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ON THE INITIAL VALUE PROBLEMS FOR NEUTRAL INTEGRO-DIFFERENTIAL SYSTEM WITHIN EXPONENTIAL KERNEL*

Stanislaus Jasmin Swetha† Velusamy Kavitha‡
Mani Mallika Arjunan§ Dumitru Baleanu¶

Abstract

This paper deals with the existence results for neutral integro-differential system (NIDS) through Caputo-Fabrizio (CF) operator. The solution to our addressing system, which can be considered innovative, is explained and proven in preliminary section. Certain novel existence findings are shown using fixed point approaches. Finally, two numerical examples are provided in the work to demonstrate the application of our theoretical findings.

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†jass.stanis27499@gmail.com, Department of Mathematics, School of Sciences, Arts, Media & Management, Karunya Institute of Technology and Sciences, Karunya Nagar, Coimbatore-641114, Tamil Nadu, India.

‡kavi_velubagyam@yahoo.co.in, Department of Mathematics, School of Sciences, Arts, Media & Management, Karunya Institute of Technology and Sciences, Karunya Nagar, Coimbatore-641114, Tamil Nadu, India.

§Corresponding author:arjunphd07@yahoo.co.in, Department of Mathematics, School of Arts, Science and Humanities, SASTRA Deemed to be University, Thanjavur-613401, Tamil Nadu, India.

¶Department of Computer Science and Mathematics, Lebanese American University, Beirut, Lebanon. E:mail: dumitru.baleanu@lau.edu.lb

1 Introduction

Due to the memory and nonlocal features that it contains, the study of fractional-order calculus has garnered a large amount of attention from a number of research organizations in recent years. Over several centuries, these characteristics have changed as mathematical perspectives have changed. In point of fact, fractional-order models are superior to integer-order models in terms of its capacity to appropriately solve complex mathematical problems and offer a description of the nature of dynamical systems [5, 31]. As a direct consequence of this fact, the fractional-order calculus has been applied in a wide number of fields, including electrical engineering, biological systems, and viscoelastic systems, amongst others.

It is conceivable to place the advent of fractional calculus in a time frame that is not too distant from the emergence of classical calculus. In a letter to l'Hospital that Leibniz composed and delivered to the l'Hospital, it was first referenced in 1695. Leibniz also provided in this letter a suggestion for the concept of a semi-derivative. Many well-known mathematicians, including Liouville, Grunwald, Riemann, Euler, Lagrange, Heaviside, Fourier, and Abel, among others, contributed to the development of the formal foundations for fractional calculus over time. These mathematicians all significantly improved the subject. Many of them proposed unique approaches, which are included in [33] in chronological order. The unique features of both classical and fractional differential representations can be found in the monographs [11, 24, 31, 34], and related research papers on fractional differential systems can be found in [8–10, 14–18, 21–23, 27–30].

The notion Riemann-Liouville fractional derivatives and Caputo fractional derivatives are used to describe fractional derivatives with a singular kernel. Because the existence of the singularity in the operators for fractional derivatives has a number of issues, Caputo and Fabrizio [13] proposed a novel fractional derivative without a singular kernel in 2015. The CF fractional derivative is widely acknowledged to be highly advantageous and useful when exploring real-world difficulties. The CF fractional derivative has a wide range of practical applications, some of which are given in [14]. As may be seen from the publications [1–4, 6, 8, 12, 17, 20, 25, 26, 32] and references thereto, several mathematicians have contributed to the development of CF fractional differential equations.

Neutral differential equations can be found in a wide variety of applications of mathematics, which is one of the primary reasons why researchers have focused so much emphasis on these equations over the past few decades. For additional details on this theory, we suggest the reader to refer [19].

In recent years, the authors have discovered several fascinating findings on fractional differential equations by the use of CFO with impulses, as shown in

[3, 4, 12, 20, 32], and without impulses, as seen in [1, 2, 6–8, 13, 14, 17, 25, 26]. In specifically, the authors of [4] investigated existence results of fractional differential system could be found in Banach spaces by using the CF derivative. The coupled system of fractional differential system through CF derivative in Banach spaces was studied by A. Boudaoui and A. Slama [12]. Eiman et al. [17] analyzed the Caputo-Fabrizio fractional differential equations under krasnoselskii fixed point theorem. After then, authors conducted research on the initial value issues for the CF impulsive fractional differential equations and published their findings in [3]. Very recently, Kanimozhi et al. [20] studied the existence results for CF fractional differential system with impulses and boundary value conditions in Banach spaces. According to a survey of numerous recent studies, the topic of neutral differential and integro-differential equations via CFO of the form (1.1)-(1.3) has not yet been addressed by anyone. This is the primary motive for this work.

In light of the foregoing, we examine the existence and uniqueness findings for a class of NDS through CF operator of the model

$$ {}^{CF}D_{\varsigma}^{\vartheta}[p(\varsigma)-h(\varsigma,p(\varsigma))] = f(\varsigma,p(\varsigma)), \quad \varsigma\in[0,\xi], \tag{1.1}$$

$$ p(0) = p_0, \tag{1.2}$$

where ${}^{CF}D_{\varsigma}^{\vartheta}$ is the CFO of order $\vartheta\in(0,1)$, $\xi>0$, $f:[0,\xi]\times\mathbb{R}\to\mathbb{R}$ is continuous, $h:[0,\xi]\times\mathbb{R}\to\mathbb{R}$ is continuously differentiable and $p_0\in\mathbb{R}$.

Further, we examine the existence and uniqueness findings for a class of NIDS via CF operator of the model

$$ {}^{CF}D_{\varsigma}^{\vartheta}[p(\varsigma)-h(\varsigma,p(\varsigma))] = f\left(\varsigma,p(\varsigma),\int_0^{\varsigma} h_1(\varsigma,s)p(s)ds\right), \quad \varsigma\in[0,\xi] \tag{1.3}$$

with the condition (1.2), where $f:[0,\xi]\times\mathbb{R}\times\mathbb{R}\to\mathbb{R}$ is continuous and $h_1(\varsigma,s)$ is continuous for all $(\varsigma,s)\in[0,\xi]\times[0,\xi]$ and we can find a positive constant H in a way that $\max\limits_{\varsigma,s\in[0,\xi]}\|h_1(\varsigma,s)\| = H$.

