## PARTIAL STABILITY IN A MODEL FOR ALLERGIC REACTIONS INDUCED BY CHEMOTHERAPY OF ACUTE LYMPHOBLASTIC LEUKEMIA\*

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Dedicated to Dr. Dan Tiba on the occasion of his 70<sup>th</sup> anniversary

## Abstract

A new model that captures the cellular evolution of patients undergoing maintenance therapy for acute lymphoblastic leukemia in connection with allergic reactions is considered. A previous model from is modified to include the cells involved in allergies induced by chemotherapy and desensitization.

Delay differential equations are used to model cell evolution. General properties of solutions are deduced, eventually proving partial stability of certain equilibria with respect to some of the variables. The immune system's functioning, as well as the therapeutic role for cancer cure without interference of allergic reactions caused by this treatment, are also evaluated using numerical simulations.

MSC: 34K20; secondary 34K12, 34K25, 92C37, 92C50.

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## **1** Introduction

Leukemia is a type of blood and bone marrow cancer characterized by an overabundance of abnormal white blood cells.

Acute lymphoblastic leukemia (ALL), also known as acute lymphocytic leukemia, is a type of cancer that arises from the early stages of lymphocyte cells known, as lymphoblasts, in the bone marrow. Leukemic cells normally infiltrate the bloodstream swiftly. They can spread to other organs such the lymph nodes, liver, spleen, etc[7].

Chemotherapy's goal is that, by using drugs, to stop or decrease the growth of cancerous cells. Following the first round of chemotherapy, front line of maintenance therapy is to give the patient 6-MT (mercaptopurine) orally[7, 21].

The more and more use of chemotherapy in recent time enhances hypersensitive responses (HSRs). Drug allergies can be lethal, limiting the use of first-line medicines and threatening patients' survival chance but also the quality of life. The reactions can range from minor cutaneous symptoms like itching and hives to potentially lethal anaphylaxis, which causes hypotension, oxygen deficiency, and cardiovascular collapse [9].

Mercaptopurine is a popular antimetabolite used to treat, besides acute lymphoblastic leukemia, the inflammatory bowel illness[26]. Mercaptopurine's side effects include myelosuppression, hepatotoxicity and hyperpigmentation. The example of a 36-year-old man with Philadelphia chromosome-negative pre-B-cell acute lymphoblastic leukemia who experienced a severe mercaptopurine-induced hypersensitivity reaction that required prolonged hospitalization as well as intensive laboratory tests and imaging is described in the literature[10].

According to [29], administration of oral 6-MP was associated with a 21 % increase in the percentage of CD4+ T cells, restoring the CD4/CD8 ratio. Prior