operating regime. AQ factors are determined based on simplified reactive power flow (which implies neglecting resistances longitudinal and transverse conductances network elements, the renunciation of active power flow and consider all angles equal to 0 - the angle of slack bus voltage).

$$\mathrm{Q}/\mathrm{U} = -\mathrm{B}\cdot\mathrm{U} \qquad (1)$$

where,

Q/*U* – vector of reactive power injected at system bus divided to voltage bus value;
U – vector of nodal voltage;
B – matrix of nodal susceptances (the imaginary parts of nodal admittances matrix $\underline{Y}_n$.

If the voltage that appears in the left side of relation (1) is considered equal to 1 pu, the system (2) becomes linear:

$$\mathbf{Q} = -\mathbf{B}\cdot\mathrm{U} \qquad (2)$$

By solving the linear system (2) results the value of nodal voltage:

$$\mathbf{U} = -\mathbf{B}^{-1}\cdot\mathbf{Q} \qquad (3)$$

which means (noting with $\mathrm{b}_{\mathrm{ji}}^{-1}, \mathrm{j}\in\mathrm{N}, \mathrm{i}\in\mathrm{N},$ matrix elements B^{-1})

$$\mathrm{U}_\mathrm{j} = -\sum_{\mathrm{i}\in\mathrm{N}}(\mathrm{b}_{\mathrm{ji}}^{-1}\cdot\mathrm{Q}_\mathrm{i}), \quad \mathrm{j}\in\mathrm{N} \qquad (4)$$

In same condition obtain:

$$Q_\ell / U = -B_\ell \cdot U_\ell \tag{5}$$

where

P_ℓ / U - vector of reactive power through the network elements divided by the voltage node value;
U_ℓ - vector of voltage of network elements from initial bus;
B_ℓ - diagonal matrix of longitudinal susceptances of network elements.

If the voltage that appears in the left side of relation (4) is considered equal to 1 pu, the system (5) becomes linear:

$$Q_\ell = -B_\ell \cdot U_\ell \tag{6}$$

Writing in extended variant the relation (6) lead to:

$$Q_{\ell jk} = -B_{\ell jk} \cdot (U_j - U_k), \quad jk \in R \tag{7}$$

Using the relation (5), relation (8) becomes:

$$Q_{\ell jk} = B_{\ell jk} \cdot \left[\sum_{i \in N} (b_{ji}^{-1} \cdot Q_i) - \sum_{i \in N} (b_{ki}^{-1} \cdot Q_i) \right] = B_{\ell jk} \cdot \sum_{i \in N} \left[(b_{ji}^{-1} - b_{ki}^{-1}) \cdot Q_i \right], \; jk \in R \tag{8}$$

Relation (8) is linear and the modification of power through network element, $\Delta Q_{\ell jk}$, can be expressed without problems due to changing of power injected in bus *i*, ΔQ_i:

$$\Delta Q_{\ell jk} = B_{\ell jk} \cdot (b_{ji}^{-1} - b_{ki}^{-1}) \cdot \Delta Q_i \tag{9}$$

Comparing the relations (9) and (1), the expression of AQ factors for network elements *jk*, corresponding changing of generated power in bus *i*:

$$AQ_{jk,i} = B_{\ell jk} \cdot (b_{ji}^{-1} - b_{ki}^{-1}), \quad jk \in R, \quad i \in N \setminus e \tag{10}$$

Analogous to generalized generation distribution factors for active power, DQ factors determine the impact of each generator on active power flow on network elements (so, they can have negative values).

They are determined in conditions of DC power flow too, being defined by the relation:

$$Q_{\ell,jk} = \sum_{i \in N} (DQ_{jk,i} \cdot Q_{gi}), \quad jk \in R \tag{11}$$

where,

$Q_{\ell,jk}$ – reactive power flow on network elements *jk;*
Q_{gi} – power generated in bus *i*;

$DQ_{jk,i}$ – DQ factor of a network elements *jk*, corresponding to power generated in bus *i*, heaving the expression:

$$DQ_{jk,i} = DQ_{jk,e} + AQ_{jk,i} = \frac{Q^0_{jk} - \sum\limits_{i \in N \backslash e} (AQ_{jk,i} \cdot Q_{gi})}{\sum\limits_{i \in N} Q_{gi}} + AQ_{jk,i} \tag{12}$$

where, Q^0_{jk} – reactive power flow on network elements *jk* from the previous iteration; *e* – slack bus.

