TWO-PARAMETER SECOND-ORDER DIFFERENTIAL INCLUSIONS IN HILBERT SPACES*

Gheorghe Moroşanu[†]   Adrian Petruşel[‡]

Dedicated to Dr. Vasile Drăgan on the occasion of his 70th anniversary

Abstract

In a real Hilbert space H, let us consider the boundary-value problem $-\varepsilon u''(t) + \mu u'(t) + Au(t) + Bu(t) \ni f(t)$, $t \in [0,T]$; $u(0) = u_0$, $u'(T) = 0$, where $T > 0$ is a given time instant, ε, μ are positive parameters, $A : D(A) \subset H \to H$ is a (possibly set-valued) maximal monotone operator, and $B : H \to H$ is a Lipschitz operator. In this paper, we investigate the behavior of the solutions to this problem in two cases: **(i)** $\mu > 0$ fixed, $0 < \varepsilon \to 0$, and **(ii)** $\varepsilon > 0$ fixed and $0 < \mu \to 0$. Notice that if $\mu = 1$ and ε is a positive small parameter, the above problem is a Lions-type regularization of the Cauchy problem $u'(t) + Au(t) + Bu(t) \ni f(t)$, $t \in [0,T]$; $u(0) = u_0$, which was recently studied by L. Barbu and G. Moroşanu [Commun. Contemp. Math. 19 (2017)]. Our abstract results are illustrated with examples related to the heat equation and the telegraph differential system.
MSC: 34G25, 47J35, 47H05, 35K20, 35L50

keywords: Lions regularization, approximation, maximal monotone operator, Lipschitz operator, heat equation, telegraph differential system.