

## TWO-PARAMETER SECOND-ORDER DIFFERENTIAL INCLUSIONS IN HILBERT SPACES\*

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DOI <https://doi.org/10.56082/annalsarscimath.2020.1-2.274>

Dedicated to Dr. Vasile Drăgan on the occasion of his 70<sup>th</sup> anniversary

### Abstract

In a real Hilbert space  $H$ , let us consider the boundary-value problem  $-\varepsilon u''(t) + \mu u'(t) + Au(t) + Bu(t) \ni f(t)$ ,  $t \in [0, T]$ ;  $u(0) = u_0$ ,  $u'(T) = 0$ , where  $T > 0$  is a given time instant,  $\varepsilon, \mu$  are positive parameters,  $A : D(A) \subset H \rightarrow H$  is a (possibly set-valued) maximal monotone operator, and  $B : H \rightarrow H$  is a Lipschitz operator. In this paper, we investigate the behavior of the solutions to this problem in two cases: **(i)**  $\mu > 0$  fixed,  $0 < \varepsilon \rightarrow 0$ , and **(ii)**  $\varepsilon > 0$  fixed and  $0 < \mu \rightarrow 0$ . Notice that if  $\mu = 1$  and  $\varepsilon$  is a positive small parameter, the above problem is a Lions-type regularization of the Cauchy problem  $u'(t) + Au(t) + Bu(t) \ni f(t)$ ,  $t \in [0, T]$ ;  $u(0) = u_0$ , which was recently studied by L. Barbu and G. Moroşanu [Commun. Contemp. Math. 19 (2017)]. Our abstract results are illustrated with examples related to the heat equation and the telegraph differential system.

**MSC:** 34G25, 47J35, 47H05, 35K20, 35L50

**keywords:** Lions regularization, approximation, maximal monotone operator, Lipschitz operator, heat equation, telegraph differential system.

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\*Accepted for publication on May 11, 2020

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