DQ factors reflect the utilization rate of electricity transmission capacity depending on reactive generated power (unlike the A factors, which indicated the incremental rate of use). They depend on network elements and operating regime and not on the choice of reference bus.

Generalized load distribution factors (CQ factors) are very similar to DQ factors and determine the contribution of each load to network elements (so, they can have negative values). They are defined by the relation:

$$Q_{\ell,jk} = \sum_{i \in N} (CQ_{jk,i} \cdot Q_{ci}), \quad jk \in R \tag{13}$$

where, $Q_{\ell,jk}$ – reactive power flow on network elements *jk;* Q_{ci} – reactive power consumed in bus *i*; $CQ_{jk,i}$ – CQ factor of a network elements *jk*, corresponding to power generated in bus *i*, heaving the expression:

$$CQ_{jk,i} = CQ_{jk,e} - AQ_{jk,i} = \frac{Q^0_{jk} - \sum\limits_{i \in N \backslash e} (AQ_{jk,i} \cdot Q_{ci})}{\sum\limits_{i \in N} P_{ci}} - AQ_{jk,i} \tag{14}$$

where,

Q^0_{jk} – reactive power flow on network elements *jk* from the previous iteration;

e – slack bus.

CQ factors reflect the utilization rate of electricity transmission capacity depending on consumed power. They depend on network elements and operating regime and not on the choice of reference bus.

3. Description of test power system analyzed

The test system with 12 buses is shown in Fig. 1. Bus 1 is slack bus. The network elements parameters are presented in Table 1. Table 2 contains the initial data of buses and the results of power flow for considered operating regime. Table 3 presents the results of the power flow for line system.

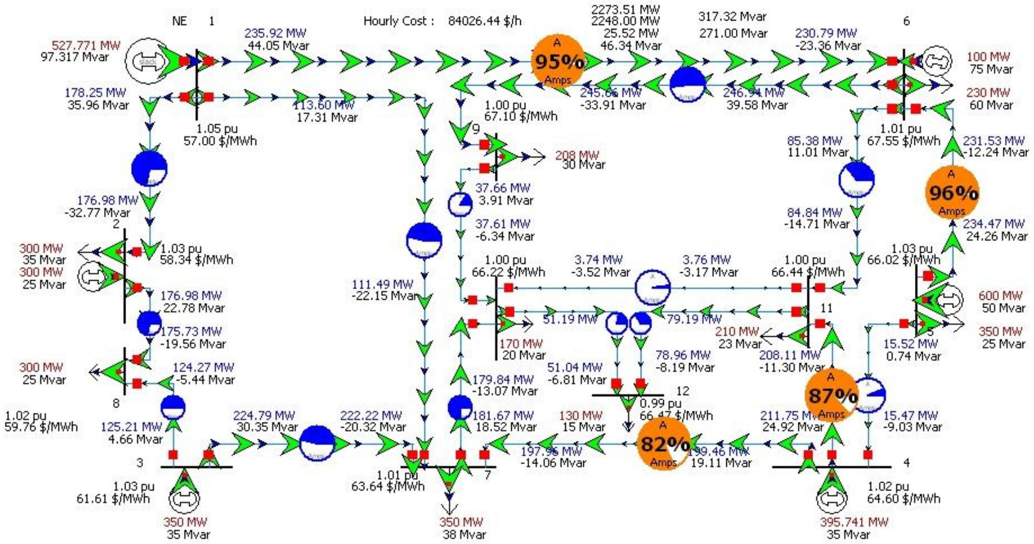

Fig. 1. Test system with 12 buses, normal operation regime.

Table 1. Network elements parameters

Bus i	*Bus j*	*R [u.r]*	*X [u.r]*	*B [u.r]*	*L [km]*	*Bus i*	*Bus j*	*R [u.r]*	*X [u.r]*	*B [u.r]*	*L [km]*
1	2	0.00415	0.025	0.04	30	6	5	0.00554	0.03335	0.05379	40
1	6	0.00969	0.05838	0.0949	70	6	9	0.002075	0.0125	0.02	30
1	7	0.0166	0.1	0.16132	120	6	11	0.00692	0.0417	0.06725	50
2	8	0.00415	0.025	0.04	30	10	7	0.00554	0.03335	0.05379	40
3	7	0.00526	0.03169	0.0511	38	9	10	0.00277	0.01667	0.0269	20
8	3	0.00623	0.03752	0.06	45	10	11	0.00692	0.0417	0.06725	50
5	4	0.0083	0.05	0.08	60	10	12	0.00484	0.02912	0.047	34
7	4	0.00387	0.02335	0.03765	28	11	12	0.00346	0.0208	0.0336	25
11	4	0.0083	0.05	0.08	60						

