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# A BEREZIN-TYPE MAP ON $L_a^2(\mathbb{C}_+)^*$

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**Abstract**

In this paper we introduce a map $E$ defined on the Bergman space $L_a^2(\mathbb{C}_+, d\widetilde{A})$ as $(Ef)(w) = \int_{\mathbb{C}_+} f(s)|b_{\overline{w}}(s)|^2 d\widetilde{A}(s), w \in \mathbb{C}_+$, where $\mathbb{C}_+$ is the right half plane, $d\widetilde{A}(s) = dxdy$ is the area measure and $b_{\overline{w}}(s) = \frac{1}{\sqrt{\pi}} \frac{1+w}{1+\overline{w}} \frac{2Rew}{(s+w)^2}, s \in \mathbb{C}_+$. We refer the map $E$ as a Berezin-type map on $L_a^2(\mathbb{C}_+)$. In this work we first investigate the boundedness of the map $E$ on various $L^p$ space and show that the sequence $\{E^n\}$ converges to 0 in norm in the space $L^2(\mathbb{C}_+, d\mu)$ where $d\mu(w) = |B(\overline{w}, w)|d\widetilde{A}(w), w \in \mathbb{C}_+$. We then discuss certain algebraic and ergodicity properties of the map $E$ involving subharmonic functions.

**MSC**: 47B35, 32M15

**keywords:** Bergman space, the right half plane, Berezin transform, automorphisms, subharmonic functions.

## 1 Introduction

Let $\mathbb{C}_+ = \{s = x + iy \in \mathbb{C} : \text{Re}s > 0\}$ be the right half plane. Let $d\widetilde{A}(s) = dxdy$ be the area measure. Let $L^2(\mathbb{C}_+, d\widetilde{A})$ be the space of complex-valued, square-integrable, measurable functions on $\mathbb{C}_+$ with respect to the area measure. Let $L_a^2(\mathbb{C}_+)$ be the closed subspace [1] of $L^2(\mathbb{C}_+, d\widetilde{A})$ consisting of those functions in $L^2(\mathbb{C}_+, d\widetilde{A})$ that are analytic. The space $L_a^2(\mathbb{C}_+)$ is called the Bergman space of the right half plane. The functions $H(s, w) = \frac{1}{(s+\overline{w})^2}, s \in \mathbb{C}_+, w \in \mathbb{C}_+$ are the reproducing kernels [2] for

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