Ann. Acad. Rom. Sci.
Ser. Math. Appl.
ISSN 2066-6594 Vol. 15, No. 1-2/2023

ON THE STABILITY AND MEAN SQUARE STABILIZATION OF A CLASS OF LINEAR STOCHASTIC SYSTEMS CONTROLLED BY IMPULSES*

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DOI https://doi.org/10.56082/annalsarscimath.2023.1-2.45

Dedicated to Dr. Dan Tiba on the occasion of his 70^{th} anniversary

Abstract

This paper is devoted to the analysis of the mean-square stability of a class of linear time-varying impulsive Itô-type stochastic systems. We succeed to obtain necessary and sufficient stability conditions in the general time-varying case. This result is obtained thanks to the theory of positive operators on Hilbert spaces. The problem of state-feedback stabilization is treated as well.

MSC: 93E15, 93E55, 93D15, 47B65

*Accepted for publication on November 21-st, 2022

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keywords: impulsive controlled linear stochastic systems; mean square stability, mean square stabilization.

1 Introduction

In many applications, systems are subject to some abrupt changes at certain time moments and cannot be considered continuously. Such a phenomenon is called the impulsive effect. More formally, impulsive systems are a class of dynamic systems in which the state propagates according to continuous-time dynamics except for a countable set of times at which the state can change instantaneously. These systems are useful in representing a number of real world applications from engineering, environmental to mathematical finance applications [1, 4, 5, 9, 13, 16, 18, 19, 20, 22]. A privileged mathematical tool used in the analysis of such systems is the theory of impulsive differential equations. It has been extensively used in the investigation of dynamical impulsive systems in the past several years [2, 10, 11, 12, 15, 17]. For a recent overview on the subject, one can refer to the review article [21].

In this paper, we first address the problem of mean-square stability of a class of linear time-varying impulsive Itô-type stochastic systems. We succeed to obtain necessary and sufficient stability conditions in the general time-varying case (*i.e.* without any assumption on the time-variation nature of the system). This result is obtained thanks to the theory of positive operators on Hilbert spaces. Indeed, the stability conditions are formulated as a global existence condition for the solution of an adequately defined backward jump Lyapunov differential equation. These very general conditions allow us, when the time variation nature of the system is constrained, to obtain several auxiliary results (viewed as particular cases), namely: the periodic case and the time invariant case. Another auxiliary result covered by the theory developed in our paper is the case of impulsive linear time invariant systems with ranged dwell-time. By specializing the obtained stability conditions to this particular case, one can obtain necessary and sufficient stability conditions for such a class of systems. This has to be directly compared to the result obtained in [3] where only sufficient stability conditions have been proposed.
Finally, by using the obtained stability conditions, we address the problem of mean square stabilization (via state-feedback) of time-varying impulsive Itô-type stochastic systems. We succeed to obtain necessary and sufficient stabilization conditions under the linear matrix inequalities framework.