Table 2. Dates and results of buses power flow

Number	*U [u.r]*	*U [kV]*	*d [grd]*	*d [grd]*	*P_C [MW]*	*Q_C [MVar]*	*P_g [MW]*
1	1.05	231	0	0	0	527.77	97.32
2	1.03468	227.629	-2.27	300	35	300	25
3	1.03182	227.001	-2.08	0	0	350	35
4	1.02382	225.241	-3.37	0	0	395.74	35
5	1.02747	226.043	-2.97	350	25	600	50
6	1.0088	221.937	-7.21	230	60	100	75
7	1.01245	222.739	-5.9	350	38	0	0
8	1.0224	224.929	-4.61	300	25	0	0
9	0.99914	219.81	-8.92	208	30	0	0
10	0.99724	219.392	-9.27	170	20	0	0
11	0.99757	219.466	-9.18	210	23	0	0
12	0.99332	218.531	-10.12	130	15	0	0

Table 3. Power flow results for network elements

Bus i	*Bus j*	P_{ij} *[MW]*	Q_{ij} *[Mvar]*	$S_{ij}^{\max}$ *[MVA]*	*ΔP* *[MW]*	*ΔQ* *[MVAR]*
1	2	178.3	36	75.8	1.27	3.19
1	6	235.9	44.1	100	5.14	20.69
1	7	113.6	17.3	47.9	2.11	-4.84
2	8	177	22.8	74.4	1.25	3.23
3	7	224.8	30.3	47.3	2.57	10.03
8	3	-124.3	-5.4	52.2	0.94	-0.78
5	4	15.5	0.7	7.5	0.05	-8.29
7	4	-198	-14.1	83.5	1.5	5.06
11	4	-208.1	-11.3	88.8	3.65	13.62
6	5	-231.5	-12.2	98.2	2.94	12.02
6	9	246.9	39.6	52.1	1.29	5.68
6	11	85.4	11	35.9	0.53	-3.7
10	7	-179.8	-13.1	76.1	1.83	5.45
9	10	37.7	3.9	15.9	0.05	-2.44
10	11	-3.7	-3.5	2.1	0.03	-6.68
10	12	51.2	2.9	21.5	0.15	-3.88
11	12	79.2	6.2	33.1	0.23	-2.01

4. Numerical results

For the calculation we used Mathematica® environment, application software DFPQ (Distribution Factor for Active and Reactive Power) being developed by the author. Using the relations (12) and (13) we obtain DQ factors (Table 4) and CQ factors (Table 5).

Table 4. Generalized generation distribution factors (DQ factors)

Line j-k	$D_{jk,1}$	$D_{jk,2}$	$D_{jk,3}$	$D_{jk,4}$	$D_{jk,5}$	$D_{jk,6}$
1-2	-0.449787	-1.79374	-0.23392	0.684412	0.83575	0.936693
1-6	-0.671947	-0.368645	0.402364	0.782211	0.783145	0.783769
1-7	-0.395048	-0.266395	0.0606469	0.304861	0.39825	0.460541
2-8	-0.289615	0.0238408	-0.17934	0.289781	0.367091	0.418657
3-7	0.0964449	0.0462214	-0.0814494	0.369913	0.218238	0.117071
3-8	0.733339	0.112715	-0.464943	-0.104448	-0.375162	-0.555729
4-5	0.0316719	-0.00615358	-0.102308	-0.246827	0.319719	0.0306042
4-7	-0.101657	-0.16929	-0.341218	0.26661	0.0623564	-0.0738809
4-11	-0.44855	-0.321843	0.000252289	0.313903	0.179898	0.0905158
5-6	-0.161478	-0.123652	-0.0274981	0.117021	0.550476	-0.16041
6-9	-0.823823	-0.558162	0.117163	0.554581	0.820685	0.998176
6-11	-0.260547	-0.185081	0.00675793	0.093705	0.261991	0.374237
7-10	-1.01215	-0.637673	0.31426	0.635423	0.601869	0.579489
9-10	-0.946538	-0.680877	-0.00555242	0.431866	0.69797	0.875461