In general, we analyze the existence results of the model (1.1)-(1.3), when $p_0\in\mathbb{X}$, $f:[0,\xi]\times\mathbb{X}\to\mathbb{X}$; $f:[0,\xi]\times\mathbb{X}\times\mathbb{X}\to\mathbb{X}$, and $h:[0,\xi]\times\mathbb{X}\to\mathbb{X}$ are given functions and $\mathbb{X}$ is real or complex Banach space with a norm $\|\cdot\|$.

The study's important findings may be summed up as follows:

1. This is the first attempt, as far as we know, to handle the NIDS with CFO for the system (1.1)-(1.3).

2. By utilizing the CFO along with the Laplace transform, we have introduced the solution of the given system (1.1)-(1.3) for the first time in literature.

In Lemma 2.2, we present and prove the solution to our addressing system, which can be considered a novelty.

3. The fundamental insights are derived with the help of Banach's and Krasnoselskii's fixed point theorems. In the final part of our discussion, we will give a few examples to illustrate how our major findings might be applied.

4. Additionally, the outcomes of this work generalized and enhanced earlier research that have been published in the literature, including those by [3, 4, 20].

The plan for carrying out this research has been laid out in the following manner: In Section 2, we explore the notion of a solution to our problem, along with some notations, a review of certain ideas, and a summary of the results from prior studies. The first result is based on the Banach contraction principle, while the second result is based on Krasnoselskii fixed point theorem, which we present in Section 3. In the final section of this paper, which is Section 4, we will demonstrate how our primary findings may be implemented by presenting a few examples.

2 Preliminaries

This section provides an overview of the essential concepts and outcomes of the CFO, which will be of use to us in establishing our primary conclusions.

The functions designated by the notation $L^1([0,\xi],\mathbb{X})$ that are integrable in the Bochner concept with reference to the Lebesgue measure and come furnished with the notation

$$\|p\|_{L^1} = \int_0^{\xi} \|p(x)\| dx$$

are referred to as $p : [0,\xi] \to \mathbb{X}$.

Let $AC([0,\xi])$ be the space of all absolutely continuous functions from $[0,\xi]$ into $\mathbb{X}$.

Definition 2.1. *[26] The CFO of order $0 < \vartheta < 1$ for a function $g \in AC([0,\xi])$ is described by*

$$({}^{CF}D_0^{\vartheta} g)(\zeta) = \frac{(2-\vartheta)N(\vartheta)}{2(1-\vartheta)} \int_0^{\zeta} e^{\left(-\frac{\vartheta}{1-\vartheta}(\zeta - x)\right)} g'(x)dx, \ \zeta \in [0,\xi], \tag{2.1}$$

where $N(\vartheta)$ is a constant depending on ϑ that satisfies the condition $N(0) = 1$ and $N(1) = 2$. Note that ${}^{CF}D_0^{\vartheta} g = 0$ iff g is a constant function.

Definition 2.2. *[26] The CFO of integral order $0 < \vartheta < 1$ for a function $g \in L^1([0,\xi])$ described by*

$$({}^{CF}I_0^{\vartheta} g)(\zeta) = \frac{2(1-\vartheta)}{N(\vartheta)(2-\vartheta)} g(\zeta) + \frac{2\vartheta}{N(\vartheta)(2-\vartheta)} \int_0^{\zeta} g(x)dx, \quad \zeta \geq 0. \tag{2.2}$$

Remark 2.1. *(i) The fractional integral of CF type of a function of order $\vartheta \in (0,1)$ is an average of function g and its integral of order one, according to the authors of [26].*

We get an explicit formula for $N(\vartheta)$ by imposing

$$\frac{2(1-\vartheta)}{N(\vartheta)(2-\vartheta)} + \frac{2\vartheta}{N(\vartheta)(2-\vartheta)} = 1.$$

Then

$$N(\vartheta) = \frac{2}{2-\vartheta}, \quad 0 \leq \vartheta \leq 1.$$

(ii) If we take $N(\vartheta) = \frac{2}{2-\vartheta}$, then (2.2) becomes

$$({}^{CF}I_0^{\vartheta} g)(\zeta) = (1-\vartheta)g(\zeta) + \vartheta \int_0^{\zeta} g(x)dx, \quad \zeta \geq 0.$$

By substituting $N(\vartheta)$ in (2.1), we obtain the definition of the CFO of order $0 < \vartheta < 1$ for a function g as follows:

Definition 2.3. *Let $0 < \vartheta < 1$. The CFO of order ϑ of a function g is given by*

$$({}^{CF}D_0^{\vartheta} g)(\zeta) = \frac{1}{(1-\vartheta)} \int_0^{\zeta} e^{\left(-\frac{\vartheta}{1-\vartheta}(\zeta - x)\right)} g'(x)dx, \zeta \in [0,\xi].$$

Lemma 2.1. *[6, Theorem 2] Let $g \in L^1([0,\xi])$. Then a function $p \in \mathscr{C}([0,\xi])$ is a solution of the following system*

$$\begin{aligned} ({}^{CF}D_0^{\vartheta} p)(\varsigma) &= g(\varsigma), \quad \varsigma \in [0,\xi], \\ p(0) &= p_0, \end{aligned} \tag{2.3}$$

iff p fulfills the subsequent integral equation

$$p(\varsigma) = p_0 + \frac{2(1-\vartheta)}{(2-\vartheta)N(\vartheta)} g(\varsigma) + \frac{2\vartheta}{(2-\vartheta)N(\vartheta)} \int_0^{\varsigma} g(s)ds. \tag{2.4}$$

From this point onwards, for simplicity, we take

$$A_\vartheta = \frac{2(1-\vartheta)}{(2-\vartheta)N(\vartheta)} \quad \text{and} \quad B_\vartheta = \frac{2\vartheta}{(2-\vartheta)N(\vartheta)}.$$

Then (2.4) can be written as

$$p(\varsigma) = p_0 + A_\vartheta g(\varsigma) + B_\vartheta \int_0^\varsigma g(s)ds. \tag{2.5}$$

Based on the above Lemma 2.1, we can define the solution of the given system (1.1)-(1.3) in the subsequent Lemma.

