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A REPRESENTATION OF DE MOIVRE'S FORMULA OVER PAULI-QUATERNIONS*

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Abstract

We investigate algebraic and analytic properties of Pauli-quaternions. Also, we compose a polar form and De Moivre's formula over Pauli-quaternions and research their characteristics by using the isomorphism of the Pauli matrices.

MSC: 32A99, 32W50, 30G35, 11E88

keywords: Pauli matrix, quaternions, De Moivre's formula, isomorphism

1 Introduction

In mathematical physics, the Pauli matrices are Hermitian and unitary which are elements of a set of three 2×2 complex matrices as follows:

Definition 1. [*5*] [*p.*213] *The Pauli matrices are real* (2×2)*-matrices which are linear combinations of the basis matrices*

$$\mathbf{1} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \quad \sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix},$$

whose multiplication rules are $\sigma_1^2 = \sigma_2^2 = \sigma_3^2 = \mathbf{1}$*,* $\sigma_1\sigma_2 = i\sigma_3 = -\sigma_2\sigma_1$*,* $\sigma_2\sigma_3 = i\sigma_1 = -\sigma_3\sigma_2$ *and* $\sigma_3\sigma_1 = i\sigma_2 = -\sigma_1\sigma_3$*.*

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