Line j-k	$D_{jk,1}$	$D_{jk,2}$	$D_{jk,3}$	$D_{jk,4}$	$D_{jk,5}$	$D_{jk,6}$
10-11	0.336729	0.226573	-0.0534492	-0.271719	-0.290396	-0.302854
10-12	0.329447	0.23743	0.00351698	-0.178811	-0.194414	-0.20482
11-12	-0.282208	-0.19019	0.0437224	0.226051	0.241653	0.25206

Table 5. Generalized Load Distribution Factors (CQ factors)

Line j-k	$C_{jk,2}$	$C_{jk,5}$	$C_{jk,6}$	$C_{jk,7}$	$C_{jk,8}$	$C_{jk,9}$	$C_{jk,10}$	$C_{jk,11}$	$C_{jk,12}$
1-2	2.09779	-0.531696	-0.632639	-0.253538	1.4751	-0.903282	-1.2677	-0.889115	-1.04686
1-6	0.880437	-0.271353	-0.271977	-0.281812	0.572644	-0.678025	-1.22477	-0.743912	-0.944268
1-7	0.532779	-0.131866	-0.194157	0.0397837	0.402221	-0.36117	-0.58605	-0.352427	-0.44977
2-8	0.178612	-0.164639	-0.216204	-0.0225438	0.860515	-0.35446	-0.54062	-0.347223	-0.427805
3-7	-0.137224	-0.309241	-0.208073	0.0552317	-0.0862569	-0.169854	-0.118391	-0.22207	-0.178871
3-8	-0.644218	-0.156341	0.0242253	-0.110612	-0.0144048	0.311535	0.698395	0.254083	0.439213
4-5	-0.0465516	-0.372424	-0.0833094	0.0983949	-0.0081661	-0.0155522	0.0756824	0.0596382	0.0663233
4-7	0.0863156	-0.145331	-0.0090936	0.345486	0.154951	0.0423748	0.111677	-0.0279422	0.0302324
4-11	0.471525	-0.030216	0.0591657	-0.0140145	0.342942	-0.044516	-0.184123	0.118104	-0.007824
5-6	0.106293	-0.567835	0.143051	-0.0386532	0.0679078	0.0752939	-0.0159407	0.0001035	-0.006585
6-9	0.929712	-0.449135	-0.626626	-0.0882986	0.660117	-0.11091	-0.762994	-0.591674	-0.663057
6-11	0.295506	-0.151566	-0.263812	0.0063203	0.218922	-0.253335	-0.239227	0.0863527	-0.049306
7-10	1.16546	-0.0740868	-0.0517062	-0.269525	0.785436	-0.305506	-0.647245	-0.359844	-0.479594
9-10	1.1841	-0.194744	-0.372235	0.166093	0.914508	-0.856518	-0.508603	-0.337283	-0.408666
10-11	-0.423646	0.0933229	0.105781	-0.0015303	-0.311859	0.156564	0.224944	0.427736	0.343239
10-12	-0.433347	-0.0015042	0.0089024	-0.0807386	-0.339968	0.0513236	0.108443	0.277844	0.623927
11-12	0.322683	-0.10916	-0.119567	-0.0299256	0.229304	-0.161988	-0.219108	-0.388508	0.265409

In order to make comparisons, it will be presented only the situations considered more representative. In this sense, it analyzed the contribution of source G4

(Q_{G4} = 35 MVar) to reactive power flow and reactive power losses through network elements.

Figure 2 shows the contribution of generator G4 to reactive power flow. The participation of this source leads to negative values on lines 3-8 (-3.66 Mvar), 4-5 (-8.64 Mvar), 10-12 (-6.26 MVar) and 10-11 (-9.51 Mvar). The most significant contribution is detected on line 2-8, for a value of 27.38 MVar. In contrast to distribution factors method, Bialek method leads to different values, the source G4 representing the contributions only on tracings 4-7-10-12 and 4-11-10-12.