Lemma 2.2. *A function $p \in \mathscr{C}([0,\xi],\mathbb{X})$ is a solution of the system* (1.1)-(1.2) *iff p fulfills the subsequent integral equation*

$$p(\varsigma) = p_0 - h(0,p_0) + h(\varsigma,p(\varsigma)) + A_\vartheta f(\varsigma,p(\varsigma)) + B_\vartheta \int_0^\varsigma f(s,p(s))ds, \quad \varsigma \in [0,\xi]. \tag{2.6}$$

Note 2.1. *In order to prove the above Lemma, first we need to recall the Laplace transform of CFO. From [13], we have*

$$L\{{}^{CF}D^\vartheta p(\varsigma)\}(s) = \frac{(2-\vartheta)N(\vartheta)}{2(s+\vartheta(1-s))}[sL\{p(\varsigma)\}(s) - p(0)]$$
$$L\{{}^{CF}D^\vartheta h(\varsigma,p(\varsigma))\}(s) = \frac{(2-\vartheta)N(\vartheta)}{2(s+\vartheta(1-s))}[sL\{h(\varsigma,p(\varsigma))\}(s) - h(0,p(0))].$$

Proof. We use the Laplace transform on both the left and right sides of (1.1):

$$\begin{aligned}
&L\{{}^{CF}D^\vartheta[p(\varsigma) - h(\varsigma,p(\varsigma))]\}(s) = L\{f(\varsigma,p(\varsigma))\}(s)\\
&\implies \frac{(2-\vartheta)N(\vartheta)}{2(s+\vartheta(1-s))}[sL\{p(\varsigma)\}(s) - p(0)]\\
&\quad - \frac{(2-\vartheta)N(\vartheta)}{2(s+\vartheta(1-s))}[sL\{h(\varsigma,p(\varsigma))\}(s) - h(0,p(0))]\\
&= L\{f(\varsigma,p(\varsigma))\}(s)\\
&\implies L\{p(\varsigma)\}(s) - L\{h(\varsigma,p(\varsigma))\}(s) = \frac{1}{s}p(0) - \frac{1}{s}h(0,p(0))\\
&\quad + \frac{2\vartheta}{s(2-\vartheta)N(\vartheta)}L\{f(\varsigma,p(\varsigma))\}(s)\\
&\quad + \frac{2(1-\vartheta)}{(2-\vartheta)N(\vartheta)}L\{f(\varsigma,p(\varsigma))\}(s).
\end{aligned} \tag{2.7}$$

Applying inverse Laplace transform on both sides of (2.7), we get

$$p(\varsigma) = p_0 - h(0,p_0) + h(\varsigma,p(\varsigma)) + A_\vartheta f(\varsigma,p(\varsigma)) + B_\vartheta \int_0^\varsigma f(s,p(s))\,ds.$$

Now, we are going to show that the solution (2.6) satisfies the given system (1.1). For this, we rewrite the solution (2.6) as follows.

$$\begin{aligned} p(\varsigma) - h(\varsigma,p(\varsigma)) = p_0 - h(0,p_0) &+ \frac{2(1-\vartheta)}{(2-\vartheta)N(\vartheta)} f(\varsigma,p(\varsigma)) \\ &+ \frac{2\vartheta}{(2-\vartheta)N(\vartheta)} \int_0^\varsigma f(s,p(s))\,ds. \end{aligned}$$

Then

$$p'(\varsigma) - h'(\varsigma,p(\varsigma)) = \frac{2(1-\vartheta)}{(2-\vartheta)N(\vartheta)} f'(\varsigma,p(\varsigma)) + \frac{2\vartheta}{(2-\vartheta)N(\vartheta)} f(\varsigma,p(\varsigma)),$$

if $\quad f(0,p_0) = 0.$

Multiply by $\dfrac{(2-\vartheta)N(\vartheta)}{2(1-\vartheta)}$ and integrate from 0 to ς, we have

$$\begin{aligned} &\frac{(2-\vartheta)N(\vartheta)}{2(1-\vartheta)} \int_0^\varsigma p'(s)ds - \frac{(2-\vartheta)N(\vartheta)}{2(1-\vartheta)} \int_0^\varsigma h'(s,p(s))ds = \int_0^\varsigma f'(s,p(s))ds \\ &+ \int_0^\varsigma \frac{\vartheta}{1-\vartheta} f(s,p(s))ds. \end{aligned}$$

Multiply the integrand by $e^{-\frac{\vartheta(\varsigma-s)}{1-\vartheta}}$ in the above equation, we have

$$\begin{aligned} &\frac{(2-\vartheta)N(\vartheta)}{2(1-\vartheta)} \int_0^\varsigma p'(s)e^{-\frac{\vartheta(\varsigma-s)}{1-\vartheta}}ds - \frac{(2-\vartheta)N(\vartheta)}{2(1-\vartheta)} \int_0^\varsigma h'(s,p(s))e^{-\frac{\vartheta(\varsigma-s)}{1-\vartheta}}ds \\ &= \int_0^\varsigma f'(s,p(s))e^{-\frac{\vartheta(\varsigma-s)}{1-\vartheta}}ds + \int_0^\varsigma \frac{\vartheta}{1-\vartheta} f(s,p(s))e^{-\frac{\vartheta(\varsigma-s)}{1-\vartheta}}ds \\ &= \int_0^\varsigma \frac{d}{ds}\left[f(s,p(s))e^{-\frac{\vartheta(\varsigma-s)}{1-\vartheta}}\right]ds. \end{aligned}$$

By thinking definition of CFO and if $f(0,p_0) = 0$, then the above equation becomes

$${}^{CF}D^\vartheta[p(\varsigma) - h(\varsigma,p(\varsigma))] = f(\varsigma,p(\varsigma)), \quad \varsigma \in [0,\xi], \quad 0 < \vartheta < 1.$$

