

# ROBUST STATIC OUTPUT FEEDBACK STACKELBERG STRATEGY FOR MARKOV JUMP DELAY STOCHASTIC SYSTEMS \*

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DOI <https://doi.org/10.56082/annalsarscimath.2020.1-2.476>

Dedicated to Dr. Vasile Drăgan on the occasion of his 70<sup>th</sup> anniversary

## Abstract

In this study, a robust static output feedback (SOF) Stackelberg strategy for a class of uncertain Markov Jump linear stochastic delay systems (UMJLSDSs) is investigated. After introducing certain preliminaries, a SOF Stackelberg strategy is derived. It is shown that the strategy set is established by solving two constraint optimization problems and cross-coupled stochastic matrix equations that consist of bilinear matrix inequalities (BMIs). In order to obtain the corresponding solutions of the constraint optimization problems and cross coupled stochastic matrix equations (CCSMEs), an algorithm based

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\*Accepted for publication in revised form on July 20, 2020

\*This paper was presented in part in a paper accepted for presentation at 2020 IFAC World Congress [1].

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on the Krasnoselskii iterative algorithm is proposed instead of solving BMI. It is also shown that weak convergence can be achieved using this approach. A practical example is provided to demonstrate the effectiveness and convergence of the proposed algorithm.

**MSC:** 49N90, 70E60, 93B36, 93E20

**keywords:** Stackelberg strategy, uncertain Markov Jump linear stochastic delay systems (UMJLSDSs), BMIs, Krasnoselskii iterative algorithm.

## 1 Introduction

Over the past decade, various control problems, stabilization problems, and dynamic games for Markov jump linear stochastic systems (MJLSSs) have been studied. Researchers have been challenged to solve problems for MJLSSs with different characteristics such as time delays, deterministic uncertainties, external disturbances, and information availability in designing strategies. Control and stabilization problems for MJLSSs with delays have been studied [2, 3, 4, 5, 6]. A Pareto suboptimal control and a Nash equilibrium via state feedback strategy have been derived for Markov jump delay stochastic delay systems (MJDSSs) [7]. Moreover, static output feedback (SOF) control problems for MJLSSs have been investigated [8], and some SOF dynamic games for MJLSSs with external disturbances have also been studied [9].

Over the past twenty years, researchers have addressed challenges related to numerical computation methods to solve various cross coupled stochastic matrix equations (CCSMEs) and cross-coupled stochastic matrix inequalities (CCSMIs), which are well known NP-hard problems [10, 11]. There are often difficulties involved in solving such CCSMEs and CCSMIs when SOF strategies are considered in a problem. In [12, 13], an iterative computational algorithm has been presented to solve high-order cross coupled matrix equations involving SOF strategies. Several theoretical results have been established regarding the existence conditions of the solutions; however, further study is required to develop effective numerical algorithms with an appropriate convergence property. Furthermore, uncertain Markov jump linear stochastic systems (UMJLSSs) are often used to describe various practical systems with deterministic uncertainties, such as the modeling errors in system matrices and stochastic changes in operating points [14]. Although there have been recent advances in the robust SOF Nash and the state feedback Stackelberg strategies for uncertain Markov Jump linear stochas-