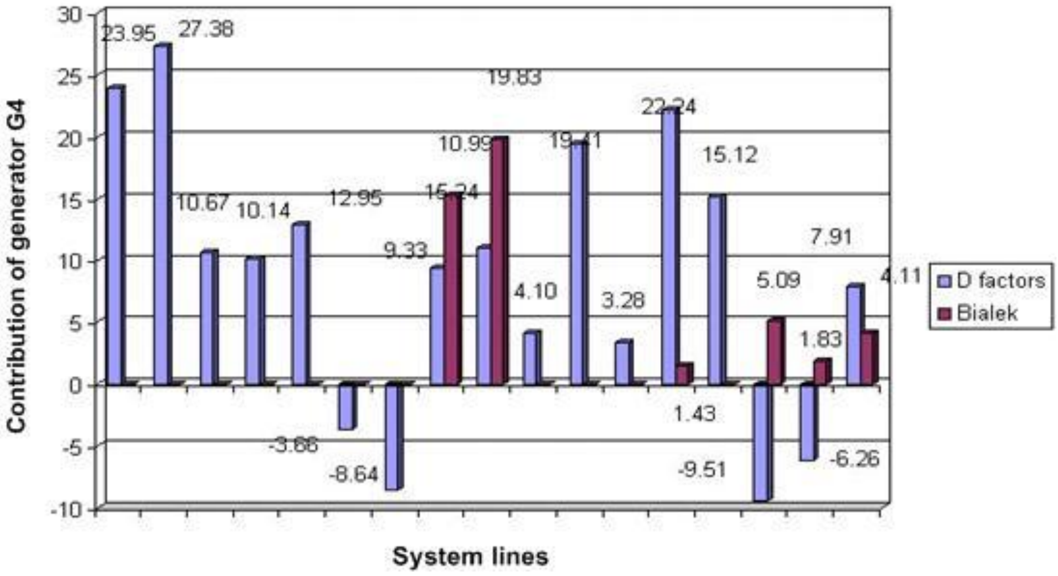

Fig. 2. Contribution of source G4 to reactive power flow.

After that it goes on to present the allocation of electricity transmission costs to energy market participants - in our case, for producers and consumers. It is considered as a basis for calculating a unit cost of transport lines 2 \$/MW·km. In order to calculate the transfer cost by the MW-km method, the following formula will be used [12]:

$$C_{tu} = C_T \cdot \frac{\sum\limits_{ij\in L} (c_{ij} \cdot \ell_{ij} \cdot Q_{ij\,u})}{\sum\limits_{k\in U} \sum\limits_{ij\in L} (c_{ij} \cdot \ell_{ij} \cdot Q_{ij\,k})} \qquad (16)$$

where:

C_{tu} – transmission hourly cost for transaction *u* [\$/h];

L_{ij} – length of line *ij* [km];

c_{ij} – unit transmission cost of line *ij* [\$/MW · km];

$Q_{ij\,u}$ – reactive power flow on line *ij*, due to transaction *u* [MW];
U – set of transactions,
L – set of lines.

Using DQ factors, transmission costs allocated to generators (Table 6) are determined. Using CQ factors, transmission costs allocated to generators (Table 7) are determined. The values obtained are compared with those resulted with Bialek methods. All values are calculated in presents of power losses.

Table 6. Transmission costs allocated to generators

	Cost allocated to generator 1 [$]	Cost allocated to generator 2 [$]	Cost allocated to generator 3 [$]	Cost allocated to generator 4 [$]	Cost allocated to generator 5 [$]	Cost allocated to generator 6 [$]
Distribution Factors	297.34	32.34	66.41	354.92	633.91	887.59
Bialek	358.93	23.04	176.99	198.22	709.95	819.00

Table 7. Transmission costs allocated to consumers

	Cost allocated to consumer 2[$]	Cost allocated to consumer 5 [$]	Cost allocated to consumer 6 [$]	Cost allocated to consumer 7 [$]
Distribution Factors	108.76	166.34	320.79	84.82
Bialek	16.48	0.00	262.93	146.23

	Cost allocated to consumer 8 [$]	Cost allocated to consumer 9 [$]	Cost allocated to consumer 10 [$]	Cost allocated to consumer 11 [$]	Cost allocated to consumer 12 [$]
Distribution Factors	33.63	240.34	258.16	185.53	141.63
Bialek	44.88	159.95	439.65	97.72	457.19

Conclusions

Reactive power plays crucial role in power system security and reliability. An accurate reactive power allocation method is significant to have correct cost for each participant. The authors proposed a method for reactive power allocation using distribution factors, in presence of active and reactive power losses. Conclusions are achieved from study over more complex and different systems. The two methods compared have more differences, given by using different principles. Distribution factors method is based on superposition principle and produced negative flow. Bialek method uses the proportional sharing principle and the value obtained are always positive.

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