□

Remark 2.2. *(i) The above Lemma 2.2 is true only when $f(0,p_0) = 0$.*

(ii) *If* $f(0,p_0)\neq 0$, *then*

$$p(\varsigma)=p_0-h(0,p_0)+h(\varsigma,p(\varsigma))-A_\vartheta f(0,p_0)+A_\vartheta f(\varsigma,p(\varsigma)) \\ +B_\vartheta\int_0^\varsigma f(s,p(s))\,ds$$

is the solution of the following system

$$^{CF}D^\vartheta[p(\varsigma)-h(\varsigma,p(\varsigma))]=f(\varsigma,p(\varsigma))-f(0,p_0)e^{-\frac{\vartheta}{1-\vartheta}\varsigma}, \\ p(0)=p_0,$$

where $\varsigma\in[0,\xi],\quad 0<\vartheta<1.$

Definition 2.4. *A function* $p\in\mathscr{C}([0,\xi],\mathbb{X})$ *is said to be a solution of* (1.1)-(1.2) *if it fulfills* $p(0)=p_0, {}^{CF}D_\varsigma^\vartheta[p(\varsigma)-h(\varsigma,p(\varsigma))]=f(\varsigma,p(\varsigma))$ *with* $f(0,p(0))=0$.

Definition 2.5. *A function* $p\in\mathscr{C}([0,\xi],\mathbb{X})$ *is said to be a solution of* (1.3) *with condition* (1.2) *if it fulfills* $p(0)=p_0, {}^{CF}D_\varsigma^\vartheta[p(\varsigma)-h(\varsigma,p(\varsigma))]=f\left(\varsigma,p(\varsigma),\int_0^\varsigma h_1(\varsigma,s)p(s)ds\right)$ *with* $f(0,p(0),0)=0$.

Lemma 2.3. *A function* $p\in\mathscr{C}([0,\xi],\mathbb{X})$ *is a solution of the system* (1.3) *with the condition* (1.2) *iff p fulfills the subsequent integral equation*

$$p(\varsigma)=p_0-h(0,p_0)+h(\varsigma,p(\varsigma))+A_\vartheta f\left(\varsigma,p(\varsigma),\int_0^\varsigma h_1(\varsigma,s)p(s)ds\right) \\ +B_\vartheta\int_0^\varsigma f\left(s,p(s),\int_0^s h_1(s,\tau)p(\tau)d\tau\right)ds,\quad \varsigma\in[0,\xi].$$

Define the mapping $\overline{\Upsilon}:\mathscr{C}([0,\xi],\mathbb{X})\to\mathscr{C}([0,\xi],\mathbb{X})$ by

$$(\overline{\Upsilon}p)(\varsigma)=p_0-h(0,p_0)+h(\varsigma,p(\varsigma))+A_\vartheta f\left(\varsigma,p(\varsigma),\int_0^\varsigma h_1(\varsigma,s)p(s)ds\right) \\ +B_\vartheta\int_0^\varsigma f\left(s,p(s),\int_0^s h_1(s,\tau)p(\tau)d\tau\right)ds,\quad \varsigma\in[0,\xi]. \tag{2.8}$$

3 Existence Results

In this part, we will give and establish the existence findings for the system (1.1)-(1.3) under the Banach contraction principle, and Krasnoselskii fixed point theorems.

In order to apply above fixed point theorems, we need to list the subsequent conditions:

(A1) (i) The function $f : [0,\xi] \times \mathbb{X} \to \mathbb{X}$ is continuous and we can find a positive constant $\mathcal{M}_f$ in a way that

$$\|f(\varsigma,u) - f(\varsigma,\bar{u})\| \leq \mathcal{M}_f \|u - \bar{u}\|, \quad \text{for each} \quad \varsigma \in [0,\xi],\, u,\bar{u} \in \mathbb{X}.$$

(ii) There exist positive constants $\widehat{\mathcal{M}_f}, \widetilde{\mathcal{M}_f}$ in ways that

$$\|f(\varsigma,p)\| \leq \widehat{\mathcal{M}_f} + \widetilde{\mathcal{M}_f}\|p\|, \quad \varsigma \in [0,\xi],\, p \in \mathbb{X}.$$

$(A1^*)$ (i) The function $f : [0,\xi] \times \mathbb{X} \times \mathbb{X} \to \mathbb{X}$ is continuous and we can find a positive constant $\mathcal{M}_f$ in a way that

$$\|f(\varsigma,u,v) - f(\varsigma,\bar{u},\bar{v})\| \leq \mathcal{M}_f[\|u-\bar{u}\| + \|v-\bar{v}\|],$$

for each $\varsigma \in [0,\xi],\ u,\bar{u},v,\bar{v} \in \mathbb{X}$.

(ii) There exist positive constants $\mathcal{M}_f, \widetilde{\mathcal{M}_f} > 0$ in ways that

$$\|f(\varsigma,u,v)\| \leq \mathcal{M}_f[\|u\| + \|v\|] + \widetilde{\mathcal{M}_f}, \quad \varsigma \in [0,\xi],\ p \in \mathbb{X}.$$

(A2) (i) The function $h : [0,\xi] \times \mathbb{X} \to \mathbb{X}$ is continuously differentiable and we can find a positive constant $\mathcal{M}_h$ in a way that

$$\|h(\varsigma,u) - h(\varsigma,\bar{u})\| \leq \mathcal{M}_h \|u - \bar{u}\|, \quad \text{for each} \quad \varsigma \in [0,\xi],\, u,\bar{u} \in \mathbb{X}.$$

(ii) There exist positive constants $\widehat{\mathcal{M}_h}, \widetilde{\mathcal{M}_h}$ in ways that

$$\|h(\varsigma,p)\| \leq \widehat{\mathcal{M}_h} + \widetilde{\mathcal{M}_h}\|p\|, \quad \varsigma \in [0,\xi],\, p \in \mathbb{X}$$

and $\widehat{\mathcal{M}_h} = \|h(\varsigma,0)\|$.

