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TOEPLITZ OPERATORS WITH BOUNDED HARMONIC SYMBOLS*

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Abstract

In this paper we show that if A and B are two bounded linear operators on the Bergman space $L^2_a(\mathbb{D})$ and $AT_\phi B = T_\phi$ for all $\phi \in h^\infty(\mathbb{D})$ then $A = \alpha I$ and $B = \beta I$ for some $\alpha, \beta \in \mathbb{C}$ and $\alpha\beta = 1$. Here $h^\infty(\mathbb{D})$ is the space of all bounded harmonic functions on the open unit disk $\mathbb{D}$.

MSC: 47B35

Keywords: Toeplitz operators, Bergman space, bounded harmonic function, Bergman shift, Hardy space

1 Introduction

Let $n \in \mathbb{N}$ and $L^{2,n}_a(\mathbb{D})$ be the Hilbert space of all analytic functions f on $\mathbb{D}$ with finite norm

$$\|f\|^2_{L^{2,n}_a(\mathbb{D})} = \lim_{r\to 1} \int_{\overline{\mathbb{D}}} |f(rz)|^2 d\mu_n(z).$$

The measure $d\mu_1$ is the normalized Lebesgue arc length measure on the unit circle $\mathbb{T}$ and for $n \geq 2$ the measure $d\mu_n$ is the weighted Lebesgue area measure given by $d\mu_n(z) = (n-1)(1-|z|^2)^{n-2}dA(z), z \in \mathbb{D}$, where $dA(z) = \frac{dxdy}{\pi}, z = x + iy$, is the planar Lebesgue area measure normalized so that the unit disk $\mathbb{D}$ has area 1. The space $L^{2,1}_a(\mathbb{D}) = H^2(\mathbb{D})$, the standard Hardy space, the space $L^{2,2}_a(\mathbb{D}) = L^2_a(\mathbb{D})$, is the unweighted Bergman space and

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