Ann. Acad. Rom. Sci.
Ser. Math. Appl.
ISSN 2066-6594 Vol. 9, No. 2/2017

JENSEN'S INTEGRAL INEQUALITY IN LOCALLY CONVEX SPACES*

Libor Vesely†

Abstract

We prove generalizations of Jensen's integral inequality for proper convex functions defined on a finite- or infinite-dimensional convex set in a locally convex topological vector space.

MSC: 26D15, 26B25, 46N10, 52A41

keywords: Convex function, locally convex space, Jensen's integral inequality

1 Introduction

The well-known Jensen's (finite) inequality asserts that if C is a convex set (in a real vector space), $f\colon C \to (-\infty, \infty]$ a convex function, $x_1, \dots, x_n$ points of C, and $\lambda_1, \dots, \lambda_n$ positive numbers whose sum equals 1, then the point $\sum_{i=1}^n \lambda_i x_i$ belongs to C, and

$$f\left(\sum_{i=1}^n \lambda_i x_i\right) \le \sum_{i=1}^n \lambda_i f(x_i)\,.$$

This inequality can be reformulated in any of the following two forms:

as

$$f\left(\int_C x\, d\mu(x)\right) \le \int_C f\, d\mu \tag{1}$$

*Accepted for publication on April 5-th, 2017

†libor.vesely@unimi.it Dipartimento di Matematica, Università degli Studi di Milano, Via C. Saldini 50, 20133 Milano, Italy.