(A3) $h_1(\varsigma,s)$ is continuous for all $(\varsigma,s) \in [0,\xi] \times [0,\xi]$ and we can find a positive constant H in a way that $\max\limits_{\varsigma,s\in[0,\xi]} \|h_1(\varsigma,s)\| = H$.

Theorem 3.1. *Suppose f,h, and h_1 are satisfy the conditions $(A1^*)(i)$, (A2)(i) and (A3). If*

$$\widetilde{\mu}_1 = [\mathcal{M}_h + \mu^* \mathcal{M}_f[1 + H\xi]] < 1, \tag{3.1}$$

where $\mu^* = (A_\vartheta + B_\vartheta \xi)$, then the system (1.3) *with the condition* (1.2) *has a unique solution on* $[0,\xi]$.

Proof. Now, we show that $\overline{\Upsilon}B_Q \subset B_Q$, where $\overline{\Upsilon} : \mathscr{C}([0,\xi],\mathbb{X}) \to \mathscr{C}([0,\xi],\mathbb{X})$ is described by (2.8). To do this, let $f(\cdot,0,0) = 0$, and let $B_Q = B(0,Q) = \{p \in \mathscr{C}([0,\xi],\mathbb{X}) : \|p\|_{\mathscr{C}} \leq Q\}$ with radius $Q \geq \dfrac{\|\Omega_1\|}{1-\widetilde{\mu}_1}$, where $\widetilde{\mu}_1 = \mathcal{M}_h + \mu^* \mathcal{M}_f[1+H\xi]$ and $\mu^* = (A_\vartheta + B_\vartheta \xi)$.

By thinking of $(A1)^*(i), (A2)(i)$ and $(A3)$, for each $\varsigma \in [0,\xi]$ and $p \in B_Q$, we sustain

$$
\begin{aligned}
&\|(\overline{\Upsilon}p)(\varsigma)\| \\
&= \left\| p_0 - h(0,p_0) + h(\varsigma, p(\varsigma)) + A_\vartheta f\left(\varsigma, p(\varsigma), \int_0^\varsigma h_1(\varsigma,s)p(s)ds\right)\right. \\
&\quad \left. + B_\vartheta \int_0^\varsigma f\left(s, p(s), \int_0^s h_1(s,\tau)p(\tau)d\tau\right) ds \right\| \\
&\leq \|p_0\| + \|h(0,p_0)\| + \|h(\varsigma,p(\varsigma)) - h(\varsigma,0)\| + \|h(\varsigma,0)\| \\
&\quad + A_\vartheta \left[\left\| f\left(\varsigma, p(\varsigma), \int_0^\varsigma h_1(\varsigma,s)p(s)ds\right) - f(\varsigma,0,0)\right\| + \|f(\varsigma,0,0)\| \right] \\
&\quad + B_\vartheta \int_0^\varsigma \left[\left\| f\left(s, p(s), \int_0^s h_1(s,\tau)p(\tau)d\tau\right) - f(s,0,0)\right\| + \|f(s,0,0)\| \right] ds \\
&\leq \|\Omega_1\| + \mathcal{M}_h Q + A_\vartheta \mathcal{M}_f[1+H\xi]Q + B_\vartheta \xi \mathcal{M}_f[1+H\xi]Q \\
&\leq \|\Omega_1\| + [\mathcal{M}_h + (A_\vartheta + B_\vartheta\xi)\,\mathcal{M}_f[1+H\xi]]\,Q \\
&\leq Q,
\end{aligned}
$$

where $\|\Omega_1\| = \|p_0\| + \|h(0,p_0)\| + \widehat{\mathcal{M}_h}$.

Thus, for $\varsigma \in [0,\xi]$, and $p \in \mathscr{C}$, we have

$$\|\overline{\Upsilon}(p)\|_{\mathscr{C}} \leq \|\Omega_1\| + (\mathcal{M}_h + \mu^* \mathcal{M}_f[1+H\xi])Q \leq Q.$$

This demonstrates that the operator $\overline{\Upsilon}$ causes the ball B_Q to be transformed into itself. Next, for $p, \overline{p} \in \mathscr{P}\mathscr{C}$ and $\varsigma \in [0,\xi]$, we sustain

$$
\begin{aligned}
&\|(\overline{\Upsilon}p)(\varsigma) - (\overline{\Upsilon}\overline{p})(\varsigma)\| \\
&\leq \|h(\varsigma,p(\varsigma)) - h(\varsigma,\overline{p}(\varsigma))\| \\
&\quad + A_\vartheta \left[\left\| f\left(\varsigma, p(\varsigma), \int_0^\varsigma h_1(\varsigma,s)p(s)ds\right) - f\left(\varsigma, \overline{p}(\varsigma), \int_0^\varsigma h_1(\varsigma,s)\overline{p}(s)ds\right)\right\| \right] \\
&\quad + B_\vartheta \int_0^\varsigma \left[\left\| f\left(s, p(s), \int_0^s h_1(s,\tau)p(\tau)d\tau\right) - f\left(s, \overline{p}(s), \int_0^s h_1(s,\tau)\overline{p}(\tau)d\tau\right)\right\| \right] ds \\
&\leq \mathcal{M}_h\|p-\overline{p}\|_{\mathscr{C}} + A_\vartheta \mathcal{M}_f(1+H\xi)\|p-\overline{p}\|_{\mathscr{C}} + B_\vartheta\xi\mathcal{M}_f(1+H\xi)\|p-\overline{p}\|_{\mathscr{C}} \\
&\leq [\mathcal{M}_h + (A_\vartheta + B_\vartheta\xi)\,\mathcal{M}_f(1+H\xi)]\,\|p-\overline{p}\|_{\mathscr{C}}.
\end{aligned}
$$

Thus, for all $\varsigma \in [0,\xi]$, we obtain

$$\|(\overline{\Upsilon}p)-(\overline{\Upsilon}\overline{p})\|_{\mathscr{C}} \leq [\mathcal{M}_h+\mu^*\mathcal{M}_f(1+H\xi)]\,\|p-\overline{p}\|_{\mathscr{C}}.$$

Based on (3.1) and the Banach fixed point theorem [17], we conclude that $\overline{\Upsilon}$ contains a unique fixed point $p \in \mathscr{C}$ that is a solution of the equation (1.3) with the condition (1.2) on $[0,\xi]$.

