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TWO-TIME-SCALE REGIME-SWITCHING STOCHASTIC KOLMOGOROV SYSTEMS WITH WIDEBAND NOISES*

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Dedicated to Dr. Vasile Drăgan on the occasion of his 70th anniversary

Abstract

In our recent work, in lieu of using white noise, we examined Kolmogorov systems driven by wideband noise. Such systems naturally arise in statistical physics, biological and ecological systems, and many related fields. One of the motivations of our study is to treat more realistic models than the usually assumed stochastic differential equation models. The rationale is that a Brownian motion is an idealization used in a wide range of models, whereas wideband noise processes are much easier to be realized in the actual applications. This paper further investigates the case that in addition to the wideband noise process, there is a singularly perturbed Markov chain. The added Markov chain is used to model discrete events. Although it is a more realistic formulation, because of the non-Markovian formulation due to the wideband noise and the singularly perturbed Markov chain, the analysis is more

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difficult. Using weak convergence methods, we obtain a limit result. Then we provide several examples for the utility of our findings.
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keywords: Wideband noise, Kolmogorov system, non-Markov model, Markov chain.

1 Introduction

Recently, much effort has been devoted to stochastic Kolmogorov systems because their wide range of applications in statistical physics, ecology, and mathematical biology, among others. For example, there have been resurgent and emerging interests in studying Ginzburg-Landau equations [10] (see also [37]) in statistical physics, Lotka-Volterra models in statistical mechanics of population [21] (see also [9] for such equations with functional responses), the study of ecological models [7, 24], and the work on infectious disease modeling [25] among others. For multi-dimensional Kolmogorov systems, the paper [14] nearly completely classified the "threshold" of coexistence and extinction.

In contrast to the recent development, to the best of our knowledge, most of the stochastic models considered thus far have been concentrated on the "Markovian" formulation. The formulation has been confined to the treatment of Brownian motion and/or jump type noise processes. That is, the resulting systems are Markov processes. Owing to the use of the Markovian formulation, we have good technical machineries to handle the systems. One would naturally ask what if the systems are non-Markovian? Unfortunately, for non-Markovian systems, we lose all the usual analytic tools. There are generally no operators or generators associated to the underlying processes.

In our recent work [38], we examined a class of such non-Markovian systems, from the consideration of approximation and scaling limit. Although the mathematical idealization enables us to take advantage of the Markovian structure, in reality, very often, one does not have true "white" noise, but only has something close to "white" noise. In our paper, we used weak convergence methods to obtain limit systems. The limit systems are driven by a Brownian motion. In this paper, taking the result of [38] as a point of departure, we further examine Kolmogorov systems in which in addition to the wideband noise, the systems are hybrid with a switching component that are subject to both strong and weak interactions. Mathematically, the