□

Theorem 3.2. *Suppose f and h satisfy the conditions (A1)(i) and (A2)(i). If*

$$\widetilde{\mu}_1 = [\mathcal{M}_h+\mu^*\mathcal{M}_f] < 1, \tag{3.2}$$

where $\mu^* = (A_\vartheta + B_\vartheta \xi)$, then the system (1.1)-(1.2) has a unique solution on $[0,\xi]$.

Proof. The proof of this theorem is similar to the proof of Theorem 3.1, hence, we omit it here. □

Finally, using Krasnoselskii fixed point theorem (KFPT) [17], we demonstrate the existence results to the equation (1.3) with the condition (1.2).

Theorem 3.3. *Suppose that the conditions $(A1^*)$, $(\widetilde{A2})$ and $(A3)$ hold with $[\mathcal{M}_h+A_\vartheta\mathcal{M}_f(1+H\xi)] < 1$ and $1-\widetilde{\mu}_2 > 0$, where $\widetilde{\mu}_2 = \widetilde{\mathcal{M}_h}+\mu^*\mathcal{M}_f[1+H\xi]$. Then the system (1.3) with the condition (1.2) has at least one solution on $[0,\xi]$.*

Proof. Allow us to define two operators from (2.8) as follows:

$$\begin{aligned}(\overline{\Upsilon}_1 p)(\varsigma) &= p_0 - h(0,p_0) + h(\varsigma,p(\varsigma)) \\ &\quad + A_\vartheta f\left(\varsigma, p(\varsigma), \int_0^\varsigma h_1(\varsigma,s)p(s)ds\right), \quad \varsigma \in [0,\xi]\end{aligned} \tag{3.3}$$

and

$$(\overline{\Upsilon}_2 p)(\varsigma) = B_\vartheta \int_0^\varsigma f\left(s, p(s), \int_0^s h_1(s,\tau)p(\tau)d\tau\right) ds, \quad \varsigma \in [0,\xi]. \tag{3.4}$$

Let $B_Q = \{p \in \mathscr{C}([0,\xi],\mathbb{X}) : \|p\|_{\mathscr{C}} \leq Q\}$ with radius $Q \geq \dfrac{\|\Omega_1\|+\mu^*\widetilde{\mathcal{M}_f}}{1-\widetilde{\mu}_2}$, where $\widetilde{\mu}_2 = \widetilde{\mathcal{M}_h}+\mu^*\mathcal{M}_f(1+H\xi)$ and $\mu^* = (A_\vartheta + B_\vartheta \xi)$.

In view of $(A1^*)(ii), (A2)(ii)$ and $(A3)$, for each $\varsigma \in [0,\xi]$ and $p, p_1 \in B_Q$, we find that

$$\begin{aligned}&\|\overline{\Upsilon}_1 p(\varsigma)+\overline{\Upsilon}_2 p_1(\varsigma)\| \\ &\leq \left\| p_0 - h(0,p_0) + h(\varsigma,p(\varsigma)) + A_\vartheta f\left(\varsigma, p(\varsigma), \int_0^\varsigma h_1(\varsigma,s)p(s)ds\right)\right.\end{aligned}$$

$$
\begin{aligned}
&+B_\vartheta \int_0^\varsigma f\left(s, p_1(s), \int_0^s h_1(s,\tau)p_1(\tau)d\tau\right) ds \Bigg\| \\
&\le \|p_0\| + \|h(0,p_0)\| + \widehat{\mathcal{M}_h} + \widetilde{\mathcal{M}_h} Q + A_\vartheta \widetilde{\mathcal{M}_f} + A_\vartheta \mathcal{M}_f Q + A_\vartheta \mathcal{M}_f H\xi Q + B_\vartheta \xi \widetilde{\mathcal{M}_f} \\
&\quad + B_\vartheta \mathcal{M}_f \xi Q + B_\vartheta \mathcal{M}_f H \xi^2 Q \\
&\le \|\Omega_1\| + (A_\vartheta + B_\vartheta \xi)\,\widetilde{\mathcal{M}_f} + \left[\widetilde{\mathcal{M}_h} + (A_\vartheta + B_\vartheta\xi)\,\mathcal{M}_f[1+H\xi]\right] Q \\
&\le Q,
\end{aligned}
$$

where $\|\Omega_1\| = \|p_0\| + \|h(0,p_0)\| + \widehat{\mathcal{M}_h}$.

Thus, for $\varsigma \in [0,\xi]$, and $p \in B_Q$, we have

$$\|\overline{\Upsilon}_1(p) + \overline{\Upsilon}_2(p_1)\|_{\mathscr{C}} \le \|\Omega_1\| + \mu^* \widetilde{\mathcal{M}_f} + \widetilde{\mu}_2 Q \le Q.$$

Thus $\overline{\Upsilon}_1(p) + \overline{\Upsilon}_2(p_1) \in B_Q$. Next, we prove that $\overline{\Upsilon}_1$ is contraction. Since f and h_1 are continuous, so is $\overline{\Upsilon}_1$, and letting $p, \overline{p} \in B_Q$, from (3.3), $(A1^*)(i), (A2)(i)$ and $(A3)$, for each $\varsigma \in [0,\xi]$, we have

$$
\begin{aligned}
&\|(\overline{\Upsilon}_1 p)(\varsigma) - (\overline{\Upsilon}_1 \overline{p})(\varsigma)\| \\
&\le \|h(\varsigma, p(\varsigma)) - h(\varsigma, \overline{p}(\varsigma))\| \\
&\quad + A_\vartheta \left[\left\| f\left(\varsigma, p(\varsigma), \int_0^\varsigma h_1(\varsigma,s)p(s)ds\right) - f\left(\varsigma, \overline{p}(\varsigma), \int_0^\varsigma h_1(\varsigma,s)\overline{p}(s)ds\right)\right\|\right] \\
&\le [\mathcal{M}_h + A_\vartheta \mathcal{M}_f (1+H\xi)] \|p-\overline{p}\|_{\mathscr{C}}.
\end{aligned}
$$

Thus, for all $\varsigma \in [0,\xi]$, we obtain

$$\|(\overline{\Upsilon}_1 p) - (\overline{\Upsilon}_1 \overline{p})\|_{\mathscr{C}} \le [\mathcal{M}_h + A_\vartheta \mathcal{M}_f (1+H\xi)]\,\|p-\overline{p}\|_{\mathscr{C}}.$$

The continuous nature of the operator $\overline{\Upsilon}_2$ is deduced from the fact that the function f and h_1 are continuous. Also $\overline{\Upsilon}_2$ is uniformly bounded on B_Q as

$$
\begin{aligned}
\|(\overline{\Upsilon}_2 p)(\varsigma)\| &\le \left\| B_\vartheta \int_0^\varsigma f\left(s, p(s), \int_0^s h_1(s,\tau)p(\tau)d\tau\right) ds \right\| \\
&\le B_\vartheta \xi (\widetilde{\mathcal{M}_f} + \mathcal{M}_f[1+H\xi]Q) = A,
\end{aligned}
$$

which implies that $\|\overline{\Upsilon}_2 p\| \le A$. Thus $\overline{\Upsilon}_2$ is uniformly bounded. To prove that the operator $\overline{\Upsilon}_2$ is compact, it remains to show that $\overline{\Upsilon}_2$ is equi-continuous. Now, for any $\tau_1, \tau_2 \in [0,\xi]$ with $\tau_1 < \tau_2$ and $p \in B_Q$, we find that

$$
\begin{aligned}
&\|(\overline{\Upsilon}_2 p)(\tau_2) - (\overline{\Upsilon}_2 p)(\tau_1)\| \\
&\le \Bigg\| B_\vartheta \int_0^{\tau_2} f\left(s, p(s), \int_0^s h_1(s,\tau)p(\tau)d\tau\right) ds
\end{aligned}
$$

$$
\begin{aligned}
& -B_\vartheta \int_0^{\tau_1} f\left(s, p(s), \int_0^s h_1(s,\tau)p(\tau)d\tau\right) ds \Bigg\| \\
\leq & \Bigg\| B_\vartheta \int_0^{\tau_2} f\left(s, p(s), \int_0^s h_1(s,\tau)p(\tau)d\tau\right) ds \\
& + B_\vartheta \int_{\tau_1}^{0} f\left(s, p(s), \int_0^s h_1(s,\tau)p(\tau)d\tau\right) ds \Bigg\| \\
\leq & B_\vartheta \int_{\tau_1}^{\tau_2} \|f\left(s, p(s), \int_0^s h_1(s,\tau)p(\tau)d\tau\right) \|ds \\
\leq & B_\vartheta(\widetilde{\mathcal{M}_f} + \mathcal{M}_f[1+H\xi]Q)(\tau_2-\tau_1).
\end{aligned}
\tag{3.5}
$$

From (3.5), we see that if $\tau_2 \to \tau_1$, then the right-hand side of (3.5) goes to zero, so $\|(\overline{\Upsilon}_2 p)(\tau_2) - (\overline{\Upsilon}_2 p)(\tau_1)\| \to 0$ as $\tau_1 \to \tau_2$. Thus, $\overline{\Upsilon}_2$ is equi-continuous. Due to the fact that $\overline{\Upsilon}_2(\mathbb{X}) \subset \mathbb{X}$ as well, $\overline{\Upsilon}_2$ is considered to be compact, and the Arzela-Ascoli theorem states that $\overline{\Upsilon}$ possesses at least one fixed point. Therefore, there is at least one solution to the problem posed by the related system.

□

Theorem 3.4. *Suppose that the conditions* $(A1)$ *and* $(A2)$ *hold with* $[\mathcal{M}_h + A_\vartheta \mathcal{M}_f] < 1$ *and* $1 - \widetilde{\mu}_2 > 0$*, where* $\widetilde{\mu}_2 = \widetilde{\mathcal{M}_h} + \mu^* \widetilde{\mathcal{M}_f}$*. Then the system* (1.1)-(1.2) *has at least one solution on* $[0, \xi]$.

Proof. The proof of this Theorem is similar to Theorem 3.3, hence, we omit it here. □

4 Applications

Example 4.1.

Consider the subsequent impulsive neutral system through CFO of the form

$$
{}^{CF}D^{\frac{1}{4}}\left[p(\varsigma) - \frac{e^{-\varsigma}}{9+e^{\varsigma}} \cdot \frac{p(\varsigma)}{1+p(\varsigma)}\right] = \frac{e^{-\varsigma}\|p(\varsigma)\|}{(16+e^{\varsigma})(1+\|p(\varsigma)\|)}, \quad \varsigma \in [0,1], \tag{4.1}
$$

$$
p(0) = 0. \tag{4.2}
$$

Set $\vartheta = \frac{1}{4}, \xi = 1, A_\vartheta = \frac{3}{4}, B_\vartheta = \frac{1}{4}, N(\vartheta) = \frac{8}{7}$, $h(\varsigma, p) = \frac{e^{-\varsigma}p}{(9+e^{\varsigma})(1+p)}, (\varsigma, p) \times [0,1] \times [0,\infty)$ and $f(\varsigma, p) = \frac{e^{-\varsigma}p}{(16+e^{\varsigma})(1+p)}, (\varsigma, p) \times [0,1] \times [0,\infty)$.

Let $u, \overline{u} \in [0,\infty)$ and $\varsigma \in [0,1]$. Then, we have

$$\begin{aligned} \|f(\varsigma,u) - f(\varsigma,\overline{u})\| &\leq \frac{e^{-\varsigma}}{(16+e^{\varsigma})} \left\| \frac{u}{1+u} - \frac{\overline{u}}{1+\overline{u}} \right\| \\ &\leq \frac{1}{17} \|u - \overline{u}\| \end{aligned}$$

and

$$\begin{aligned} \|h(\varsigma,u) - h(\varsigma,\overline{u})\| &\leq \frac{e^{-\varsigma}}{(9+e^{\varsigma})} \left\| \frac{u}{1+u} - \frac{\overline{u}}{1+\overline{u}} \right\| \\ &\leq \frac{1}{10} \|u - \overline{u}\|. \end{aligned}$$

Thus, assumptions $(A1)(i)(ii), (A2)(i)(ii)$ hold with $\mathcal{M}_f = \widetilde{\mathcal{M}_f} = \frac{1}{17}, \mathcal{M}_h = \widetilde{\mathcal{M}_h} = \frac{1}{10}$, and $\widehat{\mathcal{M}_f} = \widehat{\mathcal{M}_h} = 0$.

Furthermore

$$\begin{aligned} \mathcal{M}_h + \mu^* \mathcal{M}_f &= \mathcal{M}_h + (A_\vartheta + B_\vartheta \xi)\, \mathcal{M}_f \\ &= \frac{1}{10} + \left(\frac{3}{4} + \frac{1}{4}\right) \frac{1}{17} \\ &= 0.159 < 1. \end{aligned}$$

As a result, the criterion (3.2) satisfied when $\mathcal{M}_h + \mu^* \mathcal{M}_f = 0.159 < 1$. Accordingly, the provided impulsive fractional neutral system (4.1)-(4.2), has a unique solution in $[0,1]$ in light of Theorem 3.2.

Moreover

$$\begin{aligned} \mathcal{M}_h + A_\vartheta \mathcal{M}_f &= \frac{1}{10} + \frac{3}{4}\left(\frac{1}{17}\right) \\ &= 0.144 < 1 \end{aligned}$$

and $1 - \widetilde{\mu}_2 = 0.159 > 0$.

From the above, we note that all the assumptions of Theorem 3.4 are also satisfied. Hence, the given system (4.1)-(4.2) has at least one solution in $[0,1]$.

Example 4.2.

Consider the subsequent impulsive neutral system through CFO of the form

$$ {}^{CF}D^{\frac{1}{4}} \left[p(\varsigma) - \frac{e^{-\varsigma}}{9+e^{\varsigma}} \cdot \frac{p(\varsigma)}{1+p(\varsigma)} \right] = \frac{2 + \|p(\varsigma)\| + \left\| \int_0^1 e^{\varsigma - s} p(s)\,ds \right\|}{2e^{\varsigma+1}\left(1 + \|p(\varsigma)\| + \left\| \int_0^1 e^{\varsigma - s} p(s)\,ds \right\|\right)}, \tag{4.3}$$

$$p(0) = 0, \tag{4.4}$$

where $\varsigma \in [0,1]$. Set $\vartheta = \frac{1}{4}, \xi = 1, A_\vartheta = \frac{3}{4}, B_\vartheta = \frac{1}{4}, N(\vartheta) = \frac{8}{7}$, $h(\varsigma, p) = \frac{e^{-\varsigma} p}{(9+e^{\varsigma})(1+p)}, (\varsigma, p) \times [0,1] \times [0,\infty)$ and $f(\varsigma, u, v) = \frac{2+\|u\|+\|v\|}{2e^{\varsigma+1}(1+\|u\|+\|v\|)}, (\varsigma, u, v) \in [0,1] \times [0,\infty) \times [0,\infty)$.

Let $u, v, \overline{u}, \overline{v} \in [0,\infty)$ and $\varsigma \in [0,1]$. Then, we have

$$\|f(\varsigma, u, v) - f(\varsigma, \overline{u}, \overline{v})\| \leq \frac{1}{2e^2}[\|u-\overline{u}\| + \|v-\overline{v}\|]$$

and

$$\|f(\varsigma, u, v)\| \leq \frac{1}{2e^2}[2+\|u\|+\|v\|].$$

Thus, assumptions $(A1^*)(i)(ii)$ and $(A3)$ hold with $\mathcal{M}_f = \frac{1}{2e^2}$, $\widetilde{\mathcal{M}_f} = 0$ and $H = e$ respectively. Assumptions $(A2)(i)(ii)$ hold in light of Example 4.1.

Furthermore

$$\begin{aligned} \widetilde{\mu}_1 = \mathcal{M}_h + \mu^* \mathcal{M}_f[1+H\xi] &= \frac{1}{10} + 1\left(\frac{1}{2e^2}\right)(1+e) \\ &= 0.3516 < 1 \end{aligned}$$

and

$$\begin{aligned} \mathcal{M}_h + A_\vartheta \mathcal{M}_f(1+H\xi) &= \frac{1}{10} + \frac{3}{4} \cdot 1 \left(\frac{1}{2e^2}\right)(1+e) \\ &= 0.2886 < 1. \end{aligned}$$

From the above, we note that all the assumptions of Theorem 3.1 and 3.3 are fulfilled. Therefore, the given system (4.3)-(4.4) has a unique and at least one solution in [0,1] respectively.

5 Conclusion

We have provided a definition for the existence theory of solutions to fractional order exponential kernel-type differential equations. During the process of formulating the aforementioned theory, we relied entirely on the well-established theorems on fixed points that Banach and Krasnoselskii had derived. Lemma 2.2 presents and proves the solution to our addressing system, which is a new development and may be thought of as a novelty. In Theorem 3.2, we analyse the existence of the addressing model (1.1)-(1.2) by means of a contractive map. Theorem 3.4 is designed to investigate the existence results of the system (1.1)-(1.2) under condensing map conditions. The addressing model (1.3) with the criteria (1.2) is examined for existence and uniqueness in Theorem 3.1 using a contractive map. Under condensing map conditions, Theorem 3.3 is employed to explore the existence outcomes of the considered system (1.3) with the condition (1.2). There are fascinating instances offered to support the facts that have been obtained. These conclusions are completely original when considered in the context of neutral differential and integro-differential equations that include CFO. The usefulness of the present research might be enhanced in the not-too-distant future by employing an appropriate fixed point theorem to approximate controllability with instantaneous and non-instantaneous impulses for a number of different models.

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